Abstract

This paper explores the private equity (PE) and venture capital (VC) markets and demonstrates that unavoidable principal-agent problems result in equilibrium competitive equity prices that are decreasing in the amount of idiosyncratic risk. The structure of information in these markets means that idiosyncratic risk will be priced even if investors can fully diversify and the private capital markets are competitive. VCs are agents who help investors (the principals) find positive NPV projects. To ensure that VCs screen properly, they must receive compensation based on the performance of their recommendations. Significant time is required to determine if a project is NPV positive, which means that VCs will identify only a small number of investments, exposing them to idiosyncratic risk. Furthermore, VC compensation represents a significant fraction of their wealth. Therefore, they demand returns for the risk they hold. As a result, we show that VC investments have positive alphas while investors in VC funds earn zero alphas. In addition, some positive NPV projects with significant idiosyncratic risk will not be financed. Furthermore, projects or funds that have more idiosyncratic risk will earn higher returns. This last result can be used to empirically distinguish our idea from fixed compensation or a lack of competition. The results also extend to other situations in which price-setting agents hold idiosyncratic risk, such as hedge funds and investment decisions inside a firm. We are currently collecting VC fund and project return data to test our model.

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ABSTRACT

This paper explores the private equity (PE) and venture capital (VC) markets and demonstrates that unavoidable principal-agent problems result in equilibrium competitive equity prices that are decreasing in the amount of idiosyncratic risk. The structure of information in these markets means that idiosyncratic risk will be priced even if investors can fully diversify and the private capital markets are competitive. VCs are agents who help investors (the principals) find positive NPV projects. To ensure that VCs screen properly, they must receive compensation based on the performance of their recommendations. Significant time is required to determine if a project is NPV positive, which means that VCs will identify only a small number of investments, exposing them to idiosyncratic risk. Furthermore, VC compensation represents a significant fraction of their wealth. Therefore, they demand returns for the risk they hold. As a result, we show that VC investments have positive alphas while investors in VC funds earn zero alphas. In addition, some positive NPV projects with significant idiosyncratic risk will not be financed. Furthermore, projects or funds that have more idiosyncratic risk will earn higher returns. This last result can be used to empirically distinguish our idea from fixed compensation or a lack of competition. The results also extend to other situations in which price-setting agents hold idiosyncratic risk, such as hedge funds and investment decisions inside a firm. We are currently collecting VC fund and project return data to test our model.
The Price of Diversifiable Risk in VC and Private Equity

Venture capitalists (often called VCs) are known to use high discount rates in assessing potential investments. This may just be a fudge factor that offsets optimistic entrepreneurial projections, but VCs claim to use high discount rates even in internal projections. Furthermore, Cochrane (2001) and Gompers and Lerner (1997) find that VCs are surprisingly successful, and earn large positive alphas. Why? In this paper, we argue that these alphas are not rents but are the equilibrium outcome in a competitive model with principal-agent problems.

The basic setting is that the private equity (PE) and venture capital markets are characterized by entrepreneurs with ideas, and outside investors who are well-diversified, but have little ability to screen potential investments. Investors hire VCs who have considerable expertise in assessing entrepreneurial ideas. Typically VCs have little capital of their own, so they are in essence money managers, helping investors determine which ideas are good investments. Because of standard incentive problems, VCs receive an interest in the firms they fund. They are unable to monetize their holdings and are instead forced to hold a substantial amount of their wealth in the form of these contingent stakes. Furthermore, significant time is required to determine if a project is NPV positive, which means that a VC can identify only a few investments. This means that VCs hold considerable idiosyncratic risk. This idiosyncratic risk must be priced, because no market participant is able to diversify the risk away. As a result, CAPM or multiple factor alphas appear to be positive, but the higher observed returns are required returns, not excess returns, in order to compensate VCs for the total risk they hold.

There could be other reasons for positive alphas on VC investments. For example, the VC and PE markets might be simply uncompetitive. However, if our story is right, then projects with greater idiosyncratic risk should earn higher returns. We also provide a number of other testable implications.

Finally, we are able to perform a calibration exercise to assess the economic importance of idiosyncratic risk using the data in Cochrane (2001) and Gompers and Lerner (1997). We find that if VCs have mean-variance utility and need the same Sharpe ratio as the overall public market, then the deadweight loss from the lack of diversification is on the order of 12% of the total amount invested. This is economically significant considering that Lerner (2000) estimates that private equity funds managed $175 billion in 1999.

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1 For example, the average entrepreneurial idea may have a negative NPV, but the VC eliminates projects in the left tail making the average VC chosen project NPV positive.
2 See Chan (1983) for a formal model of this idea.
The trade-off between risk and incentives is a classic feature of contracts between a principal and an agent. Because the principal does not have perfect information about the agent’s type and/or complete information about the agent’s actions, the principal-agent contract must leave the risk-averse agent with too much risk relative to the first-best solution. The standard problem is that the agent could be a bad type or that the agent must take an action that is costly and unverifiable, such as expending effort or choosing the optimal number of projects. To combat either problem, the principal commits to a contract where the agent’s payoff depends on an observable output. In the screening problem, only good agents are willing to accept the contract. If the agent’s effort is costly and unverifiable, then a contract that depends on output induces greater effort.\textsuperscript{3} If output is also subject to shocks that are beyond the agent’s control, then these contingent contracts impose risk onto the agent.

Much of the relevant work on this aspect of the principal-agent problem has focused on either the optimal contract (for example see Holmstrom and Weiss (1985) or Holmstrom and Milgrom (1987)) or on the attempt to see the resulting trade-offs empirically (see Prendergast (1999) for a recent survey). In this paper, our main contribution is to examine the effect of the principal-agent problem on equilibrium asset prices.

In our model, the venture capitalist is an agent who must be compensated for the opportunity cost of his time. Due to the investor’s (principal’s) lack of information about the type of the VC or the VC’s actions, the VC’s compensation must depend on the returns of his chosen projects. This matches reality, as the standard compensation scheme in PE and VC is a fixed payment (usually 2% of the fund per year) and a fraction of the return above some benchmark (this benchmark is often zero). Since the VC has limited wealth, a significant portion of his wealth is the present value of his portion of the project returns.\textsuperscript{4,5} The VC must invest significant time and effort (including reading business plans, meeting with management/customers/suppliers, studying the potential market, and performing other due diligence) to determine if a project is NPV positive. Therefore, the VC will only be able to find a limited number of investments. Gompers and Lerner (1999) note that funds typically invest in at most two dozen firms over about three years. In addition, a VC’s expertise

\textsuperscript{3}Holmstrom and Ricart-I-Costa (1986) offer the idea that even if contracts do not explicitly depend on output, principals use the outcome of agents’ decisions as a signal of the agents quality. Since principals promote the high quality agents (they cannot commit to provide full insurance), agents hold risk and their incentives are distorted.

\textsuperscript{4}In practice, when VCs have significant wealth, they are typically required to invest a large fraction of it (perhaps 30-70%) in the fund to show that they “believe” in what they are doing. In other words they must invest in the fund either as a costly signal that they are good or to ensure greater effort.

\textsuperscript{5}Furthermore, the VC’s ability to raise future funds depends on the success of his first fund (See Chevalier and Ellison (1995) and Gompers (1996)). Therefore, the VC’s future income stream depends on the success of the fund. This compounds the effect of any idiosyncratic risk held by the VC. This idea is similar to Holmstrom and Weiss (1985) who focus on future career concerns rather than specifically contingent contracts.
may be limited to a particular sector or industry, which means that the VC may remain exposed to sector risk no matter how many projects he selects. Even if a VC can diversify across the entire VC industry, he may not be fully diversified because all VC projects may contain a common idiosyncratic risk component. For these reasons, the VC is exposed to significant idiosyncratic risk.\(^6\)

In evaluating investment opportunities, VCs take their compensation and compensation risk into account by applying discount rates that are higher than beta-based discount rates. Therefore, prices of VC and PE investments are lower than expected, resulting in expected returns that exhibit positive alphas. These alphas, however, do not represent excess returns. Rather they represent required returns both for the services performed by the VC and for the idiosyncratic risk borne by the VC. These positive alphas occur because financing is unavoidably wrapped with compensation.

If this is correct, then anything that increases required VC compensation should be empirically associated with higher average returns.\(^7\) Specifically, all else equal, projects with higher idiosyncratic risk should have higher returns. Furthermore, specialized funds that invest in only a limited area expose the relevant VCs to sector risk, and these should generate higher average returns than an otherwise identical diversified fund.\(^8\) Also, specialized funds that invest in riskier areas should generate higher average returns.

Why not just combine a large number of VC investments into one much larger fund and compensate the VC based on total fund performance? The answer is that this would eliminate the link between a venture capitalist’s compensation and his chosen projects. Therefore, the principal agent problem would result in low effort or incompetent VCs. Said another way, the principal-agent problem remains regardless of how VC investments are aggregated into funds, so removing risk from the VC is not optimal. Furthermore, with aggregation VCs may still hold idiosyncratic risk because even in a larger fund the individual VC’s career would depend on the projects he chose rather than

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\(^6\)Meulbroek (2001) expresses a related idea that managers of internet firms who receive stock cause a deadweight loss because outsiders can diversify. In our model, we assert that the relevant outsider is probably a venture capitalist who cannot diversify either. We agree that there is a deadweight loss relative to first-best, but we show that it manifests itself in venture capital asset prices.

\(^7\)The general issue that VCs must hold idiosyncratic risk is further compounded if VCs can also increase the value of the project. It is not a requirement that the VC’s effort to improve the project be intertwined with an investment. However, given that the VC has already spent the fixed costs to determine if the project is positive NPV they are now the low cost provider of advice. The VC could be paid in cash, but with unverifiable effort he will not work hard enough. Thus, the price of the deal may also be set to pay the VC for his additional time. Therefore, the deal has more priced idiosyncratic risk if the VC is expected to help the firm. Gorman and Sahlman (1989) report that VCs spend more than one hundred hours per year with each firm. This would lead us to expect significant priced idiosyncratic risk.

\(^8\)This assumes that the marginal fund in these areas is specialized. For example, biotech requires considerable specialization, and some funds invest in only biotech projects. However, if biotech projects are also undertaken by diversified VCs, biotech sector risk may not be priced.
the overall portfolio.

Our theory says that prices should be lower in VC and PE even if there is intense competition among VCs for projects. Prices get bid up until the VC is just indifferent, but this price is still below the beta-based price. Since the VC needs to be compensated, gross expected returns on the venture capital investment are higher than the factor risks would suggest.

However, the well-diversified investors who give money to the VCs do not earn excess returns. The contract between the investor and VC awards part of the fund’s return to the VC. Since the investor can easily diversify, competition between investors results in a contract that gives all expected returns above beta to the VC. Competition between the VCs for projects results in prices that give the VC his required compensation. Thus, although the fund earns positive alphas neither the VC nor the investor can earn more than their required amount.

Since VCs correctly use a higher discount rate to evaluate projects, some projects will not get done that are positive NPV based on factor risk alone. This is in line with earlier work, as the principal-agent problem has consistently been shown to distort investment. In papers such as Holmstrom and Ricart-I-Costa(1986), and Harris, Kriebel and Raviv (1982) principal-agent problems lead to underinvestment by the principal. In papers such as Lambert(1986) and Holmstrom and Weiss (1985) the agent’s investment choice is distorted. However, none of these papers explicitly consider prices. In our model the necessary pricing of idiosyncratic risk can result in the decision not to invest. The principal-agent problem leads to a contract that may cause a competitive VC and an entrepreneur to be unable to find a price to fund the project even though the project is NPV positive.

The impact of idiosyncratic risk introduces a different notion of competition. For example, low risk aversion is one dimension of competitive advantage in the VC industry. A VC who is less risk-averse than the marginal VC could earn excess returns. Furthermore, in our model, competition between VCs is not about the ability to find ‘better’ (higher NPV) projects because competition among investors ensures all excess return goes to the entrepreneur. Instead, a VC who could find more positive NPV projects than the marginal VC would hold less idiosyncratic risk. This would leave excess return for either the VC or investors.

This suggests that VCs have may have the incentive to spend too little time with any one investment in the attempt to find more projects. In light of this incentive to over-diversify, our model can explain the seemingly odd standard VC contract. The standard VC contract has an option-like payoff (typically 20% of all positive returns). It would seem that this type of contract would encourage excessive risk taking by the VC. However, the ‘value’ of this option is not strictly increasing in volatility. This is because the ‘value’ stems from the utility of the risk-averse VC. Increasing the volatility of the portfolio increases the VCs expected payoff but may decrease his utility because he is risk-averse. Taking on more projects reduces idiosyncratic risk, but taking
on fewer projects increases the VCs expected return. Thus, an option-like payoff for a risk-averse undiversified VC can provide incentives to take the right number of projects.

The ideas that we present extend both to hedge funds and to investment decisions made inside a firm, although in each case, the level of idiosyncratic risk that must be priced is most likely less. Therefore, the effects of the principal-agent problem should be less pronounced in those settings. We believe that there are many situations in which price-setting agents hold idiosyncratic risk. Thus, idiosyncratic risk should be a component of many different asset prices.

It is probable that other aspects of the VC and PE markets also affect returns, such as illiquidity or a lack of competition. However, we abstract from these issues and focus on the effect of the principal-agent problem on asset prices.

Our paper is organized as follows. Section I develops the model and examines the effect of an equity contract between the investors and VC. Section II calibrates the model to determine the economics significance. Section III considers a multi-dimensional option-like contract. Section IV examines the VC’s incentive to choose more or fewer projects. Section V explores extensions to hedge funds and to inside a firm, while Section VI concludes.

I The Model

The model has three participants: investors, venture capitalists and entrepreneurs. Entrepreneurs have project ideas that require an investment of $I$ dollars and produce random output of $I(1 + R_i)$. The investment has both systematic and idiosyncratic risk and an uncertain return $R_i = \alpha_i + \beta_i R_m + \varepsilon_i$, where $R_m$ is the return on the Market and $\alpha_i + \varepsilon_i$ is the idiosyncratic return. $R_i$ and $R_m$ are jointly normal, with $E[\varepsilon_i] = 0$, $E[\varepsilon_i R_m] = 0$, and $E[\varepsilon_i \varepsilon_j] = 0$, for all project pairs $i$ and $j$.

There is a risk free asset with zero return. However, entrepreneurs have no money. Investors have money, but do not have the skill to determine whether a project is positive or negative NPV. There are many more negative NPV projects than positive ones or, equivalently, the losses from the negative NPV projects are larger than the gains from the positive NPV projects. Therefore, if the investor invests in a randomly chosen project he will lose money in expectation.

The VC can determine if a project is NPV positive, but must exert effort with an opportunity cost of $e_{vc}$ in order to do so (as in Grossman and Hart (1981)). Therefore, the VC must be compensated in order to choose investments for the investors. Furthermore, this compensation must provide the VC with the incentive to provide effort. The standard compensation scheme in PE and VC is a

\[ E[\varepsilon_i \varepsilon_j] = 0 \] is not required, but it simplifies the exposition of the results.

Or equivalently they can eliminate the left tail of the project distributions resulting in an average project that is NPV positive.
fixed payment (usually 2% of the fund each year) and a fraction of the return above some benchmark (this benchmark is often zero). Holmstrom and Milgrom (1987) show that a linear sharing rule is optimal when effort choice and output are continuous, but monitoring by the principal is periodic. The idea is “that optimal rules in a rich environment must work well in a range of circumstances and will therefore not be complicated functions of the outcome” (Holmstrom and Milgrom (1987) p 325. Even if a linear rule is not optimal it may be to costly to write and enforce a more complicated contract. We do not want to focus on the actual effort choice (see Gompers and Lerner (1999), Gibbons and Murphy(1992) and, of course, Holmstrom and Milgrom (1987) for interesting work which focuses on the effort decision). Instead we simply wish to motivate the use of a sharing rule rather than fixed compensation. In our work we will take the form of the contract as given and focus instead on its implications for asset prices.

The use of a rule with a payment which depends on the performance of the fund can also be motivated with a signaling story. For example, the contract can be used to separate the VCs that are able to accurately screen projects from those who cannot. Those VCs that have no skill would be less willing to take a contract that rewards them only if the fund does well. Again, we do not wish to focus on adverse selection, but instead our goal is to motivate the necessity of a compensation scheme that depends on the outcome of the project.

Although the VC's compensation must depend on the output of the project, the VC has limited wealth and therefore limited liability. We will assume that the VC has zero outside wealth and therefore can only receive a positive payoff from the project. This assumption also ensures that the VCs compensation is a significant fraction of the VC’s wealth and therefore its impact cannot be diversified away with outside wealth. In actuality VCs with significant wealth are often required to put between 30-70% of their total net worth in their fund, thereby signaling their competence and ensuring their effort by reducing their ability to diversify.

We assume that the VC invests in \( N \) projects. The idea is that the VC requires significant time and effort to determine if a project is NPV positive. As a result, the VC is constrained in his ability to locate more than \( N \) projects that have a large enough NPV. In Section IV we will examine the VCs incentives over the choice of \( N \). The portfolio of projects has a mean return \( \mu_p = E[\alpha] + \beta E[R_m] \). Because of normality, the return to this portfolio is also normal with return variance \( \sigma^2_p = \beta^2 \sigma^2_m + N^{-1} \sigma^2_e \).

The timing of the model is a two-stage game. In the first stage, the investors and VCs agree on a contract that will govern their relationship. And in the same stage the VC negotiates the fraction of the output that the entrepreneurs will give up to get \( I \) dollars. For simplicity we will initially consider only equity contracts. Thus, the negotiations are over the fraction, \( \theta \), of the company \( I \)
dollars will purchase, and \( \phi \), the fraction of the firm given to the VCs.\(^{11}\) The investments also occur at this stage. Then, in the second stage project value is realized and payoffs are distributed.

I.1 Benchmark: No Principal-Agent Problem.

In order to provide a benchmark to compare to the more interesting results to follow, we will first consider the pricing when the investors can profitably invest directly in the entrepreneurial projects. Thus, for a moment we will remove the VC from the problem, but assume that the project is still positive NPV.

Given this setup, mean-variance preferences plus perfect competition among investors ensures that investors are willing to fund the project as long as \( \alpha_i \geq 0 \). That is, investors are willing to fund positive NPV projects, where discount rates are determined using the CAPM. Perfect competition among well-diversified outside investors implies that the entrepreneur retains all the economic rents from the project. That is, outside investors are willing to fund the project on terms such that their expected return on investment just compensates them for the systematic risk they bear. Thus, we will show that outside investors earn a zero alpha in expectation.

In the absence of a VC, investors fund a project directly. They put up \( I \) dollars and receive a fraction \( \theta_i \) of the firm. The firm’s random payoff is \( (1 + R_i)I \), where, as described earlier, project returns follow the single factor model

\[
R_i = \alpha_i + \beta_i R_m + \varepsilon_i. \tag{1}
\]

Thus the investors receive \( \theta_i(1+R_i)I \). This implies that the beta of the investors’ returns with respect to the market return is equal to \( \theta_i \beta_i \). Given this setup, the entrepreneur minimizes \( \theta_i \) subject to giving the investors a fair return:

\[
\min \theta_i \text{ subject to } \frac{\theta_i(1 + \alpha_i + \beta_i E[R_m])I}{1 + \theta_i \beta_i E[R_m]} = I, \quad 0 \leq \theta_i \leq 1. \tag{2}
\]

The constraint ensures that the expected payoff to the investors discounted at the appropriate CAPM rate generates an NPV of exactly zero.

There is only one \( \theta_i \) that satisfies the constraint, and it is given by:

\[
\theta_i = \frac{(1 + \alpha_i + \beta_i E[R_m])I}{1 + \theta_i \beta_i E[R_m]} = (1 + \alpha_i)^{-1}, \tag{3}
\]

provided that \( \alpha_i \geq 0 \). Since the fraction \( \theta_i \) of the firm is worth \( I \) dollars, the so-called post-money implied value of the whole firm is \( I \theta_i^{-1} \) or simply \( I(1 + \alpha_i) \), which simply reflects the investment plus the expected value added by taking on this positive NPV project. The so-called pre-money value of

\(^{11}\)As already mentioned, it is possible that no \( \theta \) exists that is acceptable to both the VC and entrepreneur. We will examine this more thoroughly in a moment.
the firm is just the post-money value less the amount contributed by investors, or in this case $I\alpha_i$. Thus, all the rents accrue to the entrepreneur.\footnote{A simple example will help clarify the terms post-money and pre-money value. If the entrepreneur convinces the investor that his firm/idea is worth $2$ million and the investor invests $1$ million at that valuation, then the value of the firm pre-money was $2$ million and the post-money value is $3$ million. These terms are used to make it clear that although the investment increases the value of the firm it does not change the price of the stock (there is more money but also more shares). Note that the alpha in this example must be 200\%.}

\subsection*{I.2 The Venture Capitalist’s Impact on Prices.}

Now we address the more interesting case where there is a VC present. As before, the entrepreneur gives up a fraction of the firm $\theta$ to the investors, but now the investor gives the VC a fraction of the firm $\phi$ and retains the fraction $\theta - \phi$.

The VC is risk-averse with exponential utility over terminal wealth $w$:

$$u(w) = -\exp(-Aw),$$

where $A$ is the VC’s coefficient of absolute risk aversion. If terminal wealth is normally distributed, then maximizing expected utility is equivalent to

$$\max \mu_w - \frac{1}{2}A\sigma_w^2,$$

where $\mu_w$ and $\sigma_w^2$ are the mean and variance of wealth. This functional form for utility allows for a closed-form solution but does not drive the results. All that is necessary is a risk-averse VC.

The VC has no other wealth but has outside opportunities that give him a certain payoff $e_{vc}$. Thus, in order to work on screening potential investments, the VC requires compensation that generates at least as much utility as $e_{vc}$, the opportunity cost of his effort.

In offering a contract to the VC, the investors offer a fraction of the firm $\phi$ that maximizes the present value of their payoff subject to the compensation constraint:

$$\max_{\phi} \frac{(\theta - \phi)(1 + \mu_i)I}{1 + (\theta - \phi)\beta E[R_m]}$$

s.t. $\phi I\mu_i - \frac{1}{2}AI^2\phi^2\sigma_p^2 \geq e_{vc}$,

where $\sigma_p^2 = \beta^2\sigma_m^2 + \sigma_x^2$ is the total variance of payoffs for the project being considered, and $\mu_i = \alpha_i + \beta_iE[R_m]$ is the expected return on the project.

The investor wants to minimize the VC’s take subject to this constraint, and since the VC market is competitive, the offered contract will provide a certainty equivalent of exactly $e_{vc}$. The binding constraint is quadratic in $\phi$ and yields the following expression for the fraction of the firm offered to the VC in equilibrium:

$$\phi^* = \frac{\mu_i - (\mu_i^2 - 2Ae_{vc}\sigma_p^2)^{\frac{1}{2}}}{AI\sigma_p^2}$$
Note that the VC’s take depends only on exogenously given values, and in particular it does not depend on the contract struck between the investors and the entrepreneur. This is because we have assumed perfect competition between VCs for the investors. Note also that the model implicitly assumes that the VC has no access to the capital markets. If he could, the VC might want to hedge out his market risk by trading in the market portfolio. Based on our interactions with venture capitalists, we do not believe that such hedging is common. If the VC were able to eliminate all market risk, pricing would still be affected by idiosyncratic risk rather than total project risk.

As in the benchmark case, the entrepreneur minimizes the fraction $\theta$ subject to giving the investor a fair return. The only difference from the benchmark case is that the investors’ share of the firm is $\theta - \phi^*$ because some equity is given to the VC. Formally, the minimization problem is

$$\min \theta \quad \text{s.t.} \quad \frac{(\theta - \phi^*)(1 + \mu_i)I}{(1 + (\theta - \phi^*)\beta E[R_m])} \geq I$$

(8)

As in the benchmark case, $\theta$ is minimized when the constraint binds, which implies:

$$(\theta^* - \phi^*)(1 + \mu_i) = 1 + (\theta^* - \phi^*)\beta E[R_m]$$

(9)

$$(\theta^* - \phi^*)(1 + \mu_i - \beta E[R_m]) = 1$$

$$\theta^* = \phi^* + (1 + \beta E[R_m])^{-1}$$

$$\theta^* = \phi^* + (1 + \alpha)^{-1}$$

Note that $\theta^* - \phi^*$, the fraction of the firm received by the investors net of payments to the VC, is unchanged from the benchmark case. This is because investors are perfectly competitive and well-diversified. $\theta^*$ increases because they must earn enough to compensate the VC for his services.

This simple model provides a number of important insights. In pricing and in capital budgeting, the presence of the VC introduces wedges:

**Theorem 1** Venture capital investments have positive alphas. Venture capital funds have zero alphas.

**Proof.** In return for an investment of $I$, the investors and VC as a group receive $\theta^*$ of the firm. This fraction $\theta^*$ is larger than the fraction they receive in the benchmark case, which had an alpha equal to zero. Thus, venture capital investments have positive alphas. In contrast, venture capital funds provide the initial investment of $I$ in return for a fraction $\theta^* - \phi^*$ of the firm. Because this fraction satisfies the constraint in Equation (8), by definition it gives the investors an NPV of zero, or equivalently an alpha equal to zero. ■

Relative to the benchmark case, it appears that the entrepreneur must give away too much of the firm in return for the investment. Equivalently, it appears that the entrepreneur is getting too low
a price for selling part of his firm, and it appears that the investment has expected returns that are too high. This isn't so. Nobody obtains excess returns in this model. The reason the entrepreneur must give up more of the firm is to pay the VC for his services. This leads directly to the following corollary.

Corollary 1 Even if VCs receive fixed compensation, alphas would still be positive, but not as large as with contingent compensation.

Proof. If the VC received fixed compensation then the constraint in the investor’s problem, Equation (6), would be s.t. \( \phi I \mu_i \geq e_{vc} \). Thus, the resulting equilibrium \( \phi \) would be \( \phi' = \frac{e_{vc}}{\phi I \mu_i} < \frac{\mu_i - (\mu_i^2 - 2Ae_{vc} \sigma_p^2)^{1/2}}{2AI \phi \sigma_p^2} \). Thus, \( \theta' = \phi' + (1 + \alpha)^{-1} \) would be smaller than \( \theta^* = \phi^* + (1 + \alpha)^{-1} \) but still greater than \( (1 + \alpha)^{-1} \). Therefore, alphas would still be positive but smaller than with contingent compensation.

Our theory predicts positive alphas and furthermore, that these alphas should be larger than the alphas resulting from fixed compensation. However, empirically Theorem 1 will not help us determine if our theory is correct.

Theorem 2 Some positive NPV projects cannot get done.

Proof. Fix all the parameters in the benchmark model, and suppose that \( \alpha_i > 0 \), so that the project is positive NPV and would be taken if VCs were not needed, leaving the entrepreneur with \( 1 - \theta \) of the firm. Now consider \( \theta \) as a function of the project’s alpha. Because \( \theta(\alpha_i) \) is a continuous function and \( \theta(0) = 1, \theta(\alpha_i) \) can be made arbitrarily close to unity by choosing \( \alpha_i \) sufficiently small. Now add in the VC, and consider a given VC fraction \( \phi^* \) based on other parameter values. For a given \( \alpha_i \), the entrepreneur must part with a fraction \( \theta^* = \phi^* + \theta(\alpha_i) \). For all \( \alpha_i \) sufficiently small, \( \theta^* < 1 \), which is infeasible, and the investor cannot profitably invest.

When a paper claims that the NPV rule is no longer valid, it is important to ask which NPV rule and whether a suitably adjusted NPV calculation might restore order. In this context, it is useful to think of the VC’s share of the firm as consisting of two parts. One part is pure compensation to the VC for his effort, and the other part is compensation to the VC for the idiosyncratic risk that he must hold. The value of the VC’s “salary” could and perhaps should just be taken out of the net cash flows. This would go part of the way toward restoring the NPV rule. Compensation to the VC for risk is not quite the same, however. In principle, this too could simply come out of the net cash flows, and the NPV rule would be completely restored. But a higher hurdle rate that accounts for the VC’s idiosyncratic risk also makes intuitive sense, because more risk borne by the VC is associated with a higher implied hurdle rate. Gross expected returns on the investment really do have to be higher because of the added risk.
Corollary 2  Even if VCs receive fixed compensation, some positive NPV projects could not get done, but with contingent compensation more projects could not get done.

Proof. Theorem 2 showed that for a given $\alpha_i$, the entrepreneur must part with a fraction $\theta^* = \phi^* + \theta(\alpha_i)$. Corollary 1 showed that $\phi^* > \phi'$ and $\theta^* > \theta'$. Therefore, the $\alpha_i$ such that $\theta^* > 1$ is larger than the $\alpha_i$ such that $\theta' > 1$. ■

Since VCs correctly use a higher discount rate to evaluate projects, some projects will not get done that are positive NPV based on factor risk alone. However, it is also the case than any fixed investment cost has the same although reduced affect. Therefore, as with Theorem 1, Theorem 2 will not help us empirically determine if our theory is correct. However, the next Theorem will allow us to determine if idiosyncratic risk is priced.

Theorem 3  All else equal, the price the entrepreneur receives is decreasing in the amount of idiosyncratic risk.

Proof. In equilibrium,

$$\phi^* I \mu_i - \frac{1}{2} AI^2 (\phi^*)^2 \sigma_p^2 = e_{vc}. \quad (10)$$

An increase in idiosyncratic risk $\sigma^2$ causes a one-for-one increase in $\sigma_p^2$, which increases the magnitude of the second term on the LHS. In order to continue to satisfy the constraint, the first term on the LHS must experience an offsetting increase, so $\phi^*$ must increase. This greater fraction given up by the entrepreneur is equivalent to receiving a lower implicit value for the firm. ■

This theorem is the main result of the paper. If unavoidable principal-agent problems make diversification impossible, then idiosyncratic risk must be priced. Therefore, VC returns should be correlated with total risk not just beta risk. Projects with higher risk should have higher returns. Specialized funds, that are less diversified should have higher returns. And, funds in higher risk areas should have higher returns.

The following corollary makes it clear that these results would not occur with fixed compensation.

Corollary 3  If VCs receive fixed compensation, then the price the entrepreneur receives would be independent of the idiosyncratic risk.

Proof. If the VCs receive fixed compensation then in equilibrium, $\phi' I \mu_i = e_{vc}$. Therefore, $\theta'$ would be unaffected by the risk of the project, $\sigma_p^2$. ■

Therefore, Theorem 3 will allow us to distinguish between a positive alpha that is simply the result of an uncompetitive market or simply because VC’s require compensation, and a positive alpha that is due to the pricing of idiosyncratic risk.
I.3 Multiple projects

In practice, a single venture capitalist works on and invests in more than one firm or project at a time. The results in the previous section suggest one reason why: diversification reduces the risk being held by the VC, and reducing that risk is the key to being able to offer higher prices to entrepreneurs. Clearly, from the standpoint of total surplus, a main goal is to reduce the portfolio variance held by the VC. However, as discussed above, the VC technology is time consuming and therefore limits the VC’s ability to locate more than $N$ positive NPV projects, while choosing randomly guarantees negative returns.

The model of the previous section appears to consider only one project, but it is straightforward to apply the model when there are multiple projects. To see this most clearly, suppose there are $N$ identical projects rather than just one, and investors put $I/N$ into each one, so that the total investment remains unchanged at $I$. Investors get a fraction $\theta_i$ of the payoff to each project, and the VC receives a fraction $\phi_i$ of each project.

As before, each project is subject to risk from a single common factor and idiosyncratic risk. Returns, expected returns, and project variances are given by:

\[ R_i = \alpha + \beta R_m + \varepsilon_i, \]  
\[ \mu_i = \alpha + \beta E[R_m], \]  
\[ \sigma_i^2 = \beta^2 \sigma_m^2 + \sigma_{\varepsilon}^2. \]  

All idiosyncratic shocks are mutually independent. For simplicity of exposition, projects are all identically distributed, but this is not at all important to the main results. Here, symmetry implies that the equity share in each project will be the same, so $\theta_i = \theta$ and $\phi_i = \phi$.

For the portfolio of projects, returns, expected returns, and project variances are given by:

\[ R_p = N^{-1} \sum R_i = \bar{R}_i \]  
\[ \mu_p = \alpha + \beta E[R_m] \]  
\[ \sigma_p^2 = N^{-2} \sum \sum Cov[R_i, R_j] \]  
\[ = N^{-2} \sum \sum \beta_i \beta_j \sigma_m^2 + N^{-2} \sum \sigma_{\varepsilon}^2 \]  
\[ = \beta^2 \sigma_m^2 + N^{-1} \sigma_{\varepsilon}^2 \]

Investors still have to pay the VC a certainty-equivalent of $e_{vc}$, so if the VC gets a fraction $\phi$ of each project’s payoff, investors still choose $\phi$ to maximize their NPV. Taking $\theta$ as given:

\[ \max_{\phi} \frac{(\theta - \phi)(1 + \mu_i)I}{1 + (\theta - \phi)\beta E[R_m]} \]  
\[ s.t. \phi I \mu_i - \frac{1}{2} A P^2 \phi^2 \sigma_p^2 \geq e_{vc} \]
The constraint continues to bind, and the relevant root of the resulting quadratic equation is unchanged:

$$\phi^* = \frac{\mu_i - (\mu_i^2 - 2Ae\sigma_p^2)^{1/2}}{AI\sigma_p^2}$$

(18)

The only difference is that the VC bears less idiosyncratic risk now, because he is diversified across \( N \) projects, so he unambiguously requires a lower equity fraction. Nothing else changes. The entrepreneur’s problem is the same as it was in the single project case, and the investors’ fraction \( \theta^* - \phi^* \) is also the same as before.

We can see from this discussion that for a given contract between the VC and investor, the VC has the incentive to invest in as many projects as possible. Section IV examines the countereffects that prevent investment in too many projects.

II Calibration

In the model, it is clear that idiosyncratic risk is priced into venture capital financing. But is this effect economically large? In this section, we explore this question using recent empirical data on returns to venture capital investments. We find that the presence of idiosyncratic risk is quite costly. With realistic parameter values, venture capitalists can easily value their position at less than half of its value to a fully diversified investor.

To conduct the calibration exercise, we use summary statistics from Cochrane (2001), who studies all venture capital investments in the VentureOne database from 1987 through June 2000. After correcting for selection bias, he estimates an arithmetic average annualized return of 57%, with an arithmetic annualized return standard deviation of 119%. These statistics, along with a beta estimated at close to unity, imply a CAPM alpha on the order of 40%.

Alternatively, we could have used data from Gompers and Lerner (1997) who measure returns for a single private equity group from 1972-1997 and find much lower returns, with CAPM and three-factor alphas of 8% per year.\(^{13}\) Using 8% in place of 57% would drastically increase our estimate of the impact of idiosyncratic risk. Thus, we used the higher number to be conservative.

Suppose that a venture capitalist holds all of his wealth in a single project with this representative return distribution. Under our mean-variance assumptions, the VC would value this project using a Sharpe ratio equal to the Sharpe ratio for public equity. Over the sample period, and assuming a riskless rate averaging 5% (the calculations are insensitive to the choice of a riskless rate), the venture capital investment Sharpe ratio of \((57\% - 5\%)/119\% = 0.44\) is about half the Sharpe ratio

\(^{13}\)Note that both studies report gross returns on the amount invested, not the net returns after fees and carried interest paid to the venture capitalist.
on public equity over the same interval.\textsuperscript{14}

Based on these return statistics, this representative VC investment has so much idiosyncratic risk that no investor would take it unless he were well-diversified. The good news is that venture capital fund investors are generally well-diversified. However, the VC is compensated via an equity interest, and he is forced to hold a lot of idiosyncratic risk. So we are still interested in the VC’s private valuation of his holdings compared to the value of that interest in the hands of a well-diversified investor. That difference is a measure of the deadweight loss due to lack of diversification. Continuing the earlier example, in order to get a Sharpe ratio that matches the Sharpe ratio on public equity, the venture capitalist needs twice the excess expected return, or $5\% + 2(57\% - 5\%) = 109\%$.

Now suppose a venture capital investment of $1 million and assume a horizon date of three years. The expected exit value is $1 million\times(1 + 57\%)^3 = \$3.87$ million. The venture capitalist applies a discount rate of 109\% to this expected future cash flow for a present value of $3.87 million\times(1 + 109\%)^{-3} = \$0.42$ million. Thus, the value to the venture capitalist is only 42\% of the value to a diversified investor. This 58\% haircut is substantial, and indicates that these kinds of concentrated risks can sharply reduce value.

In our model, this valuation haircut only applies to the VC’s interest, because other investors are well-diversified. If for simplicity we assume that the VC gets a pure 20\% of the exit value of the fund (rather than the 2\% of assets per year and 20\% of the upside that is the most common payment structure), then the deadweight loss from lack of diversification is $58\% \times 20\% = 12\%$ of the total amount invested. This is economically significant considering that Lerner (2000) estimates that private equity funds managed $175$ billion in 1999.

Investors still earn fair returns for the systematic risk they bear, so they won’t bear this deadweight loss. The entrepreneur bears this cost. Given these numbers, it is thus no surprise that entrepreneurs complain bitterly about the valuations that VCs apply to their businesses. There is a discount due to idiosyncratic risk. There is also a discount applied because the VC provides valuable services in return for stock rather than cash. We do not have the data to characterize the magnitude of this second effect, but the example demonstrates that the discount due to idiosyncratic risk alone can be substantial.

Why does the entrepreneur use venture capital at all? Perhaps he has no alternative if he has no wealth of his own, as we’ve assumed in the model. Even if he were wealthy enough to fund the project himself, he may still choose venture capital, because the same analysis of idiosyncratic risk also applies to the entrepreneur. If the entrepreneur has most of his financial and human capital tied up in this single source of risk, he too should apply the same discounts in arriving at his private

\textsuperscript{14}Cochrane(2000) also considers log utility maximization, in which case the goal is to maximize $E[\ln R]$. Interestingly, he finds that the average log return on venture capital investments is also about half of the average log return on diversified public equity investments over this time period.
valuation. In fact, the entrepreneur should prefer to fund via VC, all else equal, because external financing transfers (most of) the idiosyncratic risk to those best able to bear it.

### III Option-like VC contracts

Previous sections assumed that the contract between the VC and the investors was a straight equity contract. This allowed us to most simply demonstrate the basic results. We now wish to extend that result to the situation where the VC’s payoff is of the form

\[
X_{vc} = f + \max[\phi I(R_p - h), -f].
\]

(19)

This form of compensation is the standard contract in the industry. We will show that this option-like payoff increases the idiosyncratic risk held by VC, which increases his required return. Then we will go on to explore why this may be better than a straight equity contract (even in light of the extra risk held by the VC). This section is not complete. The rest of this section outlines work done to date and the direction we intend to go.

The investors want to minimize the VC’s take subject to the participation constraint, so the VC’s contract will provide a certainty equivalent of exactly \(e_{vc}\). If the project is sufficiently lucrative, the following lemma shows that there exist a continuum of possible contracts that provide the required payoff to the VC.

**Lemma 1** Taking the investors’ share \(\theta\) as given, there exist a continuum of choices for \(f, \phi\) and \(h\) that provide the VC with \(U(x_{\phi,h}) = U(e_{vc})\).

**Proof.** Not complete. ■

Note that the VC bears both systematic and unsystematic risk under any contingent contract that provides \(U(e_{vc})\).

**Lemma 2** Comparative statics for total risk, idiosyncratic risk with respect to combinations of \(f, \phi\) and \(h\) that satisfy the participation constraint \(U(x_{\phi,h}) = U(e_{vc})\).

**Proof.** Not complete. ■

This lemma is important, because the contract that exposes the VC to the least risk subject to solving the incentive problems will provide the highest price to the entrepreneur.

As before, the investors’ problem is to choose \(f, \phi,\) and \(h\):

\[
\max_{\phi, h} NPV \text{ s.t. } U(x_{f,\phi,h}) \geq U(e_{vc}),
\]

(20)

and also subject to a lower bound on the slope of the contract in order to ensure that the VC continues to expend maximal effort. Since risk held by the VC is a deadweight loss, the solution should be at the lowest possible slope consistent with maximal effort.
Perfect competition ensures that the entrepreneur can minimize $\theta$ as long as investors earn a non-negative NPV:

$$\min \theta \text{ s.t. } \alpha_I \geq 0 \quad (21)$$

Since the underlying returns are normal, we should be able to characterize both the chosen contract and payoffs in closed form, using results on utility-based option pricing in Sprenkle (1961) and Rubinstein (1976). We should also be able to show how prices are affected relative to first-best.

**IV The Number of Investments**

Until this section of the paper the number of investments chosen by the VC has been fixed. The justification for this assumption is the VC’s binding time constraint. Since choosing a positive NPV project takes time, the VC can only find a limited number of investments. On the other side, the typical VC contract restricts the amount of money that can be invested in one investment. Therefore, since the size of the fund is given, the VC must invest in a minimum number of projects. However, examining the choice of $N$ on the margin provides an interesting explanation for the odd compensation contract that is standard in the industry.

The standard VC contract pays the VC a fraction of the positive returns on the portfolio. Thus, the VC’s payoff is like an option. It might seem that this type of contract would encourage excessive risk taking by the VC. Such incentives could be removed by simply giving the VC an equity contract. Thus, why does the standard contract contain an option?

Our theory suggests that an equity contract actually gives the VC the incentive to take on too many projects. Looking at Section I.3 we see that if $\phi^*$ is set, and the VC takes on one more project than expected, then the VC’s utility would improve even if the extra project had negative NPV since the variance of the portfolio would go down. Thus, moving to an option-like contract reduces the incentive to take on too many projects by providing a benefit to volatility. The option payoff does not cause the VC to invest in too few projects because the ‘value’ of the VC’s option is not strictly increasing in volatility.\(^\text{15}\) The previous section showed that the ‘value’ stems from the utility of the risk-averse VC. Increasing the volatility of the portfolio increases the VC’s expected payoff but may decrease his utility because the VC is risk-averse.

Therefore, if idiosyncratic risk is of first order importance in the VC industry then an option-like contract is not odd at all. Rather than encourage too much risk-taking an option payoff discourages too little!

\(^\text{15}\)Decreasing the number of investments increases the volatility and therefore increases the expected return of the option. However, the value of the option is based on the utility of the VC, since neither the option nor the underlying is traded. Total utility may move up or down.
V   Extensions

The ideas that we present extend both to hedge funds and to inside a firm, although in each case, the level of idiosyncratic risk that must be priced is most likely less. Therefore, the effects of the principal-agent problem should be less pronounced.

V.1   Hedge Funds

The principal-agent problem between an investor and the manager of a hedge fund is similar to that of a PE or VC fund. The main difference is that a hedge fund typically invests in relatively liquid capital markets. But the idea is similar. In evaluating trades, the hedge fund manager may use a higher cost of capital to reflect his fees and also the risk that he bears. If certain instruments or trades require considerable expertise, hedge funds could be the marginal investor, even in liquid capital markets. For example, many convertible bonds issued over the past few years, particularly those of investment-grade issuers, have been placed almost exclusively with hedge funds that try to arbitrage the embedded option with a dynamic hedge. It seems likely that prices on these convertibles are lower as a direct result of the hedge fund principal-agent problem. Hedge funds remain the marginal investor, however. Outsiders do not buy the converts, because they do not have the trading or analytical expertise needed, say, to value or optimally exercise their American embedded options.

V.2   Inside a Firm

In the case of the firm we can think of the stockholders as investors, the executive team as the VCs and the managers in charge of different projects as the entrepreneurs. A competitive stock market ensures that the investors only receive a beta return. However, the future income of the CEO depends (implicitly or explicitly) on the success of the projects he chooses to fund. This dependence is required in order to increase unverifiable effort and/or to screen for good executives, and because the market cannot commit to not base future salary on performance. Executives have a limited ability to find positive NPV projects. Thus, even in equilibrium executives hold idiosyncratic risk and so must be compensated for holding it. This results in lower prices for corporate projects than if a diversified pool of investors could somehow access the projects directly. Thus, our model would predict the common observation that the internal cost of capital is above the market cost (see Poterba and Summers (1995)).

The impact of the success of chosen projects may be just as great on a CEO as a VC. However, projects done inside a firm probably have less impact on a manager than projects that are done outside the firm have on the entrepreneur. In either case the manager/entrepreneur must be compensated for the opportunity cost of his time. Moral hazard with respect to effort and/or adverse selection
also apply to the manager/entrepreneur. Therefore, the manager/entrepreneur’s compensation must
depend on the outcome of the project. However, the executive may know a great deal more about
the manager than a VC knows about the skill of an entrepreneur. Furthermore, greater oversight
may exist inside a firm, reducing the moral hazard. Therefore, as Prendergast (2000) suggests, a
manager’s income can be far less dependent on the project than the entrepreneur’s. As a result,
the manager can hold less risk, which implies that he needs lower returns than the entrepreneur.
Therefore, the alpha of projects done inside a firm can be smaller than projects completed by the
venture capital market.\textsuperscript{16} Therefore, although many positive NPV projects will not get financing
by the PE market, some of these projects may be completed inside a firm where the impact of
idiosyncratic risk is smaller.

This theory also helps explain why a VC is much more interested in an IPO than a private sale;
he can expect the fully diversified (beta-based) price from a public sale, but a private buyer is likely
to require compensation for some amount of idiosyncratic risk. Thus, as Black and Gilson(1999)
argue, VC and a good stock market go together. VCs don’t want to sell to a private market because
the buyer faces the same principal-agent problem. The need to get the company to a diversified
market drives everyone’s desire to IPO. Furthermore, this theory is consistent with the anecdotal
evidence that financial buyers are usually not willing to pay as much as a so-called strategic buyer
(even if no synergies exist).\textsuperscript{17}

\section*{VI Empirical Tests}

We are currently collecting data using the Venture Economics database. This database contains
comprehensive information on actual VC investments in US firms, as well as the venture capital
funds providing capital for many of these investments.

For our purposes, a key feature of the data is that investments are marked to market each quarter.
We use these marks to estimate the idiosyncratic and market risk of each investment and each fund,
and we intend to test the primary empirical implication of our model, that all else equal, valuations
are decreasing in the amount of idiosyncratic risk present.

In addition to measuring idiosyncratic risk directly, we also intend to use a number of instruments.
For example, VC funds limited to a single sector are exposed to more total risk than VC funds that are
better diversified. All else equal, we intend to test for lower valuations for single-sector investments.

\textsuperscript{16} This seems to suggest that all projects should get done inside firms or at least in partnership with firms. However,
Rhodes-Kropf and Santos (2001) suggest that the possibility that ideas can be stolen leads to some projects being
funded by investors who have a lesser ability to steal.

\textsuperscript{17} For example, Coors recently outbid two private equity firms in a contest to buy Britain’s Carling beer. See the
VII Conclusion

We find that unavoidable principal-agent problems in the private equity and venture capital markets result in idiosyncratic risk that must be priced. This leads to lower than expected prices in these markets, and some positive NPV projects with significant idiosyncratic risk cannot be completed. The higher discount rate is required not because the assets are illiquid but rather because the principal-agent problem makes diversification impossible.

This theory results in VC returns that are correlated with total risk not just beta risk. This leads us to expect: (1) above-beta returns for the PE and VC industries (as found by Cochrane(2001) and Gompers and Lerner (1997)), (2) higher alphas from projects with higher total risk, (3) higher returns from portfolios that specialize in a single sector, and (4) higher returns from portfolios that specialize in higher (total) risk sectors. However, none of these effects require any excess returns: investors receive only beta returns and VCs receive an amount that includes their ‘salary’ plus compensation for bearing risk. It is this second component that causes the correlation between idiosyncratic risk and returns.

This suggests that competition between VCs is not about the ability to find ‘better’ (higher NPV) projects, because competition among investors ensures all excess return goes to the entrepreneur. Instead, competition is about the ability to find more projects as this lowers the idiosyncratic risk held by VCs. Also, if a VC has knowledge about more disparate areas then their projects will be less correlated thereby reducing their risk. Finally, a VC should enter the market if they are less risk-averse than the marginal VC, as they could accept less compensation or earn excess return. Thus, overall we should expect this field to attract smart individuals with a high risk tolerance.

The standard contract between investors and VCs is like an option on the portfolio of chosen projects. The form of this contract suggests that idiosyncratic risk is very important in the VC and PE industry. If VCs could properly diversify then the option would only increase in value with increased volatility. This would mean that the standard contract was ensuring excessive risk taking. However, once we understand how idiosyncratic risk affects the VCs, we see that an option contract is in place to prevent over diversification.

This paper also has broader implications for any situation in which agents compete. The two extensions we briefly explore are managers inside firms and hedge funds. If our ideas extend to all private buyers then VCs should be much more interested in an IPO than a private sale, since only in an IPO will the price reflect just beta risk.

We believe that there are many situations in which price-setting agents hold idiosyncratic risk. Thus, priced idiosyncratic risk should be observable in many different asset prices.
REFERENCES


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