Talking up Liquidity:
Insider Trading and Investor Relations

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Abstract

Management ("insiders") of many corporations, especially small or newly public firms, invest considerable resources in investor relations such as voluntary disclosures and courting analyst coverage. We develop a model to explore the role of such costly investments and the incentives of insiders to undertake them. In contrast to existing theories, we point out that insiders may undertake such investments not to improve share price. Instead, they engage in investor relations to enhance the liquidity of their block of shares in case they have to sell their equity stakes for life cycle or diversification reasons. The costs of these investments are paid by all shareholders, though dispersed shareholders care little about market liquidity because of their small holdings. Our model predicts that the demographics of insiders (e.g. liquidity needs, size of equity stakes) are determinants of the extent of investor relations across firms.

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I Introduction

Many corporations, especially small or newly public firms, invest considerable resources toward investor relations. In addition to voluntary disclosures, the managements of these firms typically expend time and effort to attract analyst coverage and shape analyst expectations. Attracting analyst coverage is an especially important part of investor relations since analysts certify voluntary disclosures and get the information out to investors. But courting Wall Street attention is competitive, with many firms fighting for the attention of a limited pool of analysts (see Nocera (1997)). Hence, management has to commit to a long-term investor relations strategy. This gradual process is costly in a number of dimensions, including the time and energy of top executives and the revelation of valuable private and strategic information. Such activities can consume precious economic resources, especially for small or newly public firms.¹

There are a number of theories for why firms undertake such costly investments (see, e.g., Verrecchia (1983, 1990), Merton (1987), Fishman and Hagerty (1989), Diamond and Verrecchia (1991), Truemann (1999)). Broadly speaking, these theories argue that investor relations can boost the mean level of a firm’s stock price because certain key assumptions of efficient markets may not always hold. A general finding of a number of these theoretical studies is that expanded disclosure reduces information asymmetries between management and potential investors, which leads to more liquid markets. This, in turn, lowers a firm’s cost of capital and makes its stock more attractive to institutional investors.² As such, investments in investor relations benefit all shareholders and there is unanimity among

¹See Brennan and Tamarowski (1999) for a history of investor relations and an overview of these findings.
²There is good evidence that investor relations stimulates analyst coverage which in turn improves market liquidity. For instance, Brennan and Subrahmanyam (1995) find that large trades move prices much less in stocks with higher analyst coverage. For additional empirical evidence, see Lev (1992), Byrd, Johnson and Johnson (1993), Botosan (1997), Healy, Palepu and Sweeney (1995), Brennan and Tamarowksi (1999), Lang and Lundholm (2000)).
existing shareholders regarding a firm’s investor relations policy.

The prevailing view nowadays is that investor relations can also artificially boost stock prices because of irrational investors. Security regulators and the financial press often allege that firms engage in earnings management or other investor relations activities to get naive investors to boost a firm’s stock price. Indeed, a number of papers have found that such investor relations appear to temporarily boost stock prices (see, e.g., Teoh, Welch and Wong (1998a, b), Lang and Lundholm (2001)).

While the positive effect on stock price is no doubt important, we point out in this paper another role of investor relations. From our perspective, it is interesting to look at the insiders’ incentives to undertake such costly investments since insiders typically have discretion over investor relations policy. In contrast to existing theories, we point out that insiders may undertake such investments not to improve share price. Instead, they engage in investor relations to enhance the liquidity of their block of shares in case they have to sell their equity stakes for life cycle or diversification reasons. The costs of these investments are paid by all shareholders, though dispersed shareholders care little about market liquidity because of their small holdings. In other words, insiders may engage in investor relations even absent any positive effects on the share price.

The premise of our model is relevant for many corporations. More than ever, managements are rewarded with large equity stakes through vested option grants, which they need to then liquidate. Since changes in insider trading laws limit the selling of these stakes to only a narrow period each year, insiders frequently worry about market liquidity during these periods (see Richardson, Teoh and Wysocki (2001)). Our model is especially relevant for small or newly public firms, which comprise an increasingly larger part of the market. The management (or insiders) of these firms tend to own a substantial fraction of the equity
shares of their companies. Their equity stakes typically represent a substantial portion of their wealth. So, they especially want to liquidate their shares for various diversification or life cycle reasons. Even absent any lock-up restrictions, many insiders quickly discover that their holdings are highly illiquid. This market illiquidity is not simply a wide bid-ask spread but rather a lack of market depth in that large trades have a large price impact. It is very difficult to sell a large block of shares of many newly public companies without a significant price discount.

Our model features the following essential elements. There is an all equity firm with an insider (founder) who has a large block of shares and control rights to decide on the firm’s investor relations policy. The insider and some outside traders receive signals about future firm value with which they can speculate on. At the same time, the insider experiences a random liquidity shock and may need to sell a significant fraction of his stock holdings for purely non-informational reasons. Because traders are asymmetrically informed, trades will have price impact because of adverse selection.

The insider decides on the firm’s investor relations policy before receiving any signals about future firm value. By investing in investor relations, the insider increases the precision of a public signal about firm value, which reduces the information asymmetry among traders and increases market liquidity. Investor relations is costly, with the cost increasing in the precision of the public signal. These costs come out of firm value at the expense of all shareholders.

In deciding on these investments, the insider faces the following tradeoff. The insider shares not only the costs of a more precise public signal but also looses potential benefits

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3 For example, Field and Hanka (2000) found that for a sample of 464 non-venture financed firms that went public during 1988 and 1991, executives of these companies on average owned 33% of the total shares outstanding one year after the IPO. See also Barry et.al. (1990) and Megginson and Weiss (1991).

4 Indeed, the SEC also links the amount of selling an insider at a firm can do each day to the daily trading volume (a good proxy for liquidity) of the stock of his company. Given the relatively low volume for small or newly public firms, it may take insiders at these firms years to sell even a good portion of their stakes.
from informed trading. On the other hand, the improved market liquidity means that his liquidity trades will have less price impact. Because investor relations is costly, the insider will not always choose to make such investments. But for wide ranges of parameters of our model, we show that the benefits of improved market liquidity for his liquidity trades are substantial enough that he engages in investor relations.

In contrast, dispersed shareholders with few shares in the firm care little about market depth. In equilibrium, the insider invests too much resources (relative to the benchmark of maximizing share value) into liquidity enhancing activities at the expense of the longer-run value of the firm. This arises precisely because of his large block of shares relative to dispersed shareholders and the limited liquidity of the stock.

Our model has several implications. For instance, we ought to see more investor relations for firms in which insiders are on average net sellers, all else equal. In other words, firms with insiders who do not face liquidity needs are less likely to engage in investor relations than firms with insiders need to sell for liquidity reasons. Indeed, Richardson, Teoh and Wysocki (2001) find that earnings-guidance (a form of investor relations) is more prominent for firms whose insiders sell stocks from their personal accounts after earnings announcements, consistent with our prediction. More generally, our model predicts that demographics of insiders such as their equity stakes are important determinants of the extent of investor relations across firms.

A broader message of our model is that agency considerations are important for understanding why management focus on investor relations. More specifically, our model is reminiscent of agency theories which argue that managers, with substantial equity stakes, consider personal risk when making decisions that affect firm risk (see, e.g., Jensen and Meckling (1976)). For instance, management may engage in negative NPV diversification strategies from the perspective of shareholders because such strategies diversify their high
equity stakes. The downside of managers having too much equity is also a theme of our model.\textsuperscript{5}

Our paper proceeds as follows. We discuss the related literature in Section II. In Section III, we develop the model. We solve for the equilibrium in Section IV. In Section V, we derive the insider’s optimal investor relations policy and we discuss the empirical implications of the model. We discuss extensions to our model in Section VI. We conclude in Section VII by explaining how the economics of our model applies not only to investments in investor relations but to capital investments more generally. All proofs are in the Appendix.

II Related Literature

Most theories of investor relations explore how investor relations can boost the mean stock price when certain key assumptions for strict market efficiency do not hold. In these models, there is typically unanimity among shareholders on the optimal level of investments in investor relations. We now review a few strands of this literature.

One strand of the literature focuses on how investor relations can reduce information asymmetries in the market, thereby improving a firm’s stock price. For instance, in Merton (1987), risk averse investors do not invest in stocks that they do not know anything about. Investor relations makes investors aware of a stock, thereby increasing the stock’s investor base and the stock price through a risk sharing effect. In Diamond and Verrechia (1991), stocks are illiquid because of an adverse selection cost to trading. Since large investors invest less in illiquid stocks, this illiquidity tends to depress the stock price. Investor relations, whether it be the public disclosure of valuable information by the firm or attracting analysts

\textsuperscript{5}Another plausible agency story is that managers engage in investor relations for career concerns. By talking to analysts and being visible, a manager may increase his value in external labor markets. This hypothesis is empirically distinguishable from ours in that managers with greater equity stakes ought to be less worried about career concerns. In our model, it is these high equity stakes that lead insiders to consider the liquidity of their shares possibly at the expense of adopting a value maximizing investor relations policy.
who then dig up and disseminate information, levels the playing field for all investors. Hence, investor relations improves a stock’s liquidity, which tends to lead to a higher price.\footnote{Another paper that argues a similar point is Brennan and Tamarowski (1999). They point to investor relations leveling the playing field among investors. This lower adverse selection cost translates into a lower bid-ask spread. This lower bid-ask spread then leads to a higher stock price using an argument along the lines of Amihud and Mendelson (1986).}

Another strand of the literature focuses on the role of investor relations to boost stock prices in the presence of market mis-valuations. Verrecchia (1983, 1990) shows that when disclosure is costly and credible, managers who believe that their firms are undervalued will voluntarily provide additional information to investors if the value of reduced undervaluation exceeds the costs of expanded disclosure. Truemann (1996) offers a model in which managers pursue investor relations so as to attract investors who cannot short-sell the stock when there is uncertainty about the number of such investors trading the stock. Assume that the market anticipates that, on average, 10 non-short-selling traders will participate in a market for a firm’s shares. Suppose, however, that there are 11 potential non-short-selling traders. Then there will always be an incentive to attract the eleventh one, as a device to bias the price upward, provided that the market anticipates only 10 (and attracting the eleventh is not very costly).

A third strand of the literature focuses on financial disclosures among multiple, competing firms, the externalities that arise as a result and the appropriate disclosure regulations to adopt in the presence of such externalities. In Fishman and Hagerty (1989), investor relations involves disclosing information that lowers the fixed cost for outside traders to get valuable information about the firm. So, investor relations doesn’t level the playing field. Rather, it attracts informed traders who then make the stock price more efficient by collecting information and trading on it. This increase in stock price efficiency can raise the mean level of the stock price because it induces managers to make more efficient investment decisions. The main result is that the competition for informed traders leads firms to
over-invest in costly disclosures than is socially optimal. In Admati and Pfleiderer (2000), firms’ values are correlated and disclosures made by one firm are used by investors in valuing other firms. As such, there are also externalities associated with disclosure decisions in this context. They find that the equilibrium of a voluntary disclosure game is socially inefficient and regulation that requires a minimal precision level sometimes but not always improves welfare. Recently, Boot and Thakor (2001) answer the question of what kind of information and how much of it should firms voluntarily disclose. They answer these questions by taking into account the effects of disclosures on the incentives of outsiders to gather information and incentives for financial innovation.

Our paper is closest to the papers in first strand of the literature in that information asymmetry among investors is the source of market illiquidity. Like these papers, we assume that the insider can reduce the information asymmetry in the market through investor relations. Unlike these papers, we emphasize the incentives of the insider to pursue investor relations. More specifically, the insider has an incentive to engage in costly investor relations even absent any beneficial effects of such activities on the mean level of the stock price. Hence, there is a potential divergence of interest between insiders and dispersed shareholders on the firm’s investor relations policy. This economic mechanism for investor relations is new relative to existing theories. In addition, our model generates a number of new testable predictions on how a firm’s investor relations policy depends on the demographics of insiders.

Our paper is also related to papers in the third strand of the literature in that some of the modeling techniques that we use are similar. And like Fishman and Hagerty (1989), our model suggests that firms can over-invest in investor relations relative to a benchmark of maximizing firm value.
III The Model

Consider the following three-date model. There is an all-equity firm that is liquidated at date 2. Let $\tilde{v}$ denote the intrinsic date-2 value of the firm. $\tilde{v}$ has an unconditional mean and variance given by:

$$E(\tilde{v}) = \bar{v}, \quad \text{var}(\tilde{v}) = \sigma_v^2.$$  \hfill (1)

At date 0, the shares of the firm are owned by two groups of investors: an insider who has control rights and dispersed shareholders, each of whom owns a tiny fraction of the firm. We will often refer to the insider as management. The total fraction of the insider’s holdings of the firm’s equity at date 0 is $\theta \in (0, 1)$. We assume that $\theta$ is exogenously given. At date 0, there is no trading in the stock and no one has received any signals about $\tilde{v}$. Think of date 0 as some time in the past in which the insider (likely also the entrepreneur/founder) is allocated a block of shares for a variety of agency reasons. We could endogenize $\theta$ by assuming that introducing some benefits (due to agency reasons) for the insider to own a block of equity. Our results would be unchanged.

A Investor Relations

We assume that the insider has to commit to an investor relations policy at date 0 before receiving any signals about $\tilde{v}$. This assumption captures a realistic aspect of investor relations: it takes time for management to convince analysts to initiate coverage. In such instances, much of the future prospects of the firm are unknown to insiders when they invest in investor relations.

We model investor relations in the following parsimonious manner. We assume that there is a public signal produced between date 0 and date 1 given by

$$\tilde{s} = \tilde{v} + \tilde{e},$$  \hfill (2)
where \( \tilde{e} \) has an unconditional mean and variance given by

\[
E(\tilde{e}) = 0, \quad \text{var}(\tilde{e}) = \sigma_\tilde{e}^2. \quad (3)
\]

The precision of the public signal \( 1/\sigma_\tilde{e}^2 \) depends on investments in investor relations made by the insider. By incurring costs \( C \), the insider can increase the precision of the public signal:

\[
\left\{ \frac{1}{\sigma_\tilde{e}^2} = f(C) : f' > 0, f'' \leq 0, f(0) = 0 \right\}. \quad (4)
\]

This is a natural way to model the effects of investor relations because insiders in courting analyst coverage are essentially committing to opening up many aspects of their business for scrutiny by analysts. In this view, analysts dig up and disseminate information about the firm and hence level the playing field among all market participants.\(^7\) The cost \( C \) is simply netted from the liquidation value of the firm at date 2.

It is important to keep in mind that there are a number of other ways to model investor relations. In Section VI, we extend our model by considering a popular alternative. It turns out that the results are quite similar.

### B Liquidity and Speculative Trades

At date 1, there is trading in the stock. Before the start of trading at date 1, every market participant gets to observe the public signal \( \tilde{s} \). Since the level of \( C \) chosen by the insider at date 0 is known to all before the start of trading at date 1, market participants know the precision of the public signal. There are several sets of traders at date 1.

\(^7\)Consistent with this premise, Brennan et.al. (1993) found that the stock prices of firms with high analyst coverage respond more quickly to macroeconomic news. Hong et.al. (2000) found that stock prices of firms with more analyst coverage also respond more quickly to firm specific information, particularly bad news. Dempsey (1989) finds that the more analysts who follow a firm, the less likely is the market to be surprised by the firm’s quarterly earnings announcement. This means that when more analysts follow a firm, there is less potential for profitable informed trading ahead of earnings announcements: in other words, the more level is the playing field.
A. The Insider

The insider gets to observe $\tilde{v}$ before the start of trading at date 1. So he can speculate on his private information by selling his initial stake or by buying more shares. Additionally, he would like to liquidate a fraction $\tilde{\theta}_L$ of his initial stake of $\theta$ for liquidity reasons completely unrelated to the information that he receives about the firm. $\tilde{\theta}_L$ is known to the insider only.

Let $X_i$ denote the number of shares that the insider sells at date 1 and let $P_t$ denote the date-$t$ share price. If the insider realizes a liquidity shock of $\tilde{\theta}_L$ and doesn’t sell $X_i = \tilde{\theta}_L$, he suffers a penalty (utility loss) given by $-l(X_i - \tilde{\theta}_L)^2$, where $l \geq 0$ measures the cost of being away from an ideal liquidation level of $\tilde{\theta}_L$. The remaining shares of the insider $\theta - X_i$ are liquidated at date 2. But the insider may want to be away from $\tilde{\theta}_L$ because of his private information $\tilde{v}$. Hence, the insider is both a speculator and a liquidity trader.

More specifically, the insider chooses $X_i$ to maximize the expectation (conditional on the insider’s information set at date 1) of the following sum:

$$-l(X_i - \tilde{\theta}_L)^2 + X_i\bar{P}_1 + (\theta - X_i)\bar{P}_2.$$  

When $l = \infty$, the insider simply sets $X_i = \tilde{\theta}_L$, ignoring his private information. When $l$ is finite, the insider may want to shade $X_i$ away from $\tilde{\theta}_L$ to take into account his private information so as to increase the expected combined proceeds from his trades in date 1 and date 2.

Furthermore, we assume that $\tilde{\theta}_L$ has a mean given by

$$\mathbf{E}(\tilde{\theta}_L) \equiv \bar{\theta}_L = M\theta,$$  

where $M \geq 0$. Since $\tilde{\theta}_L$ is assumed to be positive, the insider on average wants to sell a fraction of his initial stake in the firm. Also, we assume that the variance of the liquidity
shock is proportional to the insider’s initial stake in the firm:

\[ \text{var}(\tilde{\theta}_L) \equiv \sigma^2_L = K^2 \theta^2, \]

where \( K \geq 0 \).

Hence, the four parameters \( l, M, K, \) and \( \theta \) summarize the insider’s liquidity needs. There is a simple interpretation for these liquidity needs in the context of newly public firms. The insider faces borrowing constraints which force him to finance his liquidity needs by selling a fraction of his initial stake. The penalty parameter \( l \) captures how binding these borrowing constraints are. If \( l = 0 \), even if he faces a liquidity shock, the insider does not have to sell his shares of the firm to finance it—presumably because he has external wealth or does not face any borrowing constraints. So, when we speak of the insider facing or experiencing liquidity needs, we are typically thinking of these four parameters as being non-zero. So the insider needs to sell a large fraction of his initial stake for non-informational reasons.

B. Dispersed Shareholders

The dispersed shareholders do not have any private information about \( \tilde{v} \). They may experience liquidity needs at date 1. However, since their holdings are such a small fraction of the shares outstanding of the firm, we can without loss of generality (as we will show below) assume that they do not face any liquidity needs at date 1, and hence do no trade at date 1.

C. Outside Speculator and Noise Traders

Finally, other traders without stakes in the firm at date 0 may trade the shares of the firm at date 1. First, a risk neutral speculator (who has done his homework and collected information about the firm) observes the realization of \( \tilde{v} \) and then optimally speculates on
his private information at date 1 by trading an amount equal to $X_i$.\footnote{Our results would be qualitatively similar even if we did not have a speculator in the model. See Section VI. In reality, there are many outsiders (e.g. mutual fund managers) that can obtain valuable private information about the firm. To enrich the model, we add in an outside speculator.} Additionally, noise traders want to trade $\tilde{u}$ shares with mean and variance given by:

$$E(\tilde{u}) = 0, \quad \text{var}(\tilde{u}) = \sigma^2_u. \quad (7)$$

C Price Setting Mechanism

For tractability, we will use Kyle (1985) to model the price setting at date 1. We assume that $\tilde{u}$, $\tilde{v}$ and $\tilde{\theta}_L$ are mutually independent and normally distributed. Specifically, risk neutral, competitive market makers set the price for the stock at date 1 based on the aggregate order flow $\tilde{Y}$, which comprises of the market orders of the insider, $\tilde{X}_i$, the speculator $\tilde{X}_s$, and the noise traders $\tilde{u}$:

$$\tilde{Y} = \tilde{X}_i + \tilde{X}_s + \tilde{u}. \quad (8)$$

Note that from the perspective of the market makers, these orders are all random given the market maker’s information. The price at date 1 is set in the following manner:

$$\tilde{P}_1 = E[\tilde{v} - C|\tilde{Y}, \tilde{s}]. \quad (9)$$

In using this trading mechanism, we are implicitly assuming that the insider and other liquidity traders cannot credibly communicate to the market makers that particular trades are due to purely liquidity reasons. If the insider could somehow convince that particular trades are due only to non-informational reasons, then his sale orders would have no price impact and the insider would have little incentive to undertake investor relations.\footnote{In the context of our model, if our market makers observed the market order of the insider and his realization of $\tilde{\theta}_L$, then there would be no price impact.}
IV The Security Market Equilibrium

In this section, we characterize the security market equilibrium for a fixed level of investment in investor relations at date 0 given by $C$. This investment $C$ results in a fixed level of $1/\sigma_e^2$ at date 0 given by the functional form in equation (4). The market equilibrium proceeds given $C$. In Section V, we derive the optimal level of investment $C$ chosen by the insider.

The equilibrium prices at date 2 and date 0 are easy to calculate. Since $C$ gets paid out of the terminal firm value, $\tilde{P}_2 = \tilde{v} - C$. Moreover, since the level of $C$ chosen is observable to all, it follows that

\[ P_0 = E(\tilde{P}_1) = \tilde{v} - C. \]  

(10)

The equilibrium at date 1 is described as follows in Theorem 1.

Theorem 1 At date 1, there is a unique Bayesian-Nash linear equilibrium, which is characterized by the following properties.

(i) The noise traders combine to place a market order in the amount of $\tilde{u}$.

(ii) The speculator, after observing $\tilde{v}$ and thus $\tilde{e}$ and $\tilde{P}_2 = \tilde{v} - C$, submits a market buy order in the amount of

\[ \tilde{X}_s(\tilde{v}, \tilde{e}) = \frac{2l + \lambda}{\lambda(4l + 3\lambda)} [\tilde{v} - E(\tilde{v}|\tilde{s})]. \]  

(11)

(iii) The insider, after observing $\tilde{v}$, $\tilde{e}$, and $\tilde{\theta}_L$, submits a market sales order in the amount of

\[ \tilde{X}_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L) = \frac{1}{4l + 3\lambda} [E(\tilde{v}|\tilde{s}) - \tilde{v}] + \frac{l}{l + \lambda} \tilde{\theta}_L + \frac{l\lambda}{(l + \lambda)(2l + \lambda)} \tilde{\theta}_L. \]  

(12)

(iv) Competitive, risk neutral market makers set the price as

\[ \tilde{P}_1 = E(\tilde{v}|\tilde{s}) - C + \lambda \left[ \tilde{u} + \tilde{X}_s(\tilde{v}, \tilde{e}) - [\tilde{X}_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L) - \tilde{\theta}_L] \right] - \frac{\lambda^2}{2l + \lambda} \tilde{\theta}_L, \]  

(13)
where \( \lambda \) is given by a positive solution for
\[
\frac{\lambda^2 (4l + 3\lambda)^2}{2(l + \lambda)(2l + \lambda)} \left[ \sigma_u^2 + \left( \frac{l}{l + \lambda} \right)^2 \sigma_L^2 \right] = \sigma_v^2, \tag{14}
\]
and
\[
\sigma_v^2 = \frac{\sigma_v^2 \sigma_e^2}{\sigma_v^2 + \sigma_e^2} = \text{var}(\tilde{v}|\tilde{s}). \tag{15}
\]
Equation (14) has a unique solution.

Consider the equilibrium price function given in equation (13). The first two terms \( E(\tilde{v}|\tilde{s}) - C \) is simply the publicly available forecast of terminal firm value net the investment in investor relations \( C \). The next term is simply the unexpected order flow observed by the market makers. The final term,
\[-\frac{\lambda^2}{2l + \lambda} \bar{\theta}_L < 0,
\]
is a bit harder to understand. Apparently, the stock price decreases with \( \bar{\theta}_L \), the mean of the investor’s ideal liquidation position. This term accounts for the fact that the insider will sell \( X_i < \bar{\theta}_L \) even if \( \bar{v} = E[\tilde{v}|\tilde{s}] \) and \( \tilde{\theta}_L = \bar{\theta}_L \) because the insider tries to increase the price for all the \( X_i \) shares at the expense of a small liquidity cost.

We consider a couple of special cases to further develop intuition for the equilibrium. First, consider the extreme case in which \( l = 0 \), i.e. there is no liquidity penalty. In this case, it is easy to see from equations (11) and (12) that the speculator’s optimal market buy order and the insider’s optimal market sale order collapse to
\[
\tilde{X}_s(\tilde{v}, \tilde{e}) = -\tilde{X}_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L) = \frac{1}{3\lambda} [\bar{v} - E(\tilde{v}|\tilde{s})]. \tag{16}
\]
The equilibrium price given in (13) simplifies to
\[
\tilde{P}_1 = E(\tilde{v}|\tilde{s}) + \lambda \left[ \bar{u} + \tilde{X}_s(\tilde{v}, \tilde{e}) - \tilde{X}_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L) \right], \tag{17}
\]
where

$$\lambda = \frac{\sqrt{3} \hat{\sigma}_v}{\sigma_u}. \quad (18)$$

Now consider the opposite extreme in which \( l = \infty \), i.e. there is an infinite penalty for the insider to deviate from his ideal liquidation position \( \tilde{\theta}_L \). In this limiting case, it is easy to see the insider’s optimal market sale order given in equation (12), of course, reduces to

$$X_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L) = \tilde{\theta}_L. \quad (19)$$

In order words, the insider does not speculate on his private information. The speculator’s market buy order given in equation (11) reduces to

$$X_s(\tilde{v}, \tilde{e}) = \frac{1}{2\lambda} [\tilde{v} - E(\tilde{v}|\tilde{s})]. \quad (20)$$

The equilibrium price given in (13) simplifies to

$$\tilde{P}_1 = E(\tilde{v}|\tilde{s}) + \lambda \left[ \tilde{u} + X_s(\tilde{v}, \tilde{e}) - [X_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L) - \tilde{\theta}_L] \right], \quad (21)$$

where

$$\lambda = \frac{\hat{\sigma}_v}{\sqrt{\sigma_u^2 + \sigma_L^2}}. \quad (22)$$

Notice that the speculator trades more aggressively on his private information (also observed by the insider) when \( l = \infty \) than when \( l = 0 \). The reason, of course, is that when \( l = \infty \), the insider doesn’t speculate at all, leaving the speculator more room to aggressively exploit his private information.

The key parameter in Theorem 1 is \( \lambda \), which measures the impact of trades on price or market illiquidity. In general, we will need to numerically solve equation (14) to yield \( \lambda \) as a function of the parameters \( l \) (the liquidity penalty), \( \sigma_L^2 \) (the variance of the liquidity shock), \( \sigma_v^2 \) (the variance of the firm terminal value), \( \sigma_u^2 \) (the variance of noise trades), and \( \sigma_e^2 \) (the noisiness of the public signal).
However, we can prove a couple of intuitive results regarding $\lambda$ without resorting to any numerical solutions. These results will be useful in understanding the optimal level of investments in investor relations, $C$, chosen by the insider.

**Lemma 1** The measure of market illiquidity, $\lambda$, is decreasing in the precision of the public signal, $1/\sigma^2_e$, and the variance of noise trades, $\sigma^2_u$.

When the public signal is more precise or the variance of noise trades are higher, there is less adverse selection (from the perspective of the market makers) in the market all else equal and the smaller is $\lambda$. By undertaking investments of $C$ in investor relations, the insider can increase $1/\sigma^2_e$ and thereby decrease the market illiquidity as measured by $\lambda$. It is to the determinants of these investments that we now turn.

V Optimal Investor Relations Policy

The basic premise of our paper is that absent any actions on the part of the firm, $\lambda$ is non-trivial. It is useful to note that this premise is especially applicable for small or newly public firms. For very large and liquid firms, this premise is probably less reasonable. Nonetheless, with existing trading laws which force insiders to sell at very narrow windows, even insiders at large firms have to worry about the liquidity of their block of shares.

We will show that for insiders at these firms who face stringent liquidity needs, they have a strong incentive to under-take investments in investor relations to decrease $\lambda$. Even though it is the insider who ultimately determines $C$, it is useful to first consider the optimal $C$ that dispersed shareholders prefer.

A The Interest of Dispersed Shareholders

Let $x$ be the fraction of the company held by a dispersed shareholder. Here, think of $x$ as being close to zero. Now suppose that dispersed shareholders only liquidate their shares
at date 2. Then the expected revenue for a given dispersed shareholder in this instance is simply

\[ x(\bar{v} - C). \]

Of course, dispersed shareholders would prefer that \( C = 0 \) since they do not face any liquidity needs at date 1 and the cost comes out of the terminal firm value.

Now suppose that for some reason, dispersed shareholders want to liquidate their shares at date 1. Then the expected revenue in this instance for a given dispersed shareholder is

\[ x(\bar{v} - C) - \lambda x^2. \] (23)

Notice that the penalty for market illiquidity (the term involving \( \lambda \)) is proportional to the square of the fraction of shares of the firm held by the dispersed shareholder. Since \( x \) is close to zero, it follows that \( x^2 \approx 0 \) and so again the optimal choice for the dispersed shareholder is to choose \( C = 0 \). In other words, dispersed shareholders would prefer that no investments be made in investor relations. Their preferred solution coincides with the benchmark of maximizing firm value at date 2.

**B The Interest of the Insider**

We next consider the insider’s choice of \( C \). By paying a cost \( C \), the insider increases \( 1/\sigma^2_e \) according to the functional form given in (4). The insider chooses \( C \) to maximize at date 0 the expected profits given by \( I(C) \):

\[ I(C) = \mathbb{E} \left[ -l(\hat{X}_i - \hat{\theta}_L)^2 + \hat{X}_i(\hat{P}_1 - \hat{P}_2) + \theta \hat{P}_2 \right]. \]

Using the solution given in Theorem 1, we can explicitly calculate \( I(C) \). This is given in Lemma 2.

**Lemma 2** The insider’s expected profits are given by

\[ I(C) = \theta(\bar{v} - C) + \frac{l + \lambda}{(4l + 3\lambda)^2} \sigma^2_v - \frac{l\lambda^2}{(2l + \lambda)^2} \theta^2_L - \frac{l\lambda}{l + \lambda} \sigma^2_L. \] (24)
In equation (24), the first term, $\theta(\bar{v} - C)$, is simply the expected terminal value of the firm net of the cost of investor relations. The remaining three terms reflect the expected profit associated with the insider’s informed trading and the expected utility of the insider’s liquidity trades.

Broadly speaking, these four terms capture the following tradeoff of investor relations. In investing $C$ in investor relations, the insider shares the cost of $C$ with dispersed shareholders and loses potential speculative profits as investor relations results in disclosure of private information. On the other hand, investor relations lowers market illiquidity as measured by $\lambda$, which decreases the price impact of his liquidity trades. When his liquidity needs are substantial and markets are highly illiquid, the insider may engage in costly investor relations.

To get some intuition for this tradeoff, suppose that $l = 0$. When $l = 0$, the insider does not face a penalty for being far away from his ideal liquidating position $\theta_L$. In this instance, it is easy to see that $I(C)$ of equation (24) reduces to

$$I(C) = \theta(\bar{v} - C) + \frac{\lambda}{2} \sigma_u^2.$$  

(25)

The optimal solution for this objective function is $C = 0$. By investing in investor relations, the insider shares part of the cost $C$. Also, in choosing a non-trivial $C$, $\lambda$ will be lower and hence the second term involving $\sigma_u^2$ is also lower. This second term captures the speculative profits of the insider, which he would have to give up if he engages in investor relations. Hence, he has no incentive to do so if he is not forced to liquidate his shares to finance his liquidity needs.

Next, suppose that the initial stake of the insider ($\theta = 0$). In this case, $I(C)$ takes the form of

$$I(C) = \theta(\bar{v} - C) + \frac{\lambda^2}{2(2l + \lambda)} \sigma_u^2.$$  

(26)
When $\theta = 0$, the insider has such a small stake in the firm that his market sell order is miniscule in comparison to the liquidity of the market. Again, he has no incentive to invest in investor relations. The case in which $M = K = 0$ (i.e. the insider has no liquidity needs) is equivalent to the case of $\theta = 0$.

In general, we need to numerically solve for the equilibrium $C$ by assuming a functional form for $f$. It turns out that for large parameter ranges, it is optimal for the insider to choose a non-trivial level of investments in investor relations. Below, we will solve the model for a wide range of parameters.

### C Divergence of Interests

But even without resorting to numerical calculations, we can prove a key result regarding the divergence of interests between the insider and dispersed shareholders regarding investor relations. In Theorem 1, we provide a sufficient condition for the insider to engage in costly investor relations.

**Theorem 2** Suppose $l > 0$ and $\theta > 0$. If

$$f'(0) > G(\bar{v}, \sigma_v^2, \sigma_u^2, l, \theta, K, M) > 0,$$

(27)

where $G$ is defined in the Appendix, then the insider invests $C > 0$. Since dispersed shareholders prefer that $C = 0$, there is an over-investment in investor relations relative to the benchmark of maximizing firm value.

It is not necessarily the case that the insider always chooses a non-trivial $C$ if he has liquidity needs at date 1. Whether or not he does depends on a number of factors. Certainly, the benefits of liquidity need to be weighed against the costs of diverting firm resources and losing out on speculative profits. As Equation (27) shows, this tradeoff also depends on how quickly the precision of the public signal increases with $C$. That is, it depends on the...
functional form of \( f \). When the precision of the public signal does not respond sufficiently quickly to \( C \), the insider may still optimally choose \( C = 0 \) even if he has extreme liquidity needs.

To better understand the nature of the divergence of interests pointed out in Theorem 1, we focus on a special case in which \( f \) is assumed to be linear in \( C \):

\[
f(C) = \alpha e^C, \quad (28)
\]

and the insider faces an infinite penalty for deviating from his ideal liquidation position, i.e. \( l = \infty \). While the case of \( l = \infty \) is not very realistic, we can obtain closed form solutions for \( \lambda \) and \( C \), which will serve to provide some insight into the nature of the equilibrium.

The results of this special case are summarized in Proposition 1.

**Proposition 1** Suppose \( l = \infty \) and \( f(C) = \alpha e^C \). There is a unique solution for \( C \) given by

\[
C^* = \text{Max} \left\{ \frac{K^{4/3} \theta^{2/3}}{2(2\alpha e)^{1/3} (\sigma_u^2 + K^2 \theta^2)^{1/3}} - \frac{1}{\alpha e \sigma_v^2}, 0 \right\}.
\]

The equilibrium \( \lambda \) is given by

\[
\lambda^* = \text{Min} \left\{ \left[ \frac{\theta}{2\alpha e \sigma_v^2 (\sigma_u^2 + \sigma_L^2)} \right]^{1/3}, \frac{\sigma_v}{2\sqrt{\sigma_u^2 + \sigma_L^2}} \right\}.
\]

Suppose condition (27) holds (i.e. when \( \alpha e \) is sufficiently large), then it is easy to show that \( C^* > 0 \). In this instance, it is also easy to show that \( C^* \) is increasing in \( \theta \). Intuitively, when \( \theta \) is large, the insider’s liquidity trades are larger and so he invests more in investor relations. It is important to note that this comparative static is true only at \( l = \infty \). It does not hold generally. When \( \theta \) is large, the insider also owns a larger fraction of the firm and so ends up paying for a higher fraction of the costs \( C \). In general, whether \( C^* \) increases or decreases in \( \theta \) depends on the magnitude of \( l \).
D Numerical Solution

We now solve our model for a set of parameter values. Our goal is to solve the model for a range of \( \theta \) and \( l \) while fixing all the other parameters. The basic point we want to get across is that when both \( \theta \) and \( l \) are small, there is no incentive for the insider to invest in investor relations. But when \( \theta \) and \( l \) are both large, the insider ends up spending a non-trivial fraction of the firm’s resources on investor relations.

While we do not think that such a stylized model can accurately capture the market illiquidity faced by insiders, it is interesting to think of some reasonable ranges for \( \theta \) and \( l \), as well as the other parameters, and use these parameters in our numerical solutions. We calibrate these parameters for a typical small, newly public firm. For insiders at such firms, we think that a \( \theta \) on the order of 20% is reasonable. So, we will consider solutions for \( \theta \) ranging from 0 to 0.4. It is not obvious how to calibrate \( l \) for insiders at these firms, so we have a degree of freedom here in deciding what is a sensible value for \( l \). We will provide solutions for \( l \) from 0 to 1000. We will take an \( l \) near 200 as a focal point of our discussions.

In addition, we set the mean terminal value of the firm, \( \bar{v} \), to be 100 with a standard deviation, \( \sigma_v \), to be 50. We assume that insider on average liquidates 50% of his stake of \( \theta \) (i.e. set \( M = 0.5 \)) and faces a liquidity shock with a standard deviation equal to 40% of his stake (i.e. set \( K = 0.4 \)). So, at the base case of \( \theta = 0.2 \), the insider on average liquidates \( \bar{\theta}_L = 0.5(0.2) = 0.1 \) with a standard deviation of \( \sigma_L = 0.4(0.2) = 0.08 \).

The standard deviation of the noise trades \( \sigma_u \) is set to 0.02. One justification for this number is that \( \sigma_u \) is one-fourth of \( \sigma_L \) for the case in which \( \theta = 0.2 \). So we are essentially assuming that the amount of noise trades is small compared to the liquidity trades of the insider. We think this is a sensible way to capture the illiquidity of the insider’s shares since for small, newly public firms, the float of the firm is small in comparison to the insider’s holdings.
Furthermore, we assume that $1/\sigma_e^2$ is linear in $C$ with a slope of $\alpha_e$ as given in equation (28). We choose $\alpha_e$ such that when $C = 1$ (the insider spends 1% of the firm’s value on investor relations), investor relations means that $1/2$ of the private information is revealed to the public, i.e. $\hat{\sigma}_v^2/\sigma_v^2 = 0.5$ when $C = 1$. This condition gives us an $\alpha_e = 0.0004$.

In this instance, solving for the optimal $C$ is a non-linear optimization problem in a single variable $C$ over a bounded support. It is easy to show that $C$ cannot exceed the average value of the firm $\bar{v}$. Hence, we are looking for a solution between 0 and $\bar{v}$. It is straightforward to identify the global optimum for different sets of parameters.

Figures 1(a) and (b) show the plots of the optimal level of investment in investor relations ($C$) and the accompanying equilibrium measure of market illiquidity ($\lambda$). Consider Figure 1(a). When the insider stake ($\theta$) and the liquidity penalty ($l$) are near zero, the optimal solution is $C = 0$ (consistent with the discussion above). Roughly speaking, the figure indicates that when $\theta < 0.05$ or $l < 80$, there is no investment in investor relations. More importantly, when $\theta$ and $l$ are large enough, $C > 0$.

Note that this does not mean that $C$ is increasing in $\theta$ or $l$ in general. Focus again on Figure 1(a). Fix $\theta = 0.4$ and look at how $C$ changes as $l$ goes from 80 to 1000. Note that in this region, there is already quite a bit of investment in investor relations. As $l$ gets larger, the insider actually reduces the level of investments in investor relations. This might seem surprising because as $l$ gets bigger, the insider faces more stringent liquidity needs all else equal. Hence one might expect him to invest more to reduce market illiquidity. However, in equilibrium, market makers realize that for $l$ larger, the insider is less aggressive in speculating on his private information. So all else equal, the market maker is more confident that trades are more likely to be liquidity driven and that there is less adverse selection in the market. So the market is more liquid when $l$ is large and the insider needs to invest less. The exact relationship between $C$ and $l$ depends on these two off-setting effects.
Notice also that in the parameter space where \( C > 0 \), \( C \) may increase or decrease in \( \theta \). As \( \theta \) increases, the insider owns a larger fraction of the firm and so he shares more of the costs of investor relations. All else equal, this means he is less likely to invest for \( \theta \) large. On the other hand, the higher the \( \theta \), the larger are his liquidity trades and so the more costly is market illiquidity and the more likely \( C > 0 \).

Additionally, it is interesting to note from Figure 1(a) that the optimal investor relations policy does not change smoothly with changes in \( \theta \) or \( l \). For instance, fix \( l = 100 \) and consider how \( C \) changes as \( \theta \) increases from 0 to 0.4. Roughly, there is no investment for \( \theta \) less than 0.3. Then for \( \theta \)'s above this level, investment increases to 2.5, i.e. 2.5% of the mean value of 100. As \( C \) increases from zero to a small amount, the costs increase linearly in \( C \). However, the benefits may be highly non-linear in \( C \) in that the benefits do not kick in sufficiently to outweigh the costs until \( C \) is large enough. This is in fact the case at these parameters. We have experimented with other parameter ranges and such a non-linear tradeoff appears to be a robust effect of our model.

For completeness, we consider the equilibrium \( \lambda \)'s in Figure 1(b) that result from the investments shown in Figure 1(a). For \( \theta \) and \( l \) small, \( C = 0 \) and hence \( \lambda \) is large. As one would expect, when \( C > 0 \), there is a substantial and precipitous drop in \( \lambda \).

\section{Empirical Implications}

Since our model is highly stylized, we do want to guard against over-interpreting the various implications of our model. However, the one set of predictions that we think are novel to our model and may be empirically sensible are the ones relating the intensity of investor relations across firms with certain demographic features of insiders.

To begin with, there are a number of different measures of investor relations. One often used measure is earnings-guidance or earnings-management as used in Richardson,
Figure 1: Investments in investor relations and equilibrium market illiquidity. In figure (a) the investment policy of the firm is shown for various values of the insider stake $\theta$ and the liquidity penalty $l$. In figure (b) the equilibrium lambda is shown for various values of the insider stake $\theta$ and the liquidity penalty $l$. $\theta$ ranges from 0 to 0.4 and $l$ ranges from 0 to 1000 in both figures. The remaining parameters are set at the following values: $\bar{v} = 100$, $\sigma_v = 50$, $M = 0.5$, $K = 0.4$, $\sigma_u = 0.02$, $\alpha_e = 0.0004$. 
Teoh and Wysocki (2001). Other proxies include presentations to analyst societies (Byrd et al. (1993)) and conference calls (Tasker (1998)). In addition, demographic information on insiders such as their trading activities, shares owned and other personal characteristics such as their wealth are available.

Hence, it is feasible to relate the intensity of investor relations in the cross-section to insider demographics. Perhaps the most robust of our model’s implications is that, all else equal, firms in which insiders are large net sellers of their shares ought to have a greater intensity of investor relations. One can think of the size of these sales as a measure of liquidity needs.

Relatedly, we can more precisely test the model by looking at the size of equity stakes and more primitive measures of liquidity needs. Our model predicts that a firm with large insider ownership ought to be more likely to engage in investor relations than one without (see Theorem 2 and the discussion in Section V.D). Also, to the extent that we can identify the extent of liquidity need with an insider’s outside wealth, our model predicts that a firm with an insider with little outside wealth is more likely to engage in investor relations than other firms, all else equal.

VI Further Discussions and Extensions

In this section, we discuss some of the important elements of our model and consider some extensions.

A Investor Relations Increases the Variance of Noise Trades

An alternative and more cynical way to model investor relations is that by incurring costs $C$, the insider can increase the variance of noise trades $\sigma_u^2$. In this view, analyst coverage stimulates lots of noise trading in a stock. One can think of these noise traders as individual
investors who watch shows like CNBC and end up buying stocks based on what analysts recommend on these shows.

More specifically, assume that the insider can increase the date-1 variance of noise trades by incurring costs, \( C \), at date 0 according to the following functional form:

\[
\left\{ \sigma_u^2 = g(C) : g' > 0, \ g'' \leq 0, \ g(0) = \sigma_u^2 \right\}.
\]  

(29)

The equilibrium given in Theorem 1 still holds. The objective function of the insider is the same as in Lemma 2 except that \( \lambda(C) \) depends on \( C \) indirectly through \( \sigma_u^2(C) \) as opposed to \( 1/\sigma_e^2 \). From Lemma 1, we know that as \( C \) increases, \( \lambda \) decreases. The optimal choice for the insider has to be solved numerically.

Without calculating the exact level of \( C \), we can prove the following theorem, which is the analog to Theorem 2.

**Theorem 3** Assume that investments in investor relations \((C)\) increases the variance of noise trades \((\sigma_u^2)\). If

\[
g'(0) > \frac{\theta A(l, \lambda_0, \sigma_u)}{(l + \lambda_0)^2 \sigma_L^2 + \frac{4l^2 \lambda_0}{(2l + \lambda_0)^2} \sigma_v^2} + \frac{2(l + \lambda_0)(2l + \lambda_0)}{\lambda_0^2 (4l + 3\lambda_0)^2},
\]

(30)

where \( \lambda_0 = \lambda(C = 0) \) and

\[
A(l, \lambda, \sigma_u) \equiv \frac{\lambda(4l + 3\lambda)(16l^3 + 36l^2\lambda + 27l\lambda^2 + 6\lambda^3)}{2(l + \lambda)^2 (2l + \lambda)^2} \sigma_u^2
\]

\[
+ \frac{l^3 \lambda(4l + 3\lambda)(16l^2 + 20l\lambda + 7\lambda^2)}{2(l + \lambda)^4 (2l + \lambda)^2} \sigma_L^2,
\]

(31)

then there is over-investment in investor relations relative to the benchmark of maximizing firm value. That is, the insider invests \( C > 0 \), whereas dispersed shareholders prefer \( C = 0 \).

Unlike the case in which investor relations meant disclosure of private information, the insider no longer loses speculative profits from investor relations. In fact, because the insider...
has private information which he can speculate on at date 1, he will also want to undertake a non-trivial investment in investor relations to boost noise trades so as to camouflage his trades even absent any liquidity needs. Hence, modeling investor relations in this manner only makes it easier for us to obtain our results.

B Equilibrium without an Outside Speculator

Intuitively, one might suspect that the existence of outside speculators might be driving all of our results since it is easier for the insider to disclose valuable private information because the insider is not the sole beneficiary of this information. Our results do not depend qualitatively on whether there is an outside speculator who has the same information as the insider. To see that this is the case, we report the key results of our model assuming that there is no outside speculator.

In following theorem, we describe the equilibrium without an outside speculator.

Theorem 4 Assume that there is no outside speculator. At date 1, there is a unique Bayesian-Nash linear equilibrium, which is characterized by the following properties.

(i) The noise traders combine to place a market order in the amount of $\tilde{u}$.

(ii) The insider, after observing $\tilde{v}, \tilde{e},$ and $\tilde{\theta}_L$, submits a market sales order in the amount of

$$
\tilde{X}_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L) = \frac{1}{2(l + \lambda)} \left[ \mathbb{E}(\tilde{v}\tilde{s}) - \tilde{v} \right] + \frac{l}{l + \lambda} \tilde{\theta}_L + \frac{l\lambda}{(l + \lambda)(2l + \lambda)} \tilde{\theta}_L. 
$$

(iii) Competitive, risk neutral market makers set the price as

$$
\tilde{P}_1 = \mathbb{E}(\tilde{v}\tilde{s}) - C + \lambda \left[ \tilde{u} - [\tilde{X}_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L) - \tilde{\theta}_L] \right] - \frac{\lambda^2}{2l + \lambda} \tilde{\theta}_L,
$$

where $\lambda$ is given by a positive solution for

$$
\frac{4\lambda(l + \lambda)^2}{2l + \lambda} \left[ \sigma_u^2 + \left( \frac{l}{l + \lambda} \right)^2 \sigma_L^2 \right] = \tilde{\sigma}_v^2.
$$

27
Equation (34) has a unique solution.

Proof: Available upon request.

We next provide the sufficient condition for when the insider will over-invest in investor relations.

Theorem 5 Suppose \( l > 0 \) and \( \theta > 0 \). If

\[
\frac{\theta}{\sigma_v^4} \left[ \frac{l^2}{(l + \lambda_0)^2} \sigma_L^2 + \frac{4l^2 \lambda_0}{(2l + \lambda_0)^2} \bar{\theta}^2 + \frac{1}{4(l + \lambda_0)^2} \sigma_v^2 \right]^{-1} > 0,
\]

(35)

where \( \lambda_0 = \lambda(C = 0) \), then the insider invests \( C > 0 \). Since dispersed shareholders prefer that \( C = 0 \), there is an over-investment in investor relations relative to the benchmark of maximizing firm value.

Proof: Available upon request.

We have numerically solved for the optimal investor relations policy. The results are similar to those described in Section V.D. There are a large range of parameters for which \( C > 0 \). We omit this discussion for brevity.

VII Conclusion

In this paper, we develop a model to study the incentives of insiders to undertake costly investor relations. We find that insiders have a strong incentive to allocate resources to enhance the liquidity of their stock because of potential liquidity needs. In contrast, dispersed shareholders care little about market illiquidity because of their relatively small holdings. Potentially testable implications are developed.

We allow trading in the equity market as the only means for the insider to liquidate his shares. In practice, the insider can try to unload his shares through brokerage houses in private negotiations. That is, a brokerage house tries to find a buyer to take the shares. One
can think of this alternative trading channel as an upstairs market in which block trades are worked through. In practice, the liquidity the upstairs market is tied to liquidity in the downstairs market.

In the case of small or newly public firms, we rule out the possibility that the insider might push to merge the young firm with another company. Indeed, the market illiquidity of insider stakes is a strong motivation for insiders of these firms to push for mergers with more liquid, larger firms. In fact, one can think of investor relations as an important means with which to market the firm to Wall Street in the hopes of attracting merger possibilities. Moreover, the insider has an incentive to sell the firm to an acquiring company at a price much lower than dispersed shareholders would like.

While we have focused our discussions on the effects of insider ownership and market illiquidity on investments in investor relations, we conjecture that these effects may also apply to other types of investments. For instance, imagine that management had to decide between two projects A and B with which to make capital investments. Project A has higher expected returns than B, but there is also more asymmetric information about its cash flows. One can think project A as R&D related in which insiders have lots of private information about outcome and project B as a more transparent project whose outcome is well understood or easily predicted given public information. Assuming that there is no way for management to credibly reveal their private information about project A, our theory suggests that management has strong incentives to choose project B over A even though the expected returns to A are much higher. In other words, large stakes and the illiquidity of these stakes can lead insiders to lean toward more transparent, but less profitable projects. We leave this for future research.
VIII References


IX Appendix: Proofs

Proof of Theorem 1: We look for a Nash equilibrium in which both the insider and the speculator take each other’s strategy as given and optimize. The noise traders submit their random market order. The market makers set the price to be the expected value of the time-2 price of the firm conditioning on the information they have at \( t = 1 \).

At \( t = 1 \), the speculator observes \( \tilde{v} + \tilde{e} \), and the insider observes \( \tilde{v}, \tilde{e}, \tilde{\theta}_L \). Let \( \tilde{X}_s(\tilde{v}, \tilde{e}) \) denote the speculator’s equilibrium market buy order, and let \( \tilde{X}_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L) \) denote the insider’s equilibrium market sell order.

The market maker observes \( \tilde{s} \equiv \tilde{v} + \tilde{e} \) and \( \omega \equiv \tilde{u} + \tilde{X}_s - \tilde{X}_i \). He sets the time-1 price to be

\[
\tilde{P}_1(\tilde{s}, \tilde{\omega}) = E[\tilde{v} - C|\tilde{s}, \tilde{\omega}] .
\]

(36)

We conjecture that the result of this price setting procedure leads to the following linear functional form for the price:

\[
\tilde{P}_1(\tilde{s}, \tilde{\omega}) = \gamma + \alpha \tilde{s} + \lambda \tilde{\omega}
\]

\[= \gamma + \alpha (\tilde{v} + \tilde{e}) + \lambda \left[ \tilde{u} + \tilde{X}_s(\tilde{v}, \tilde{e}) - \tilde{X}_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L) \right],
\]

(37)

and we will later confirm this conjecture.

Under the linear price rule and given the insider’s strategy, the speculator’s strategy \( \tilde{X}_s(\tilde{v}, \tilde{e}) \) should solve

\[\max_x E_{\tilde{v}, \tilde{e}} \left[ x \left( (\tilde{v} - C) - \tilde{P}_1(\tilde{v} + \tilde{e}) \tilde{u} + x - \tilde{X}_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L) \right) \right],\]

where \( E_{\tilde{v}, \tilde{e}} \) denotes taking the expectation conditioning on observing \( \tilde{v} \) and \( \tilde{e} \). Solving this problem yields

\[
\tilde{X}_s(\tilde{v}, \tilde{e}) = \frac{1}{2\lambda} \left[ (\tilde{v} - C) - \left[ \gamma + \alpha (\tilde{v} + \tilde{e}) - \lambda E_{\tilde{v}, \tilde{e}} [\tilde{X}_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L)] \right] \right].
\]

(38)
For the insider, under the linear price rule and given the speculator’s strategy, his market sell order $\tilde{X}_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L)$ should solve

$$\max_{x} \mathbb{E}_{\tilde{v}, \tilde{e}, \tilde{\theta}_L} \left[ -l(x - \tilde{\theta}_L)^2 + x \left( \tilde{P}_1(\tilde{v} + \tilde{e}, \tilde{u} + \tilde{X}_s(\tilde{v}, \tilde{e}) - x) - (\tilde{v} - C) \right) + \theta(\tilde{v} - C) \right],$$

which yields

$$\tilde{X}_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L) = \frac{1}{2(l + \lambda)} \left[ 2l\tilde{\theta}_L + \left( \gamma + \alpha(\tilde{v} + \tilde{e}) + \lambda\tilde{X}_s(\tilde{v}, \tilde{e}) \right) - (\tilde{v} - C) \right]. \tag{39}$$

Solving (38) and (39) simultaneously gives the solution of the Nash equilibrium:

$$\tilde{X}_i(\tilde{v}, \tilde{e}) = \frac{2l + \lambda}{\lambda(4l + 3\lambda)} \left[ (\tilde{v} - C) - \left( \gamma + \alpha(\tilde{v} + \tilde{e}) \right) \right] + \frac{2l}{l + \lambda} \tilde{\theta}_L \tag{40}$$

$$\tilde{X}_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L) = \frac{1}{4l + 3\lambda} \left[ \left( \gamma + \alpha(\tilde{v} + \tilde{e}) \right) - (\tilde{v} - C) \right] + \frac{l}{l + \lambda} \tilde{\theta}_L + \frac{l(2l + \lambda)}{(l + \lambda)(4l + 3\lambda)} \tilde{\theta}_L. \tag{41}$$

Given these strategies by the speculator and the insider, the market maker sets price $\tilde{P}_1$ according to (36). From (40) and (41), we have

$$\tilde{u} + \tilde{X}_s - \tilde{X}_i = \tilde{u} + \frac{2(l + \lambda)}{\lambda(4l + 3\lambda)} \left[ (\tilde{v} - C) - \left( \gamma + \alpha(\tilde{v} + \tilde{e}) \right) \right] - \frac{l}{l + \lambda} \tilde{\theta}_L + \frac{l(2l + \lambda)}{(l + \lambda)(4l + 3\lambda)} \tilde{\theta}_L.$$

So the market maker should set the price to be

$$\tilde{P}_1 = \mathbb{E} \left[ \tilde{v} - C \mid \tilde{v} + \tilde{e}, \tilde{u} + \tilde{X}_s - \tilde{X}_i \right] = \mathbb{E} \left[ \tilde{u} \left| \tilde{v} + \tilde{e}, \tilde{v} + \frac{\lambda(4l + 3\lambda)}{2(l + \lambda)} \tilde{u} - \frac{l\lambda(4l + 3\lambda)}{2(l + \lambda)^2} (\tilde{\theta}_L - \tilde{\theta}_L) \right] - C$$

$$= \frac{\tilde{u}}{\sigma^2_e} + \frac{\tilde{e} + \tilde{e}}{\sigma^2_e} + \frac{1}{\Sigma^2} \left[ \tilde{u} + \frac{\lambda(4l + 3\lambda)}{2(l + \lambda)^2} \tilde{u} - \frac{l\lambda(4l + 3\lambda)}{2(l + \lambda)^2} (\tilde{\theta}_L - \tilde{\theta}_L) \right] - C \tag{42}$$

where

$$\Sigma^2 = \left[ \frac{\lambda(4l + 3\lambda)}{2(l + \lambda)} \right]^2 \sigma^2_u + \left[ \frac{l\lambda(4l + 3\lambda)}{2(l + \lambda)^2} \right]^2 \sigma^2_{\theta_L}. \tag{43}$$

With some algebra, one can show that the price function in (42) is indeed of the linear form conjectured in (37). By comparing the two equations and matching the linearity coefficients, we obtain:

$$\gamma = -C + \frac{\sigma^2_u}{\sigma^2_e} \tilde{v} + \frac{2l\lambda}{2l + \lambda} \tilde{\theta}_L \tag{44}$$
\[ \alpha = \frac{\hat{\sigma}_v^2}{\sigma_e^2} \]  

(45)

where

\[ \hat{\sigma}_v^2 \equiv \text{var}(\tilde{v}|\tilde{v} + \tilde{e}) = \frac{\sigma_v^2 \sigma_e^2}{\sigma_v^2 + \sigma_e^2}, \]

and \( \lambda \) is given by

\[
\frac{\lambda^2(4l + 3\lambda)^2}{2(l + \lambda)(2l + \lambda)} \left[ \sigma_u^2 + \left( \frac{l}{l + \lambda} \right)^2 \sigma_L^2 \right] = \hat{\sigma}_v^2. \quad (46)
\]

With the linear pricing function solved, and using

\[ E(\tilde{v}|\tilde{v} + \tilde{e}) = \frac{\hat{\sigma}_v^2 \tilde{v} + \hat{\sigma}_e^2(\tilde{v} + \tilde{e})}{\sigma_v^2}, \]

we show that the equilibrium trading strategies for the speculator and the insider are:

\[
\tilde{X}_s(\tilde{v}, \tilde{e}) = \frac{2l + \lambda}{\lambda(4l + 3\lambda)} \left[ \tilde{v} - E(\tilde{v}|\tilde{s}) \right],
\]

\[
\tilde{X}_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L) = \frac{1}{4l + 3\lambda} \left[ E(\tilde{v}|\tilde{s}) - \tilde{v} \right] + \frac{l}{l + \lambda} \tilde{\theta}_L + \frac{\lambda}{(l + \lambda)(2l + \lambda)} \tilde{\theta}_L,
\]

and the market maker’s pricing function can be rewritten as

\[
\tilde{P}_1 = E(\tilde{v}|\tilde{s}) - C + \lambda \left[ \tilde{u} + \tilde{X}_s(\tilde{v}, \tilde{e}) - [\tilde{X}_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L) - \tilde{\theta}_L] \right] - \frac{\lambda^2}{2l + \lambda} \tilde{\theta}_L.
\]

So the conjectured equilibrium is verified.

Finally, we prove the existence and uniqueness of such an equilibrium. To do so, we need to show that Equation (46) has a unique solution. Define the left hand side of (46) as

\[
B(\lambda) \equiv \frac{\lambda^2(4l + 3\lambda)^2}{2(l + \lambda)(2l + \lambda)} \left[ \sigma_u^2 + \left( \frac{l}{l + \lambda} \right)^2 \sigma_L^2 \right]. \quad (47)
\]

Since \( B(0) = 0, B(\infty) = \infty \), and

\[
B'(\lambda) = A(\lambda, \sigma_u) = \frac{\lambda(4l + 3\lambda)(16l^3 + 36l^2 \lambda + 27l \lambda^2 + 6 \lambda^3)}{2(l + \lambda)^2(2l + \lambda)^2} \sigma_u^2
\]

\[
+ \frac{l^3 \lambda(4l + 3\lambda)(16l^2 + 20l \lambda + 7 \lambda^2)}{2(l + \lambda)^4(2l + \lambda)^2} \sigma_L^2
\]

\[ > 0, \quad (48) \]

\[ > 0, \quad (49) \]

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Equation (46) indeed has a unique solution. So Theorem 1 is proved. QED

**Proof of Lemma 1:** For given $l$ and $\sigma_L$, taking the total differential of Equation (14), we have

$$A(l, \lambda, \sigma_u) d\lambda + \frac{\lambda^2(4l + 3\lambda)^2}{2(l + \lambda)(2l + \lambda)} d\sigma_u^2 = d\sigma_v^2,$$

(50)

from which we have

$$d\lambda = \frac{1}{A(l, \lambda, \sigma_u)} \left[ -\frac{\lambda^2(4l + 3\lambda)^2}{2(l + \lambda)(2l + \lambda)} d\sigma_u^2 + d\sigma_v^2 \right],$$

(51)

which shows that $\lambda$ is decreasing in $1/\sigma_v^2$ and $\sigma_u^2$. QED

**Proof of Lemma 2:** From Theorem 1, we have

$$\tilde{X}_i - \tilde{\theta}_L = \frac{1}{4l + 3\lambda} [E(\tilde{v}|\tilde{s}) - \tilde{v}] - \frac{\lambda}{l + \lambda} (\tilde{\theta}_L - \tilde{\theta}_L) - \frac{\lambda}{2l + \lambda} \tilde{\theta}_L,$$

and

$$E \left[ (\tilde{X}_i - \tilde{\theta}_L)^2 \right] = \frac{1}{(4l + 3\lambda)^2} \tilde{\sigma}_v^2 + \frac{\lambda^2}{(l + \lambda)^2} \sigma_L^2 + \frac{\lambda^2}{(2l + \lambda)^2} \tilde{\theta}_L^2.$$

(52)

We also have

$$\tilde{P}_1 - \tilde{P}_2 = \frac{2l + \lambda}{4l + 3\lambda} [E(\tilde{v}|\tilde{s}) - \tilde{v}] + \lambda \tilde{u} - \frac{l\lambda}{l + \lambda} (\tilde{\theta}_L - \tilde{\theta}_L),$$

and

$$E \left[ \tilde{X}_i(P_1 - P_2) \right] = \frac{2l + \lambda}{(4l + 3\lambda)^2} \tilde{\sigma}_v^2 - \frac{l\lambda^2}{(l + \lambda)^2} \sigma_L^2.$$

(53)

From (52) and (53), we finally have

$$I(C) = E \left[ -l(\tilde{X}_i - \tilde{\theta}_L)^2 + \tilde{X}_i(\tilde{P}_1 - \tilde{P}_2) + \theta \tilde{P}_2 \right]$$

$$= \theta(\tilde{v} - C) + \frac{l + \lambda}{(4l + 3\lambda)^2} \tilde{\sigma}_v^2 - \frac{l\lambda^2}{(2l + \lambda)^2} \tilde{\theta}_L^2 - \frac{l\lambda}{l + \lambda} \sigma_L^2.$$

QED

**Proof of Theorem 2:** Define

$$G(\tilde{v}, \sigma_v^2, \sigma_u^2, l, \theta, K, M) \equiv \frac{\theta}{\sigma_v^4} \frac{1}{A(l, \lambda_0, \sigma_u)} \left[ \frac{l^2}{(l + \lambda_0)^2} \sigma_L^2 + \frac{1}{(2l + \lambda_0)^2} \tilde{\theta}_L^2 + \frac{2l + 3\lambda_0}{(4l + 3\lambda_0)^2} \sigma_v^2 \right] - \frac{l + \lambda_0}{(4l + 3\lambda_0)^2},$$

(54)
where \( \lambda_0 \) is the solution of (14) for \( C = 0 \), and \( A(l, \lambda, \sigma_u) \) is defined in (48). We have

\[
I'(0) = -\theta - \left[ \frac{l^2}{(l + \lambda_0)^2} \sigma_L^2 + \frac{4l^2 \lambda_0}{(2l + \lambda_0)^3} \theta_L^2 + \frac{2l + 3\lambda_0}{(4l + 3\lambda_0)^3} \sigma_v^2 \right] \frac{d\lambda}{dC} \bigg|_{C=0} - \sigma_v^4 \frac{l + \lambda_0}{(4l + 3\lambda_0)^2} f'(0).
\]

Using (51), we have

\[
\frac{d\lambda}{dC} \bigg|_{C=0} = -\frac{\sigma_v^4}{A(l, \lambda_0, \sigma_u)} f'(0).
\]

Putting (56) back into (55), we have

\[
I'(0) = \theta \left[ \frac{f'(0)}{G(\bar{v}, \sigma_v^2, \sigma_u^2, l, \theta, K, M)} - 1 \right].
\]

So if the technical condition specified in the theorem holds, then \( I'(0) > 0 \). This means that the optimal choice of \( C \) for the insider must be positive. QED

**Proof of Proposition 1:** For the case of \( l = \infty \), we have, from (14), \( \lambda = \frac{\bar{v}}{2\sqrt{\sigma_u^2 + \sigma_L^2}} \), and

\[
I(C) = \theta (\bar{v} - C) - \lambda \sigma_L^2
= \theta (\bar{v} - C) - \frac{\sigma_L^2}{2\sqrt{\sigma_u^2 + \sigma_L^2}} \frac{1}{\sqrt{\frac{1}{\sigma_u^2} + \alpha_e C}}.
\]

Taking derivatives, we have

\[
I'(C) = -\theta + \frac{\sigma_L^2}{4\sqrt{\sigma_u^2 + \sigma_L^2}} \frac{\alpha_e}{(\frac{1}{\sigma_u^2} + \alpha_e C)^{3/2}}
\]

\[
I''(C) = -\frac{3\sigma_L^2}{8\sqrt{\sigma_u^2 + \sigma_L^2}} \frac{\alpha_e^2}{(\frac{1}{\sigma_u^2} + \alpha_e C)^{5/2}} < 0.
\]

From (58) and the constraint that \( C^* \geq 0 \), we know that the optimal cost \( C^* \) is given by the maximum of zero and the solution to \( I'(C) = 0 \). Solving \( I'(C) = 0 \) and applying the constraint of \( C \geq 0 \), we have

\[
C^* = \text{Max} \left\{ \frac{K^{4/3} \theta^{2/3}}{2^{4/3} \alpha_e^{1/3} (\sigma_u^2 + K^2 \theta^2)^{1/3}} - \frac{1}{\alpha_e \sigma_v^2}, 0 \right\},
\]
and, given that $d\lambda/dC < 0$ (from (51)), the equilibrium $\lambda$ is given by

$$\lambda^* = \text{Min} \left\{ \frac{\theta}{2\alpha_e\sigma_v^2(\sigma_u^2 + \sigma_L^2)} \right\}^{1/3}, \frac{\sigma_v}{2\sqrt{\sigma_u^2 + \sigma_L^2}} \right\}.$$ 

QED

**Proof of Theorem 3:** From (51), we have

$$\frac{d\lambda}{dC} \bigg|_{C=0} = \frac{1}{A(l, \lambda_0, \sigma_u)} \frac{\lambda^2(4l + 3\lambda)^2}{2(l + \lambda)(2l + \lambda)g'(0)}, \quad (59)$$

from which we have

$$I'(0) = -\theta - \left[ \frac{l^2}{(l + \lambda_0)^2\sigma_L^2} + \frac{4l^2\lambda_0}{(2l + \lambda_0)^3\theta_L^2} + \frac{2l + 3\lambda_0}{(4l + 3\lambda_0)^3\sigma_v^2} \right] \frac{d\lambda}{dC} \bigg|_{C=0}$$

$$= -\theta + \left[ \frac{l^2}{(l + \lambda_0)^2\sigma_L^2} + \frac{4l^2\lambda_0}{(2l + \lambda_0)^3\theta_L^2} + \frac{2l + 3\lambda_0}{(4l + 3\lambda_0)^3\sigma_v^2} \right] \frac{1}{A(l, \lambda_0, \sigma_u)} \frac{\lambda^2(4l + 3\lambda)^2}{2(l + \lambda)(2l + \lambda)g'(0)}.$$ 

So if (30) holds, then $I'(0) > 0$, which means that the optimal choice of $C$ for the insider must be positive. QED