Competition and Strategic Information Acquisition in Credit Markets*

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Abstract

Regulatory changes and technological advances have profoundly affected the competitive landscape of credit markets. In this paper, we investigate how intermediaries alter their information acquisition strategies in response to increased competition. We specify a model where the severity of asymmetric information between banks and borrowers increases with their informational distance. As the number of active banks grows, investments in information acquisition initially fall with returns to informed intermediation. However, when a critical investment threshold is reached, further entry leads to specialization. Intermediaries optimally refocus informational resources in their core markets by retrenching from peripheral segments to fend off competitive threats to their captive customer base. The incentive to concentrate informational resources increases in the degree of adverse selection in the market.
1 Introduction

The recent evolution of financial intermediation has been characterized by rapidly increasing competition, both from traditional lenders such as commercial banks as well as from non-bank institutions that have expanded into credit markets. Indeed, one of the most cited driving forces behind the current consolidation in banking are the pressures exerted by competing products and services. Much of this increased competition is the result of recent deregulation in the banking industry, such as the relaxation of restrictions on inter-state banking in the US or the Single Market in financial services in Europe. Technological advances such as automated credit scoring and other improvements in banks’ data processing capabilities have also allowed banks to enter and compete in markets in which they previously had no presence. In this paper, we seek to clarify how banks respond to increasing competitive pressures in terms of their investments in information acquisition and their allocation of resources between core and peripheral markets.

Recent literature has argued that financial intermediaries arise from the need to overcome the consequences of informational asymmetries between lenders and borrowers (see, e.g., Diamond, 1984; Ramakrishnan and Thakor, 1984; or Allen, 1990). However, information about borrower quality is often relation-specific so that certain banks enjoy an information-induced competitive advantage (see Rajan, 1992; Sharpe, 1990; or von Thadden, 1998). While competition for borrowers tends to erode informational rents and relationship value, it is hindered by an adverse selection problem which is at the root of the monopolistic nature of financial intermediation. These opposing effects beg the question of how increased competition affects the nature of financial intermediation.

In our model, banks compete for borrowers by entering a loan market and investing in an information acquisition technology that generates borrower-specific information. This screening technology consists of two components: specialized expertise and broad-based screening ability. Banks then decide whether to acquire information about borrowers who are located around them. In order to capture the varying degrees of informational expertise present in modern banking, we take the success of the information generation process to be a function of the distance between bank and borrower. The closer a borrower is located to a bank in informational space, the more informative the credit screen becomes and the better a bank can assess a borrower’s credit worthiness. Given
the location-dependent screening and information acquisition choice, banks make loan offers to applicant borrowers who then choose the bank with the best quote.

We find that, when loan markets are not too competitive, banks will choose to invest resources in obtaining borrower-specific information only for a subset of their clientele. For their remaining customers, banks choose to lend without the benefit of information beyond what is publicly available. Interest rate offers decrease in the informational distance between informed bank and screened customer but increase in the distance between borrower and competitor banks. However, as competitive pressures increase, banks respond by using information acquisition strategically. In doing so, banks face two counter-vailing forces: while more competition, by reducing their rents, decreases intermediaries’ overall incentives to become informed, it may also induce them to change the scope of their information acquisition.

Our main result is that, as competition increases, banks optimally shift resources away from peripheral markets to focus on their core segment by cutting back on broad-based screening investment in favor of specialized expertise. In essence, banks respond to increased competition by specializing and using their superior knowledge about captive borrowers to limit outside competition. To the extent that more competition leads to more precise loan screening in a bank’s core markets, customers in that bank’s field of expertise also benefit through more efficient lending decisions. We also show that the degree to which banks choose to refocus their information-acquisition strategies and concentrate on borrowers closer to their core expertise depends on the nature of information asymmetries about borrowers. When adverse selection problems are large because, for example, borrowers are very heterogeneous, banks’ profits are very sensitive to increases in proprietary information and the incentive to refocus on their core segment becomes stronger.

Casting varying degrees of lending expertise in terms of differentiated information acquisition allows us to explicitly derive lending rents from informational asymmetries. Consequently, we can show how increased competition affects such rents and leads to a strategic refocusing of informational resources and sector specialization. In contrast to traditional models of financial markets with product differentiation à la Salop (1979), the locational differentiation affects the bank (supplier) through information decay and not its customers (borrowers) through transportation costs. This

\[1^1_{\text{See, for instance, Chiappori, Perez-Castrillo, and Verdier (1995).}}\]
innovation provides a natural setting for the study of strategic information acquisition, as there is no reason to assume that banks have equal access to information *ex ante*. One possible interpretation of our framework is that banks build relationships with customers, and use these relationships to acquire private information about borrowers. Hence, one of our contributions lies in the development of a tractable model for the analysis of issues related to competition and information in financial intermediation, such as the impact of entry on lending decisions.

Our main contribution, however, is to characterize the interdependence of banks’ information acquisition strategies and the industry’s competitive structure. In our analysis, the threat of adverse selection serves to limit not only competition for individual banks’ core markets but also entry into the industry. Hence, banks’ strategic response to increased competition shapes the industrial structure of banking. While related effects have been identified in the literature (see, e.g., Dell’Ariccia, 2001), to our knowledge the feedback between competition and banks’ incentives to invest in information acquisition has not been studied. Our paper attempts to fill this void by providing an analysis of competition’s impact on overall credit market equilibria in the presence of asymmetric information.

Differential information among intermediaries induces specialization in lending, a feature of financial intermediation stressed, among others, by Gehrig (1998), who focuses on banks’ incentives to invest in the production of information. However, he does not analyze the allocation of resources between different loan market segments nor endogenize the degree of competition and industry structure. Winton (1999) argues that banks may have an incentive to specialize if diversification were to lower their incentive to monitor borrowers and potentially increase their probability of failure. While our model focuses on *ex ante* screening, it also suggests that specialization may be an optimal response to changes in a bank’s environment. Closest to our work, Almazan (2002) analyzes a model in which a bank’s monitoring expertise is a decreasing function of the distance between borrower and bank. His focus, however, is on the role of bank capital as a way of providing incentives to monitor, and not on the generation of rents through information or on the organization of the industry.

Similar in spirit to our work, Sharpe (1990) studies borrowing under adverse selection, and shows that if relationship building generates inside information, competition in the refinancing
stage will be constrained. Rajan (1992) emphasizes the disadvantages of informed (relationship) vs. uninformed (“arm’s length”) debt, but does not explore how the distribution of information affects loan market equilibria and competition. In the context of credit worthiness tests, Broecker (1990) and Riordan (1993) both investigate the effects of competition on loan markets under independent loan screening, but do not consider the incentives to invest in acquiring information. Dell’Ariccia (2001) shows that information can serve to “carve out” a market between competitor banks and to block entry. He does not, however, endogenize the generation of this information, an issue that is central to this paper.

This paper is also related to the literature on banking relationships and, in particular, to recent work on how increased credit market competition changes the incentives to forge such ties. Our analysis complements and contrasts with the results of Boot and Thakor (2000) and Dinc (2000). In Boot and Thakor (2000), increased capital market competition lowers ex ante bank profits and reduces entry into banking. Less competition among commercial banks leads to greater investment in relationship building expertise, a result similar to ours but that arises for very different reasons. In our model, competition intensifies precisely because of increased entry into banking. With more banks and increased competitive pressure, there is an incentive to refocus resources and refine the bank’s expertise at the expense of peripheral markets. This strategic concentration of resources is not studied in Boot and Thakor. Moreover, our model allows us to analyze how this reallocation of resources depends on economic factors relating, for example, to the characteristics of borrowers. Dinc (2000) primarily focuses on how competition may increase the incentives for banks to maintain their good reputation by honoring loan commitments. We do not address bank reputation per se, but instead analyze rent generation in lending relationships and the equilibrium market structure.

From an empirical perspective, Petersen and Rajan (2001) find that geographical distance seems to be a good, though over time declining, indicator of the strength of a lending relationship. At the same time, they also find that competition in lending markets seems to have increased. In their study of banks’ choice between focus and diversification, Acharya, Hasan and Saunders (2001) find

\[\text{\textsuperscript{2}}\]

Boot and Thakor (2000) find that an increase in competition among banks lowers the incentive to specialize, which in turn lowers the value of relationship lending to borrowers. This occurs because they keep the size of a bank’s loan portfolio fixed as competition grows. In our model, banks’ market shares shrink as more competitors enter the market. This initially leads to a similar reduced incentive to invest resources in relationship building, but eventually forces banks to refocus fixed resources towards more specialized uses.
strong evidence that a bank’s informational effectiveness is lower in newly entered and competitive sectors. As a result, broad-based lending leads to lower loan quality which, in light of our results, could be a consequence of the higher threat of adverse selection in such markets. Finally, Degryse and Ongena (2001) present evidence that is consistent with models of differentiated loan pricing, such as ours. In particular, they find that loan rates are decreasing in the distance from lending bank to borrower but decreasing in the distance of borrower to competitor bank, as predicted by our analysis.

The paper is organized as follows. The next section presents a formal model of financial intermediation and information acquisition in the context of a locationally differentiated credit market. Section 3 derives the lending equilibrium between two neighboring and competing banks. In Section 4, we calculate the overall equilibrium for a given number of active intermediaries and study how increased competition affects their screening incentives. Section 5 analyzes the intermediation game’s free-entry equilibrium and characterizes banks’ strategic choice of informational investments and loan specialization as a function of characteristics of the market. Section 6 discusses implications of our results and concludes. Proofs are mostly relegated to the Appendix.

2 A Model of Locationally Differentiated Credit Assessment

Let a continuum of borrowers with mass $M > 1$ be uniformly distributed along a circle with circumference 1. Each potential borrower has an investment project that requires an initial outlay of $1 and generates a terminal cash flow $\xi$. This cash flow $\xi$ can be an amount $R$ with probability $p_\theta$ and 0 with probability $1 - p_\theta$, where $\theta \in \{l, h\}$ denotes the firm’s type (low or high quality). We assume that the success probability for the firm with the better investment opportunity is higher: $p_h > p_l$. Final cash flows are observable and contractible, but project type $\theta$ is unknown to either borrower or lender. The likelihood of finding a high-quality firm $h$ is $q$ and this distribution of borrower types is common knowledge. We also assume that borrowers have no private resources, and that $p_l R < 1 < p_h R$, so that it is efficient to finance good borrowers but not bad ones. Moreover, letting $\bar{p} \equiv q p_h + (1 - q) p_l$ denote the average success probability, we assume that $\bar{p} R > 1$, so that

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3 $M$ represents a scaling factor that guarantees that at least two banks are active in equilibrium.

4 Alternatively, we could assume that there are no self-selection or sorting devices such as collateral available because, for example, the borrower is wealth-constrained and the project can not serve as collateral.
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Figure 1: Banking under locationally differentiated asymmetric information

it is *ex ante* efficient to grant a loan.

$N$ banks compete for these borrowers in three stages. First, banks decide whether or not to enter the loan market and how to invest in a screening technology that generates borrower-specific information. We assume that if they enter, they will locate equi-distantly around the circle. On the basis of their investment, banks next decide whether or not to screen a loan applicant. Borrower screening leads to better credit assessments that provide banks with an informative signal $\eta \in \{l, h\}$ about a borrower’s type. However, such assessments are not perfect and depend on the distance of the borrower to the screening bank, denoted by $x$. Conditional on entry and any borrower-specific information, banks compete in the third stage by simultaneously making interest rate offers. These offers can depend on the informational distance between bank and borrower $x$. Borrowers choose last by accepting a loan from the bank quoting the lowest rate. Figure 1 summarizes the time structure of the intermediation game.

Information acquisition requires costly investments along two different dimensions. Banks must decide how much to invest in a core competency $I$, which gives them specific expertise in a financial product, borrower group (e.g., industry), or geographic market. This investment might correspond to the number of loan officers in a geographic region or financial analysts covering a particular industry. Banks also have to decide how much to invest in order to extend this expertise to other market segments ($\alpha$) by, for instance, opening new branches in previously unserved markets. The investment $\alpha$ can be alternatively thought of as the “breadth” of the bank’s expertise in screening, or as the inverse of the rate of decay of its lending expertise outside its core market. In choosing their investments, banks therefore engage in a trade-off between their degree of specialization and their breadth across different market segments. This specification captures the idea that banks enjoy an informational advantage in the market segments in which they specialize, which would
occur, for instance, if there are positive externalities in information processing across locations. The more they move outside their core competencies and the less they invest in extending their original franchise to new markets, the more severe become the informational problems they face.\textsuperscript{5}

To model these effects, we assume that credit assessments yield a signal \( \eta \in \{l, h\} \) at cost \( c \) whose quality depends on the borrower’s distance \( x \) to the bank. The following distributional assumptions capture the idea of locationally differentiated asymmetric information:\textsuperscript{6}

\[
\begin{align*}
\Pr_x(\eta = h | \theta = h) &= \phi(x) = \Pr_x(\eta = l | \theta = l) \\
\Pr_x(\eta = h | \theta = l) &= 1 - \phi(x) = \Pr_x(\eta = l | \theta = h)
\end{align*}
\] (1)

where the probability of successful screening \( \phi(x) \) decreases with bank-borrower distance.

We also assume that credit screens are informationally valuable so that \( \phi(x) \geq \frac{1}{2} \). For concreteness, suppose that for \( \alpha \geq 0 \) and \( I \in (0, 1) \) credit assessments have a location-dependent success rate of

\[
\phi(x) = \frac{1 + I}{2} - \frac{x}{\alpha}, \quad x \in \left[0, \frac{\alpha I}{2}\right],
\] (2)

and that \( \phi(x) = \frac{1}{2} \) for \( x > \frac{\alpha I}{2} \), so that banks are better at screening borrowers located close to them. By the parameter restrictions, all probabilities are well defined in equation (1).

The cost of the screening technology comprises a variable component \( C(\alpha, I) \) that depends on the specialized expertise investment \( I \) and broad-based skills investment \( \alpha \), as well as a fixed component \( T \). The latter reflects the cost of installing and maintaining the resources needed to process information or any possible regulatory cost to entering the loan market. This technology, therefore, requires an up-front total outlay of \( K(\alpha, I) \) which is increasing and convex in the investments \( I \) and \( \alpha \):

\[
K(\alpha, I) = C(\alpha, I) + T = \frac{\alpha^2}{2} + \frac{I^2}{2} + T, \quad \alpha \geq 0, \ I \in (0, 1).
\] (3)

Moreover, we assume that a minimal investment \( \bar{C} > 0 \) in \( I \) and \( \alpha \) is required for the bank

\textsuperscript{5}Acharya et al. (2001) provide evidence that higher levels of risk and expansion into new markets or industries erode banks’ informational capabilities such as monitoring.

\textsuperscript{6}All probabilities are a function of informational distance \( x \) which we suppress in the interest of notational clarity.
to effectively carry out credit assessments and to properly interpret gathered data. Since credit assessments are only informative for $\phi > \frac{1}{2}$, this assumption insures that, at a minimum, banks become informed about borrowers at their immediate vicinity. Hence, it is this minimum scale investment that defines informed intermediation in our framework. In the context of relationship lending, it would identify a relationship-building bank.

If more than one bank try to screen the same borrower, we assume that the banks located farther away can only gather information about the closer bank’s borrower-specific knowledge. For example, consider a borrower located closer to bank $n$ who is also approached by bank $n + 1$ at the screening stage. In this case, bank $n + 1$, located further away, only observes a noisy signal on the outcome of bank $n$’s credit assessment. This assumption captures the fact that information accrues over time and is only partially observable to outsiders. The bank that has the most expertise in this market segment (closest in informational distance $x$) can use this information to fend off competition by other banks. As a result, it will know at least as much as the less informed bank, which is precisely our noisy signal assumption.\(^7\)

Our specification of credit screening allows us to think of bank lending as either relationship-driven (the bank has borrower-specific information) or transaction-driven (lending without screening). Costly screening serves as a metaphor for the time, effort and resources that it takes to build lending relationships, and for the losses that a bank might incur during this period. We could equally well assume that banks first make a loan to a borrower, and learn some information about that borrower in the course of granting and maintaining the loan at cost $c$.\(^8\) The focus would then be on the competition for borrowers once (some) relationships have already been established. In this context, screening permits a bank to insure a borrower against random shocks to profitability, so that even borrowers who have had bad outcomes in the past may continue to be financed if they are known (by their inside bank) to have positive NPV projects.\(^9\) While there may be other

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\(^7\)Specifically, we assume that the information collected by the closer bank is a sufficient statistic for any signals observed by banks further away. We make this assumption so that we can model competition for borrowers as that between one informed bidder and $N − 1$ uninformed bidders. Note that our specification implies that the aggregate amount of information about a borrower remains constant as a function of the number of screening banks.

\(^8\)James (1987) and Lummer and McConnell (1989) present evidence suggesting that banks have access to private information over the course of the lending relationship.

\(^9\)It is in this sense that relationships (screening) are good for both borrowers and banks. To the extent that this insurance is valuable, relationship lending, or lending with screening, can be of value to a borrower even if it comes at the price of being somewhat locked in to their lending bank. See Petersen and Rajan (1995) for evidence to this effect.
aspects to long-term lending relationships and information acquisition, our focus is consistent with much of the recent literature (e.g., Sharpe, 1990; Rajan, 1992; or von Thadden, 1998).

Our setup is quite different from the usual treatment of competition with differentiated products, e.g., Salop (1979). In particular, such models assume that consumers (borrowers) have preferences for some suppliers (banks) over others, and so would naturally prefer to purchase (borrow) from one particular source, all other things equal.\textsuperscript{10} Here, we place no such restriction on borrowers’ preferences, and merely use the circle to model banks’ expertise in evaluating the creditworthiness of some borrowers better than others. In particular, products are not differentiated \textit{per se} so that borrowers just choose the least costly loan on offer. Instead, banks, by investing in information acquisition, add value to a lending relationship and generate rents to differing degrees depending on the proximity of the borrower.

Finally, it should be noted that we can also interpret $\phi$ as the \textit{relative} information advantage of the screening bank over other possible lenders rather than the \textit{absolute} amount of information a particular lender has about a borrower. In this context, the distance $x$ between borrower and bank on the circle admits new interpretations. We could let distance represent the duration of a lending relationship by taking $x = x(t), x' < 0$ where $t$ represent the length of time a particular firm has interacted with a single lender. Similarly, $x$ might be thought of as the age or size of firms. There usually exists more publicly available information about more established and bigger firms. In this case, firms located farther away are the ones for which their “screening” or “relationship” banks have little information advantage relative to other lenders precisely because the larger amount of publicly available information renders the firms more informationally transparent.

\section{Lending Competition}

As a preliminary step, we derive a potential borrower’s success probability in light of the bank’s screening outcome. By Bayes’ rule, the probability of a project being of high or low quality given

\textsuperscript{10}This outcome usually results from imposing a transportation cost which is proportional to the distance between consumer and supplier. Since the cost of transportation is increasing in the distance between the consumer and the supplier, consumers have a tendency to purchase from suppliers that are nearer.
a credit assessment of $\eta = h$ or $\eta = l$ is

$$\Pr(\theta = h|\eta = h) = \frac{\phi(x)q}{\phi(x)q + (1 - \phi(x))(1 - q)} \equiv H(x)$$

$$\Pr(\theta = l|\eta = l) = \frac{\phi(x)(1 - q)}{\phi(x)(1 - q) + (1 - \phi(x))q} \equiv L(x)$$

Note that location dependent screening success implies $H_x \equiv \frac{\partial}{\partial x} H(x) < 0$ and $L_x \equiv \frac{\partial}{\partial x} L(x) < 0$: the posterior distributions on borrower type $\theta$ deteriorate in informational distance. We obtain the project’s success probability conditional on having screened a borrower at location $x$ as

$$p(h; x) \equiv \Pr(\xi = R|\eta = h) = H(x)p_h + (1 - H(x))p_l \in [p, p_h]$$

$$p(l; x) \equiv \Pr(\xi = R|\eta = l) = (1 - L(x))p_h + L(x)p_l \in [p_l, \bar{p}]$$

for credit assessment outcomes $\eta = h, l$. Note that the probability of the project being successful after a positive credit assessment ($\eta = h$) decreases for borrowers located further away. Similarly, the probability of the project being successful after observing a negative signal from the credit assessment increases with the distance between bank and borrower. As the signal becomes less informative a bank should be less likely to believe that the borrower is in fact of low quality.

We now derive the equilibrium in the lending game, for which the appropriate solution concept is Perfect Bayesian Nash Equilibrium. Starting with its last stage, each bank competes for borrowers with all competitor banks that entered the market. However, a standard feature of this class of models is that the equilibrium can be fully characterized by assuming that each bank competes only with its nearest neighbor on either side.\textsuperscript{11} Hence, it suffices to study competition in the last stage for the case of two adjacent banks, one informed and one uninformed, vying for the business of borrowers located between them. We verify in the next section that this approach is indeed the appropriate, as each borrower will be screened by at most one bank. By symmetry, both banks will be informed about some borrowers and uninformed about others so that we arbitrarily label one intermediary $i$ for informed and the other one $u$ for uninformed.

As has been demonstrated in similar contexts (see, e.g., Broecker, 1990 or von Thadden, 1998),

\textsuperscript{11}For further details see, e.g., Engelbrecht-Wiggans et al. (1983).
the interest rate game between two neighboring banks does not have an equilibrium in pure strategies when one bank has superior information.\footnote{This result is well known from models of competition with asymmetric information and from much of the literature on the theory of auctions. An alternative formulation would be to assume that banks make offers sequentially to their borrowers. While this approach may allow for the existence of pure strategy equilibria (see Gehrig (1998)), it is somewhat less standard and is highly susceptible to the timing assumptions of the interest rate offers. Our specification is consistent with models of competition where firms cannot fully observe their competitors’ actions before having to make their own decisions.} However, a unique equilibrium in mixed strategies exists which we now characterize in terms of banks’ distribution functions over interest rate offers. Let $F_i(r; \eta, x)$ represent the bidding distribution of an informed bank for borrowers located at a distance $x$, conditional on the loan screening outcome $\eta$. Similarly, let $F_u(r; y)$ represent the bidding distribution of an uninformed bank for borrowers located at a distance $y$. Also, we define $\pi^*_i(\eta, x)$ as the equilibrium expected profit for an informed bank, and $\pi^*_u(y)$ as that for an uninformed bank.

Proposition 1 The bidding game for a borrower located a distance $x$ from bank $i$ and $y = \frac{1}{N} - x$ from bank $u$ has a unique, mixed-strategy equilibrium, characterized by continuous and strictly increasing distribution functions $F_i(r; \eta, x)$, $F_u(r; y)$, such that:

1. the uninformed bank breaks even: $\pi^*_u(y) = 0$;

2. the informed bank earns positive expected profits $\pi^*_i(h, x) > 0$ on borrowers with $\eta = h$, while it breaks even on borrowers with $\eta = l$, i.e., $\pi^*_i(l, x) = 0$.

Proof. See Appendix. \qed

The lending game has an outcome very reminiscent of Bertrand competition. Since, by definition, the uninformed lender has no private information about the borrower, it is unable to obtain any rents from the loans it grants. The informed bank, however, is able to use its informational advantage over competitors to extract rents on borrowers deemed to be of high quality. An informed bank’s ability to distinguish good from bad risks allows it to adjust its bidding and lending strategy accordingly, and subjects less informed lenders to problems of adverse selection. By symmetry, each bank will lend on an informed basis to borrowers located nearby and lend without prior screening to others located further away.
To provide some intuition for the location dependence of loan granting policies, consider the simple case of only two banks, i.e., \( N = 2 \), and specialization and breadth investments of \( I = 1 \) and \( \alpha = \frac{1}{2} \). The signal will then have no information content at \( x = \frac{1}{4} \), i.e., \( \phi \left( \frac{1}{4} \right) = \frac{1}{2} \), which is exactly the midpoint between two banks located at opposite extremes of the circle of circumference 1. For values of \( x \) near zero, the probability that the uninformed bank bids at all is quite low, and the informed bank obtains high profits on these borrowers. As \( x \) increases, the probability of bidding by the uninformed bank increases, since the adverse selection problem it faces is decreased. In the limit, as \( x \) approaches \( \frac{1}{4} \), the distribution functions for both the informed and the uninformed bank concentrate all mass at \( r = r_p = 1/p \), the break-even rate on an average borrower. In essence, at \( x = \frac{1}{4} \), neither lender has any information so that they compete in a symmetric way, driving all profits to zero.

Proposition 1 implies a very simple expression for \textit{ex ante} expected profits prior to the observation of the screening signal \( \eta \) that is linear in the informativeness \( \phi \) of the credit assessment:

\begin{equation}
E[\pi_i(\eta, x)] = Pr_x(\eta = h) \pi_i(h; x) + (1 - Pr_x(\eta = h)) \pi_i(l; x)
= \frac{1}{p}(p_h - p_l)q(1 - q)[2\phi(x) - 1]
\end{equation}

where \( Pr_x(\eta = h) \) is the probability of finding a borrower located at \( x \) with a signal of high quality.

Expected profits are decreasing in bank-borrower distance \( x \), i.e., \( \frac{\partial}{\partial x}E[\pi_i(\eta, x)] < 0 \) and increasing in borrower heterogeneity \( \Delta_p \equiv p_h - p_l \): \( \frac{\partial}{\partial \Delta_p}E[\pi_i(\eta, x)] \).

\textbf{Proof}. See the Appendix. \( \blacksquare \)

In addition to the signal quality \( \phi \), expected profits to an informed bank only depend on borrower attributes, i.e., the average borrower quality \( \bar{p} \), the heterogeneity of borrowers \( \Delta_p \), and their distribution \( q \). The second part of the proposition says that an informed bank makes lower profits on those borrowers that are farther away (in information space) from its core competency. Intuitively, as we move further away from a bank, the quality of information the bank has about a
particular borrower decreases, so that its rents from information should decrease as well. Conversely, as borrowers become more heterogeneous, an informed bank earns higher profits on those borrowers with good signals, even if the average success probability $\bar{p}$ remains constant. Behind this part of the proposition lies the adverse selection problem faced by the uninformed bank, as screening becomes more important the more diverse are the borrowers.

4 Equilibrium and Screening Incentives

We now turn to the banks’ decision to acquire information and the industry equilibrium when the minimum scale restriction is not binding, i.e., $C(\alpha, I) > \bar{C}$. Since all banks are ex-ante symmetric, we will focus throughout on a symmetric equilibrium in information acquisition. Since, by symmetry, every bank competes both in informed and uninformed lending, we denote a given intermediary by $n \leq N$. We first verify that, in equilibrium, borrowers are not screened by multiple banks and characterize the marginal borrower screened by an intermediary. It then follows that banks acquire information about borrowers on either side of their location up to a distance $\hat{x}$.

Lemma 1 In equilibrium, borrowers are screened by at most one bank. Intermediaries assess a borrower’s creditworthiness up to a maximal distance, $\hat{x}_n$, where, for $A \equiv \frac{1}{p} \Delta p q(1 - q)$,

$$\hat{x}_n = \min \left\{ \frac{\alpha}{2} \left[ I - \frac{c}{A} \right], \frac{1}{2N} \right\}$$

Proof. See the Appendix. □

This lemma justifies our analysis in the previous section of one informed bank competing against a number of uninformed banks. By Corollary 1, expected ex ante profits, i.e., before the signal is observed, are decreasing in the distance of the borrower to the bank. If more than one bank had screened a borrower, competition among these banks would drive profits to zero for all but

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13 One might suspect that this result is driven by changes in $p_h$, so that borrowers with a high signal should also be considered to be qualitatively better borrowers. However, keeping $p_h$ fixed and lowering only $p_l$ has the same effect. Since changes in $p_h$ and $p_l$ equally affect $\Delta p$ it is exactly the severity of the information problem that drives the result.

14 As will be seen in the next section, this condition amounts to a restriction on entry, so that the requisite free-entry condition is not yet binding. This situation may occur if the market is protected, or not sufficiently mature.
the most informed one. Only the intermediary closest in informational space could possibly recoup
its screening cost \( c \), so that it would never pay to screen a borrower that is located closer to a
competitor bank.

The requirement that \( \hat{x} \) be no greater than \( \frac{1}{2N} \) immediately follows from the preceding: in any
symmetric equilibrium with \( \hat{x}_n > \frac{1}{2N} \), there would be an interval of borrowers that both banks
screen, so that the bank located further away would fail to recover its cost of screening \( c \). Note that
a bank would like to screen more borrowers (\( \hat{x} \) is greater) the smaller the number of active banks,
the larger its expertise in its core market (\( I \)), the more transferable its skill (\( \alpha \)), and the lower the
marginal screening cost \( c \).

The success in extracting information from borrower screening clearly depends on banks’ in-
vestments in core competencies \( I \) and their willingness to extend their captive markets as reflected
by their investments in non-specific skills \( \alpha \). The precise mix of specialized expertise and broad-
based investment determines a bank’s strategy in the lending market and its response to increased
competition. To find the equilibrium investments \( I \) and \( \alpha \), we first calculate the total profits for
bank \( n \) upon entering, which, summed across all screened borrowers, can be expressed as

\[
\Pi_n = 2 \int_0^{\hat{x}_n} \left[ A(2\phi(x) - 1) - c \right] M dx,
\]

for \( A = \frac{1}{\bar{p}} \Delta p q(1 - q) \), as previously defined. Given the number of active banks \( N \), an entering
intermediary \( n \) will choose \( \alpha_n > 0 \) and \( I_n \in (0, 1) \) in its entry and investment decision so as to
maximize net profits \( V_n = \Pi_n - K_n \):

\[
\max_{\alpha, I} V_n = \max_{\alpha, I} \left\{ 2M \int_0^{\hat{x}_n} \left[ A(2\phi(x) - 1) - c \right] dx - \left[ \frac{\alpha^2}{2} + \frac{I^2}{2} + T \right] \right\}
\]

(4)

We now analyze the investment decision for a given number of active banks, assuming that the
minimal investment constraint in the screening technology is not binding, i.e., \( C(\alpha, I) > \bar{C} \). This
preliminary step allows us to see how changes in the degree of competition in the market affect
the overall incentives to invest in information acquisition. Since we are interested in the case of
increasing competition, we will assume that \( N \) is sufficiently large so that \( \hat{x} = \frac{1}{2N} \).\(^{15}\)

\(^{15}\)In the proof of Proposition 2 we verify that, once this condition is satisfied, \( \hat{x} = \frac{1}{2N} \) for all larger values of \( N \).
Proposition 2 For a given number of intermediaries $N$, the profit maximizing investment in specialized expertise $I_n^*$ and broad-based skills $\alpha_n^*$ are given by

$$I_n^* = \frac{1}{\bar{p}} \Delta_p q (1 - q) \frac{M}{N}$$

$$\alpha_n^* = \left[ \frac{1}{\bar{p}} \Delta_p q (1 - q) \frac{M}{2N^2} \right]^{\frac{1}{3}}$$

Proof. See the Appendix. ■

Once banks screen up to $\hat{x} = \frac{1}{2N}$, the entire market is “covered” by at least one bank, so that increasing the number of banks shrinks the share of the market segment screened by each intermediary. Figure 2 depicts such an equilibrium where intermediaries are located so close to each other that all borrowers are screened by some bank. As $N$ increases, banks choose a lower value of $\alpha$ so that they invest less in extending their expertise beyond their core markets. The same is true for $I$: as more banks are active, it becomes harder to sustain investments in specialized lending.
As competition increases, banks’ overall investment in the screening technology falls because competition reduces the returns to information acquisition. It follows that increasing the number of active intermediaries leads to more inefficient lending decisions. Proposition 2 also establishes intuitive comparative statics properties of the equilibrium. As the information asymmetry problem becomes more acute ($\Delta_p$ increases), banks choose higher values of $\alpha$ and $I$. When borrower characteristics are less homogeneous, better creditworthiness assessments become more important. In other words, as the risk of lending to a poor quality borrower grows, banks invest more in the screening technology, which helps them to avoid making inefficient lending decisions. This mechanism is the same as in Corollary 1 in which we establish that expected profits are increasing in our measure of information asymmetries, $\Delta_p$.

To put the above results into context, we would expect the minimum investment scale not to be binding for “young” markets where banks do not yet have long-standing relationships with customers or have not developed the expertise in evaluating borrowers far outside of their “niche”. Initially, there exists a set of borrowers between any two banks that are not screened by either bank in a symmetric equilibrium such that $\hat{x}_n < \frac{1}{2N}$. For these borrowers we observe symmetric interest rate competition among all banks. For all other borrowers - those less than a distance of $\hat{x}$ from some bank - competition among potential lenders will always be characterized as in the previous section, with one informed relationship bank and one uninformed transactional lender. The existence of non-screened segments could be interpreted as a situation where the cost of the screening technology, or of developing broad-based expertise, is too high to warrant the expense incurred in evaluating these borrowers.

As banks build up their proprietary intelligence on borrowers, the initial returns on informational investments are substantial. These profits and the existence of non-screened credit market

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16 Note that $I_n^*$ is smaller than 1 for large enough values of $N$ since $A \in (0, 1)$ and $M$ is a fixed parameter. Hence, the informativeness of the credit worthiness signal $\phi_n^*(x) = \frac{1 + I_n^*}{2} - \frac{\alpha^*}{x} = 1 + \frac{I_n^*}{2} - \frac{\alpha^*}{x}$ is well defined and one can verify that $\hat{x}_n = \min \left\{ \frac{1}{2N}, \frac{\alpha^*}{2} \left[ I_n^* - \frac{4}{A} \right] \right\} = \frac{1}{2N}$.

17 Broecker (1990) illustrates similar lending inefficiencies in the context of independent but symmetric creditworthiness tests. In his model, the probability that any given borrower passes at least one test increases with the number of banks that conduct $N$ independent but noisy tests. However, in Broecker (1990) the aggregate amount of information increases with the number of banks even if the inference problem for each bank worsens. In our model, a reduced incentive to screen rather than an increased winner’s curse drives the deterioration of overall credit quality. Shaffer (1997) provides some evidence that is consistent with both models: as the number of banks in a market grows, each bank’s provision for loan losses increases.
segments attract entry so that at one point intermediaries have extended their expertise to cover the whole market in terms of information production, i.e., $\hat{x} = \frac{1}{2N}$. As the market matures, additional entry now reduces the returns to information acquisition as described in Proposition 2.

5 Strategic Information Acquisition

As emphasized earlier, the importance of banks in our model stems from their ability to generate borrower-specific information that requires a minimal investment in the requisite screening technology. Hence, we now assume that sufficient entry has occurred so that this investment constraint binds at $C(\alpha, I) = \bar{C}$ for credit assessments to be informative. That it will eventually bind is clear from the previous section: as entry occurs, banks reduce their investment in $I$ and $\alpha$, thus lowering $C$. Consistent with our specification so far, upon entry banks costlessly relocate around the circle so that they are equi-distant from each other.

5.1 Free-Entry Equilibrium

We first establish the existence of a free-entry equilibrium and derive the number of active banks $N^\ast$. At the same time, we also characterize the impact of entry into the credit market on banks’ incentives to invest in information acquisition as technological or regulatory barriers to entry decrease, i.e., the cost of entry $T$ falls. Our focus is on the optimal allocation of resources between investments in specialized expertise $I$ and broad-based expertise $\alpha$ as competitive pressures grow.

**Proposition 3** For a fixed screening technology expenditure $\bar{C}$, there exists a free-entry equilibrium with $N^\ast$ active intermediaries.

Banks increase investment in specialized expertise ($\frac{\partial I^\ast}{\partial N} > 0$) and cut back on non-specific investments ($\frac{\partial \alpha^\ast}{\partial N} < 0$) as their number increases. In particular, the free-entry number of intermediaries $N^\ast$ increases for lower fixed costs $T$, so that investment in specialized expertise increases ($\frac{\partial I^\ast}{\partial T} < 0$) while nonsegment-specific investment decreases ($\frac{\partial \alpha^\ast}{\partial T} > 0$).

**Proof.** See the Appendix. ■
A lowering of entry barriers as represented by a reduction in $T$ leads to a larger number of banks and more competition in information acquisition. The preceding proposition shows how banks optimally respond by strategically shifting their focus. As more intermediaries crowd the market, each one finds it more difficult to appropriate the gains from screening borrowers outside their realm of expertise. Hence, with a fixed investment constraint $\bar{C}$, it is beneficial for banks to cut back on their overall scope by reducing their investment $\alpha$ in extending their franchise. The resources freed up by this retrenchment can now be reallocated to their core market, so that segment-specific expertise expenditure $I$ increases. We see that a bank’s optimal investment in specialized expertise and broad-based knowledge determines its ultimate scope in terms of informed intermediation.

Figure 3 illustrates how this shift in investments affects the informativeness of the lending relationship as a function of informational distance. As a result of growing competition banks become more specialized in their core markets at the expense of breadth. While each bank’s captive market shrinks with entry, most of the remaining customers become informationally more valuable to the bank.\footnote{The increased rent extraction that may accompany this specialization need not necessarily be bad for borrowers. Informed or relationship lending can be seen as providing an element of insurance for the borrower, which has value to her. Greater investment in the relationship should also lead to better insurance, albeit at an increased cost.} In the management literature, this interaction of competition and focus...
has long been recognized. For instance, Prahalad and Hamel (1990) stress that, in today’s world of
global competition, firms need to focus on core competencies that they transform into core products
to fend off competitive threats. Hence, core competencies go together with corporate focus.

Interpreting intermediaries at different locations on the circle as offering different but compet-
ing debt products, our results are in line with Boot and Thakor (2000), who find that growing
competition from capital market debt also increases a bank’s investment in specialization. Our
model, however, provides a very different channel for the competitive effects to play out, allowing
us to study the underlying economic forces that shape an intermediary’s strategic response to more
entry, a topic to which we turn in the next section.

Given a fixed expenditure \( C \) on the lending technology, we can now determine how the free-entry
number of banks depends on the economic attributes of banks and their customers.

Proposition 4 For fixed technology expenditure \( C \), the free-entry number of banks \( N^* \) is increasing
in the degree of borrower heterogeneity, \( \Delta_p \), and decreasing in the cost of monitoring, \( c \).

Proof. The proposition follows from the preceding one and the Implicit Function Theorem.

The free-entry equilibrium number of banks is a function of both the choice of the informa-
tion acquisition technology \((I, \alpha)\) as well as the cost of screening, \( c \), and the characteristics of the
borrowers, \( \Delta_p \). As should be expected, \( N^* \) is lower for higher values of \( c \): the cost of screening
borrowers lowers each bank’s profitability directly, and so should lead to a lower number of banks
in equilibrium. The effect of borrower heterogeneity \( \Delta_p \) is more subtle because it also affects the
investments in specific and non-specific expertise, \( I \) and \( \alpha \). However, as we showed in Corollary
1, greater borrower heterogeneity has a positive effect on bank profits for a fixed screening tech-
nology. Therefore, the market can support a greater number of banks when borrowers are more
heterogeneous.

\(^{19}\) However, Boot and Thakor (2000) find the opposite effect when analyzing growing competition among commercial
banks only: bank entry reduces their investment in sector specialization. This difference arises because they keep the
size of a bank’s loan portfolio fixed as competition grows, whereas in our model banks’ market shares shrink as more
competitors enter the market.
5.2 Determinants of Strategic Focus

In order to investigate the determinants of the banks’ choice between focus and breadth in information acquisition, we first establish that informed banks differentiate interest rate offers in terms of the quality of proprietary information. Since banks enjoy a natural advantage for borrowers in their (informational) vicinity, their competitors should bid less aggressively given their relatively large information disadvantage. The following result is one of the driving forces behind intermediaries’ decision to focus resources on their core markets.

**Proposition 5** In the loan market between two banks \( n \) and \( n+1 \), an informed bank’s interest rate offer is (stochastically) decreasing in the distance from bank to borrower \( (\frac{\partial F_i}{\partial x} > 0) \) while, conditional on bidding, an uninformed bank’s rate offer is (stochastically) increasing in the distance from bank to borrower \( (\frac{\partial F_u}{\partial y} < 0) \).

**Proof.** The result follows from \( \frac{\partial F_i}{\partial x} = \frac{\partial F_i}{\partial \phi} \frac{\partial \phi}{\partial x} \) and likewise for bank \( u \) by observing that \( \frac{\partial \phi}{\partial x} < 0 \) and \( \frac{\partial \phi}{\partial y} < 0 \) for \( y = N^{-1} - x \), and that \( \frac{\partial F_i}{\partial \phi} , \frac{\partial F_u}{\partial \phi} < 0 \). □

This proposition implies that expected interest rate offers are a decreasing function of \( x \), the distance between informed bank and borrower, and, similarly, an increasing function of \( y \), the distance between uninformed competitor and borrower (since \( y = N^{-1} - x \)). The closer a borrower is located to a competitor bank, the greater the adverse selection problem becomes for an uninformed bank. As the distance between the borrower and the informed bank increases, the screening bank’s information advantage decreases, and the uninformed bank is able to bid more aggressively.

The finding in Degryse and Ongena (2001) that loan rates decrease in the physical distance between a corporate borrower and the lending bank but increase in the distance between the firm and competing banks provides strong evidence in favor of this prediction. Furthermore, recall that, in the context of established business relationships, we can also interpret the informational distance \( x \) in terms of the duration of a lending relationship by taking it to be a decreasing function of \( t \), the time of interaction between bank and borrower. Degryse and Ongena (2001) report that interest rates increase in the duration of lending relationships which is again consistent with Proposition 5. Their finding also points to the informational capture aspects of informed intermediation and
relationship lending. As banks invest in borrower-specific information acquisition and become more informed over time (increases in $t$ correspond to decreases in $x(t)$), they obtain higher loan rates from informationally captured borrowers.

We next establish how banks’ relative informational advantage, as measured by the screening success probability $\phi(x)$, changes with the number of active banks in the free-entry equilibrium.

**Lemma 2** There exists a borrower located at distance $x_N$ from a bank such that $\frac{\partial \phi}{\partial N} > 0$ for all $x \leq x_N$ and $\frac{\partial \phi}{\partial N} < 0$ otherwise.

The proof of the lemma follows by inspection from Figure 3. Since $I^*$ increases with $N$ while $\alpha^*$ decreases, $\phi$ must increase for borrowers located closer than some distance $x_N$ to the informed bank. The result is a direct consequence of a bank’s strategic response to entry. By reallocating informational resources towards their core markets, intermediaries increase their informational advantage for borrowers located nearby while decreasing it for those further away.

One immediate consequence of this concentration of informational resources is that borrower screening should become more efficient for those borrowers located close to their specialized bank. The improved quality of the information obtained from the credit assessment allows the screening bank to more often avoid lending to poor quality borrowers, and to feel more confident when lending to borrowers with positive credit assessments. An additional strategic benefit lies in the heightened adverse selection problem that deters an uninformed bank from competing aggressively for screened borrowers.

In light of the threat of adverse selection that an uninformed intermediary faces, it will not always make loan offers to borrowers screened by other intermediaries. Hence, we can evaluate the *ex post* competitive intensity by the probability that an uninformed bank makes a loan offer to a borrower located at a distance $y$. By Proposition 1, the probability that an uninformed bank bids is given by $F_u(R; y)$, which we define as $\beta(y)$.

**Corollary 2** The probability of a loan offer by an uninformed bank decreases in the information advantage of the informed bank ($\frac{\partial \beta}{\partial \phi} < 0$). As the number of active banks $N$ increases, the probability of a bid by an uninformed bank decreases in distance from bank to borrower ($\frac{\partial \beta}{\partial y} < 0$) for borrowers located at $y > y_N$, and increases ($\frac{\partial \beta}{\partial y} > 0$) for borrowers located at $y < y_N$, where $y_N = N^{-1} - x_N$. 21
Proof. This result follows from noticing that $\frac{\partial F_u(R,y)}{\partial \phi} < 0$, and $\frac{\partial \phi}{\partial N} > 0$ only for $x < x_N$, from Lemma 2. ■

Once again, adverse selection is the driving force behind this result. The more potential clients lie in an informed bank’s core market, the more severe the adverse selection problem faced by the uninformed bank becomes. Hence, the latter needs to be careful in its loan offers and will refrain more frequently from bidding for customers that have been screened by other banks. In fact, one can interpret the uninformed bank’s bidding probability as an indicator of the competitiveness of the lending market. The fact that it decreases in the informed bank’s relative informational advantage $\phi$ illustrates the strategic motivation for reallocating resources towards core markets as a way of limiting competition in the segments in which banks have developed particular lending expertise.

We finally turn to the effect of growing competition on expected profits arising from informed intermediation.

**Proposition 6** Equilibrium expected profits per borrower of informed banks increase in the number of active banks $N$ for borrowers nearby ($x < x_N$) and decrease otherwise.

Furthermore, $\frac{\partial^2 E[\pi_i]}{\partial \Delta_p \partial N} > 0$ for borrowers nearby ($x < x_N$). For borrowers further away ($x > x_N$), $\frac{\partial^2 E[\pi_i]}{\partial \Delta_p \partial N} < 0$.

Proof. The first part of the proposition follows from the fact that $\frac{\partial}{\partial \phi} E[\pi_i(\eta, x)] > 0$ and that $\frac{\partial \phi}{\partial N} > 0$ for $x < x_N$, and $\frac{\partial \phi}{\partial N} < 0$ otherwise. The second part follows from the fact that $\frac{\partial^2 E[\pi_i]}{\partial \Delta_p \partial \phi} > 0$ and that $\frac{\partial \phi}{\partial N} > 0$ for $x < x_N$. ■

The proof of the proposition demonstrates that there is a complementarity between the heterogeneity of borrowers and the quality of information obtained by the bank. In other words, increased information asymmetry, as measured by $\Delta_p$, makes screening more valuable. This implies that, for borrowers located nearby, banks’ incentives to refocus resources are greater when information asymmetries are large. Hence, there also exists a feedback effect between information asymmetries and the number of banks, suggesting that the strategic refocusing of resources we highlighted earlier
should be more intense in markets where adverse selection problems loom larger.

Take, for instance, the financing of high-tech firms where providers of finance often specialize in one or two industries. Especially in bridge finance (e.g., Silicon Valley Bank) and IPO underwriting (investment boutiques such as the former Hambrecht & Quist or Robertson Stephens), smaller players concentrate their informational resources in terms of industry coverage to carve out a niche in particular industries as a response to increasing competitive pressures. Since high-tech firms are often informationally opaque, the concentration of informational resources in a core segment helps to fend off competition from larger, more established players through the threat of adverse selection.

5.3 Changes in Screening Technology

There has been much debate concerning the likely impact of changes in banks’ ability to gather and process information on their capacity to establish and maintain relationships with borrowers. In a recent paper, Petersen and Rajan (2001) argue that increased access to information has eroded some of the geographic boundaries that have previously defined banking markets. This erosion has allowed banks to serve not only borrowers located further away, but also those that may not have previously appeared to have the observably highest quality. Similarly, Emmons and Greenbaum (1998) suggest that improved information processing ability may lead banks to offer credit to previously unserved or underserved customers, thus expanding the market. As Wilhelm (2001) observes, technological progress has allowed banks to leverage human capital to a point where far fewer loan officers are required for the management of portfolios of unchanged size. Hence, one would expect a corresponding decrease in the cost of screening or an increase in its informativeness.

While the complete analysis of advances in information technology is beyond the scope of this paper, we can offer some perspective on the preceding observations within the context of our model. Suppose one starts from a situation where regulatory barriers to entry, or costs of initial access to the information technology, are high in banking so that our fixed cost, \( T \), is large. In this equilibrium, there are some purely transactional borrowers who are never screened by a bank. These pure transactional market segments exist because entry has not yet pushed banks to encroach on each other’s markets (\( \hat{x} < \frac{1}{2N} \)). In this case, lowering the per borrower cost of screening \( c \), which can
be interpreted as an improvement in banks’ credit assessment abilities and access to information, 
causes each bank’s core market to expand (\(\hat{x}\) increases). Improving the informativeness of the 
signal \(\phi\) through, e.g., better information processing capabilities, would have the same effect.\(^{20}\) As 
a result of the improved ability to screen borrowers, banks should find it beneficial to screen and 
serve borrowers that were previously underserved.

6 Discussion and Conclusion

In this paper, we show how informational asymmetries concerning borrower quality lead to in-
formational rents, whose magnitudes depend crucially on the information advantage of a lender 
relative to its competitors. By establishing a direct link between the degree of informational asym-
metries among intermediaries and their profitability, we are able to investigate how changes in the 
industry structure affect a bank’s incentive to invest in its core market. While informational rents 
attract competition from both potential entrants and from other established banks, the informa-
tional monopoly of informed lenders also makes competition less effective because of the adverse 
selection threat faced by competitors.

We establish that a bank’s optimal strategic response to changes in the number of competitors 
comprises two related parts. Increased competition among lenders pushes down loan profitability, 
shrinks each intermediary’s market share, and decreases their incentives to invest in information 
acquisition. Consequently, the aggregate amount of information produced in the economy falls, 
which is consistent with other studies on the effect of competition on borrower screening (see, e.g., 
Gehrig, 1998). However, as competition increases further via entry, banks strategically reallocate 
resources towards their core market in order to protect the returns from information production 
obtained in that sector. As each bank’s captive market segment shrinks further, banks specialize 
in their core market at the expense of developing lending relationships in more peripheral markets. 
Hence, intense competition leads to a strategic focus by banks on those markets or activities in 
which they have a natural advantage or expertise.

Our analysis is consistent with recent empirical findings regarding loan pricing in function of 
\(^{20}\)For an analysis of the impact of information technology on the competitiveness of financial markets, see Hauswald 
and Marquez (2001).
physical distance between lenders and borrowers and the duration of lending relationships (Degryse and Ongena, 2001). In terms of further testable implications, our results suggest that a bank’s incentive to strategically focus its resources should be greater in markets where adverse selection problems are greater such as, for instance, lending to young firms. In such markets it becomes crucial to develop expertise and carve out a niche in order to remain profitable and avoid lending to borrowers with poor prospects. Furthermore, our model predicts that, at least for some borrowers, lending competition may decrease even as the number of banks increases. This outcome occurs precisely because of the refocusing effect that serves to protect a bank’s market and increase loan profitability, which constitutes a further testable implication.

The results also shed some light on the recent debate concerning the nature and future evolution of banking. Boot and Thakor (2000) argue that increased competition, either among banks or from outside sources, will drive banks to invest more in relationship lending as these markets can be better insulated from competition and banks are uniquely equipped to add value to borrowing firms. Our analysis highlights the role of asymmetric information and informational rent seeking as the underlying economic forces in this process, and points out that the changing competitive nature of the banking market can and should have an impact on banks’ lending strategies. Since borrower-specific information translates into rents, banks are willing to protect these rents by specializing in a core expertise, differentiating loans in terms of the information obtained and sacrificing peripheral markets.
Appendix

Proof of Proposition 1. The proof proceeds in a sequence of steps: first, we establish that there does not exist a pure strategy equilibrium. Next, we show that the two competing banks offer loan rates over the same interval, then we verify that the mixed strategies are well-behaved distribution functions and, finally, we prove uniqueness by explicitly calculating the location dependent mixed strategies. To fix notation, let \( r_p = p^{-1} \) and \( r_i(x) = p(l; x)^{-1} \).

Lemma 3 (Absence of Pure Strategy Equilibria) There exist no pure strategy equilibria in the bidding game for borrowers between an informed and an uninformed lender.

Proof. Let pure strategies conditional on signal and borrower location be denoted by \( r_i(\eta) \) and \( r_u \) for the informed and uninformed bank, respectively (we suppress the dependence on \( x \) for now). Suppose that \( r_u \leq r_i(h), r_i(l) \). In order for this to be optimal for the uninformed bank, \( r_u \geq p^{-1} \). However, the informed bank could increase its profit by offering a rate \( r_i(h) = r_u - \epsilon \) and lending to all \( \eta = h \) borrowers. Therefore, \( r_u \leq r_i(h), r_i(l) \) cannot be an equilibrium.

Suppose then that \( r_i(h) \leq r_u \leq r_i(l) \). In this case, the uninformed bank only makes loans to \( \eta = l \) borrowers. In order for this to be optimal for the uninformed bank, it must be that \( r_u \geq p(l; x)^{-1} \). If \( r_i(h) < r_u \), the informed bank would be better off charging \( r_i(h) + \epsilon \). But if \( r_i(h) = r_u \), the uninformed bank would be better off charging a rate \( r_i(h) - \epsilon \) and lending to all borrowers. Therefore, \( r_i(h) \leq r_u \leq r_i(l) \) cannot be an equilibrium.

Finally, suppose that \( r_i(l) \leq r_u \leq r_i(h) \). At \( r_u \), the uninformed bank lends only to \( \eta = h \) borrowers and makes positive expected profits if \( r_i(l) \) makes non-negative profits for the informed bank. But then, as above, if \( r_u < r_i(l) \), the informed bank could increase its profits strictly by lowering its bid to \( r_u - \epsilon \) and lending to all good borrowers. Therefore, this also cannot be an equilibrium, and no equilibrium exists in pure strategies. □

Lemma 4 (Common Support) Both banks randomize their loan offers over the same interval \([r_p, r_i(x) \wedge R]\). Moreover, the informed bank earns positive expected profits and the uninformed one breaks even.

Proof. Let \( F_i(r; \eta; x) \) represent the bidding distribution for the informed bank for a borrower located at distance \( x \), with support conditional on \( \eta: r \in [r_i^n, \bar{r}_i^l] \). Similarly, let \( F_u(r; N^{-1} - x) \) represent the bidding distribution for the uninformed bank, for loan rate offers in \( r \in [r_u, \bar{r}_u] \). The expected profits for each bank from offering a rate \( r \) are for \( F(r^-) \equiv \lim_{s \to r^-} F(s) \)

\[
\pi_i(r; \eta; x) \equiv (1 - F_u(r^-; N^{-1} - x)) \left[ p(\eta; x) r - 1 \right] \tag{5}
\]

\[
\pi_u(r; N^{-1} - x) \equiv \Pr(\eta = h) \left( 1 - F_i(r^-; h; x) \right) \left[ p(h; x) r - 1 \right] + \Pr(\eta = l) \left( 1 - F_i(r^-; l; x) \right) \left[ p(l; x) r - 1 \right] \tag{6}
\]

As in the text, define \( \hat{x} \) as the value of \( x \) such that \( p(l; x)R = 1 \). We first consider a borrower located at \( x < \hat{x} \) from the informed bank so that \( p(l; x)R < 1 \).

Claim 1 Both banks offer loan rates \( r_i, r_u \in [p^{-1}, R] \). The informed bank makes positive profits on high-quality borrowers and does not offer loans to low-quality ones.

\(^{21}\)This step represents a generalization of an argument in von Thadden (1998).
Obviously, the informed bank never bids on an \( \eta = l \) borrower, so that \( F_i(r, l; x) = 0 \) for all \( x < \hat{x} \) and all \( r \). The uninformed bank will never bid less than \( r_\pi = p^{-1} \) because low quality firms switch banks at any offer so that the loan pool has a success probability of at most \( \overline{p} \). But then, the informed bank will never offer rates below \( r_\pi \) to its high quality customers (\( \eta = h \)) making positive profits on high-quality borrowers: \( \pi_i(r, h; x) > 0 \). Clearly, neither the informed nor the uninformed bank will ever offer (gross) rates higher than \( R \), the project’s pay-off in the successful state.

**Claim 2** \( F_i(r, h; x) \) is continuous on \([\underline{r}_i^h, \overline{r}_i^h]\).

Suppose not, i.e., there exists \( s \in [\underline{r}_i^h, \overline{r}_i^h] \) such that \( F_i(s^-, h; x) < F_i(s, h; x) \). Since the informed bank makes positive (expected) profits on high-quality borrowers \( p(h; x) r - 1 > 0 \) on \([\underline{r}_i^h, \overline{r}_i^h]\) so that \( \pi_u(s^-; N^1 - x) > \pi_u(s; N^1 - x) \) by (6) and the right-continuity of \( F_i(r, \eta; x) \), \( \eta = h, l \). Since \( \pi_u(r; N^1 - x) \) is also right-continuous, there exists a neighborhood \([s, s + \varepsilon], \varepsilon > 0\), on which \( F_u \) must be constant implying \( F_u(s^-; N^1 - x) = F_u(s^-; N^1 - x) \). Hence, by (5), \( \pi_i(r, h; x) \) is continuous at \( s \) and strictly increasing in the neighborhood; but then, \( F_i(r, h; x) \) cannot have any mass on \([s, s + \varepsilon] \) so that \( F_i(s^-; h; x) = F_i(s, h; x) \), contradicting the assumption of an atom at \( s \).

**Claim 3** The lower and upper bounds of the supports are equal: \( \underline{r}_u = \underline{r}_i \) and \( \overline{r}_i^h = \overline{r}_u = R \), respectively.

We first show that \( \underline{r}_i \geq \underline{r}_u \). If not, \( \pi_i(r, h; x) = 0 \) for \( r \in (\underline{r}_u, \underline{r}_i) \), contradicting an earlier result. Suppose then that the inequality is strict: \( \underline{r}_i > \underline{r}_u \). In this case, \( \pi_u(\underline{r}_i; x) > \pi_u(\underline{r}_u; x) \), since the uninformed bank has the same probability of having the lowest rate at \( \underline{r}_i \) as at \( \underline{r}_u \). Therefore, the two lower bounds must be the same.

If \( \overline{r}_u < \overline{r}_i \) then \( \pi_i(r, h; x) = 0 \) on \([\overline{r}_u, \overline{r}_i] \) which contradicts \( \pi_i(r, h; x) > 0 \) so that \( \overline{r}_i \leq \overline{r}_u \leq R \). This also establishes that \( \overline{r}_u = R \). If \( \overline{r}_i^h < R \), the uninformed bank would never choose to bid in \((\overline{r}_i^h, R) \). But by an argument similar to that above, the informed bank would be better off raising its bid above \( \overline{r}_i^h \). Therefore, we must have that \( \overline{r}_i^h = \overline{r}_u = R \).

**Claim 4** The uninformed bank breaks even: \( \pi_u(r; N^1 - x) = 0 \). Therefore, \( \underline{r}_u = \underline{r}_i = \overline{p}^{-1} \).

Since \( F_i(r, l; x) = 0 \), (6) simplifies to

\[
\pi_u(r; N^1 - x) = \Pr(\eta = h) (1 - F_i(r, h; x)) [p(h; x) r - 1] + \Pr(\eta = l) [p(l; x) r - 1]
\]

which is continuous on \([\underline{r}_u, \overline{r}_u] \) by continuity of \( F_i(r, h; x) \). To show that the uninformed bank earns 0 expected profits, recall that \( \overline{r}_u = \overline{r}_i = \overline{p} \). Now, by definition, \( F_i(\overline{r}^-, h; x) \leq 1 \). If \( F_i(\overline{r}^-, h; x) = 1 \), then \( \pi_u(\overline{r}^-; x) = 0 \), since the probability of winning for this bank is zero. This implies that equilibrium expected profits must be zero. Suppose then that \( F_i(\overline{r}^-, h; x) < 1 \). Since both banks cannot have an atom at \( \overline{p} \), we must have either that \( F_u(\overline{r}^-; x) = 1 \) or \( F_u(\overline{r}^-; x) < 1 \) and the uninformed bank sometimes refrains from bidding. Start with \( F_u(\overline{r}^-; x) = 1 \). In this case, this implies that \( \pi_i(\overline{r}^-, h; x) = 0 \), contradicting the earlier result that \( \pi_i(r, h; x) > 0 \) for all \( r \in \text{supp}(F_i) \). Therefore, it must be that \( F_i(\overline{r}^-; x) < 1 \) and the uninformed bank sometimes does not bid. But this implies that its equilibrium expected profits must be zero.

We can now calculate the lower bound. At \( \underline{r}_u \), the uninformed bank wins almost surely, and so makes a profit of \( \underline{r}_u \overline{p} - 1 = 0 \). This implies \( \underline{r}_u = \overline{p}^{-1} \). Hence, we can conclude that \( [\underline{r}_i^h, \overline{r}_i^h] = [\underline{r}_u, \overline{r}_u] = [\overline{p}^{-1}, R] \), which does not depend on \( x \).

We now turn to the case of borrowers located at \( x > \hat{x} \) from the informed bank.
Claim 5 If \( x > \tilde{x} \) the informed bank bids \( r_i (l; x) = r_i (x) = p(l; x)^{-1} \) almost surely for \( \eta = l \).

Consider \([\underline{r}^1_i, \overline{r}^1_i]\), the support of \( F_i(r,l; x) \) for \( x > \tilde{x} \). Clearly \( \underline{r}^1_i(l; x) \geq r_i(x) = \frac{1}{p(l; x)} \); otherwise the informed bank would lose money. To show that \( \overline{r}_i(l; x) \leq r_i(x) \) suppose the contrary, i.e., that \( \overline{r}_i(l; x) > r_i(x) \). By bidding \( r_u = \overline{r}_i(l; x) - \varepsilon \) for small \( \varepsilon > 0 \), the uninformed bank would make strictly positive profits: while the worst expected type of borrower now is \( \min(l, h) \), then \( F_i(r, l; x) \) is continuous in both cases. A similar argument establishes continuity of \( F_i(r, h; x) \) is continuous in both cases. A similar argument establishes continuity of \( F_u(r; x) \): suppose that there exists \( s \in [\underline{r}_u, \overline{r}_u] \) such that \( F_u(s^+; x) < F_u(s^-; x) \). Since \( F_u \) is right-continuous \( \pi_i(s^+, h; x) > \pi_i(s, h; x) \) as the informed bank’s expected profits for high-quality borrowers is strictly positive. Hence, there exists a neighborhood \([s, s + \varepsilon], \varepsilon > 0\), on which \( F_i(r, h; x) \) must be constant. Since \( F_i(r, h; x) \) is continuous so is \( \pi_u(r; x) \) at \( s \) and strictly increasing on the neighborhood by (7); but then, \( F_u(r; x) \) can not have any mass at \( s \).

To show strict monotonicity, suppose that \( F_i(r, h; x) \) is constant on some interval \( (s, \bar{s}) \subset [\underline{r}_u, r_i(x) \wedge \bar{R}] \). By continuity, \( \pi_u \) is strictly increasing on the interval. This establishes the contradiction by Claim 4, since uninformed banks must always break even.

Monotonicity of \( F_u \) is established by a completely analogous argument using the fact that \( \pi_i \) must also be constant for \( F_i \) strictly increasing.

Lemma 5 (Loan Rate Distribution Functions) For all \( x, y \in \left[0, \frac{1}{N}\right] \), \( F_i(r, h; x) \) and \( F_u(r; y) \) are strictly increasing, continuous distribution functions so that profits \( \pi_i \) and \( \pi_u \) are constant on \([\underline{r}_i(l; x) \wedge \bar{R}]\).

Proof. By construction, \( F_i(r, h; x) \) and \( F_u(r; x) \) satisfy the usual requirements of distribution functions on their common support. By the proof of Lemma 4 \( F_i(r, h; x) \) is continuous in both cases. A similar argument establishes continuity of \( F_u(r; x) \): suppose that there exists \( s \in [\underline{r}_u, \overline{r}_u] \) such that \( F_u(s^+; x) < F_u(s^-; x) \). Since \( F_u \) is right-continuous \( \pi_i(s^+, h; x) > \pi_i(s, h; x) \) as the informed bank’s expected profits for high-quality borrowers is strictly positive. Hence, there exists a neighborhood \([s, s + \varepsilon], \varepsilon > 0\), on which \( F_i(r, h; x) \) must be constant. Since \( F_i(r, h; x) \) is continuous so is \( \pi_u(r; x) \) at \( s \) and strictly increasing on the neighborhood by (7); but then, \( F_u(r; x) \) can not have any mass at \( s \).

To show strict monotonicity, suppose that \( F_i(r, h; x) \) is constant on some interval \( (s, \bar{s}) \subset [\underline{r}_u, r_i(x) \wedge \bar{R}] \). By continuity, \( \pi_u \) is strictly increasing on the interval. This establishes the contradiction by Claim 4, since uninformed banks must always break even.

Monotonicity of \( F_u \) is established by a completely analogous argument using the fact that \( \pi_i \) must also be constant for \( F_i \) strictly increasing.

Theorem 5 (Voluntary Bankruptcy) \( \pi_i(r; x) \wedge \pi_u(x) = 0 \) and \( \pi_i(r; x) \wedge \pi_u(x) = 0 \). \( \pi_i(r; x) \wedge \pi_u(x) = 0 \). \( \pi_i(r; x) \wedge \pi_u(x) = 0 \). \( \pi_i(r; x) \wedge \pi_u(x) = 0 \).

Proof. Since the mixing distributions are strictly increasing by the preceding lemma, expected profits must be constant on \([\underline{r}_i(l; x) \wedge \bar{R}]\). For \( \pi_i(r, h; x) = \pi \) and \( \pi_u(r; x) = 0 \), (5) and (7) yield the following system of equations defining the loan rate distributions:

\[
(1 - F_u(r; N-1 - x)) [p(h; x) r - 1] = \tilde{\pi}
\]

\[
\Pr(\eta = h) (1 - F_i(r, h; x)) [p(h; x) r - 1] + \Pr(\eta = l) [p(l; x) r - 1] = 0
\]

Evaluating the first equation at the lower bound of the support shows that the constant is \( \tilde{\pi} = p(h; x) r \bar{p} - 1 \). We can substitute into the above the definition of \( \Pr(\eta = h) = \phi(x) q + (1 - \phi(x))(1 - \)
\( q \), which simplifies to \( \Pr(\eta = h) = \frac{p - p(l|x)}{p(h|x) - p(l|x)} \). Solving out for \( F_u \) and \( F_i \) one finds

\[
F_i(r, h; x) = \frac{p(h; x) - p(l; x)}{p(h; x) - p(l; x)} \frac{\overline{p} r - 1}{\overline{p} p(h; x) r - 1}
\]

\[
F_u(r; N^{-1} - x) = \frac{p(h; x) - p(l; x)}{p(h; x) - p(l; x)} \frac{\overline{p} r - 1}{\overline{p} p(h; x) r - 1}
\]

The preceding distributions represent the unique equilibrium of the competitive bidding game for a given borrower. As both banks randomize over the full support of the distribution functions they can not profitably deviate from their mixed strategies. This establishes uniqueness. ■

**Proof of Corollary 1.** By the proof of Proposition 1, we know that the equilibrium profits for both banks are the same for every interest rate offered in the support of the mixing distributions. In particular, at \( r = \overline{p}^{-1} \) the informed bank wins with probability 1, but only bids if \( \eta = h \), realizing a profit of \( \pi_i(h, x) = \pi_i(r, h; x) = \overline{p}^{-1} p(h; x) - 1 > 0 \). *Ex ante* expected profits are given for \( \Pr_x (\eta = h) = \phi(x) q + (1 - \phi(x)) (1 - q) \) by

\[
E[\pi_i(\eta, x)] = \Pr_x (\eta = h) \pi_i(r, h; x) + (1 - \Pr_x (\eta = h)) \pi_i(r, l; x)
\]

Since \( \pi_i(r, l; x) = 0 \), this reduces to

\[
E[\pi_i(\eta, x)] = \frac{1}{p} (p_h - p_l) q (1 - q) [2\phi(x) - 1].
\]

This is well defined: by the informativeness restriction on screening, \( \phi(x) \geq \frac{1}{2} \); \( E[\pi_n(\eta, x)] \geq 0 \), which proves the first part of the result.

The second part of the corollary is obtained by noticing that \( \phi(x) \) is decreasing in \( x \) so that \( \frac{\partial}{\partial x} E[\pi_i(\eta, x)] < 0 \). To show that profits to the informed bank conditional on \( \eta = h \) are increasing in \( \Delta_p \), note that the profit expression above can be written as

\[
\pi_i(h, x) = \overline{p}^{-1} p(h; x) - 1 = \frac{(H(x) - q)(p_h - p_l)}{\overline{p}} = \frac{(H(x) - q)\Delta_p}{\overline{p}}
\]

so that differentiation with respect to \( \Delta_p \) yields

\[
\frac{\partial}{\partial \Delta_p} \pi_i(r, h; x) = \frac{\partial}{\partial \Delta_p} \frac{(H(x) - q)\Delta_p}{\overline{p}} = \frac{H(x) - q}{\overline{p}} > 0,
\]

because \( H(x) > q \). ■

**Proof of Lemma 1.** We first show that for \( c > 0 \), borrowers are screened by at most one bank. Suppose there are two banks, \( n \) and \( m \), and that they both screen a borrower located at a distance \( x \) from bank \( n \) and \( y \) from bank \( m \). By assumption, if \( x < y \), bank \( n \)’s signal is a sufficient statistic for bank \( m \)’s. By an argument similar to that used in Proposition 1, profits in the subsequent competition stage would be zero for bank \( m \). This occurs because, even though it is informed, its information is a subset of bank \( n \)’s information set, and so the usual zero-profit result holds (see Engelbrecht-Wiggans et al. (1983)) for a discussion of this point). If \( c > 0 \), bank \( m \) would not then recoup its cost of screening, and so, anticipating that bank \( n \) will screen, will not itself screen. Note that this also implies that a bank will never screen a borrower that is closer to another bank.

Since *ex ante* expected profits are given by

\[
E[\pi_n(\eta, x)] = \frac{1}{p} (p_h - p_l) q (1 - q) [2\phi(x) - 1],
\]

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\[ E[\pi_n(\eta, x)] = c \] now defines \( \hat{x} \) from \( \frac{1}{p}(p_h - p_l)q(1 - q) \left[ I - \frac{2c}{A} \right] = c \) as

\[ \hat{x} = \frac{\alpha}{2} \left( I - \frac{c}{A} \right), \]

where \( A \equiv \frac{1}{p} \Delta \rho q(1 - q) \) as long as \( \frac{\alpha}{2} \left[ I - \frac{c}{A} \right] \geq \frac{1}{2N} \). Otherwise, we would have an overlap in the screening interval (since banks are symmetric ex-ante), which would contradict the first result above. Therefore, it must be that

\[ \hat{x} = \min \left\{ \frac{1}{2N}, \frac{\alpha}{2} \left[ I - \frac{c}{A} \right] \right\}, \]

as desired. ■

**Proof of Proposition 2.** The result is easily obtained by differentiating equation (4) with respect to \( \alpha \) and \( I \), using the constraint that \( \hat{x} = 1/2N \). We then check that, in equilibrium, \( \frac{\alpha}{2} \left[ I - \frac{c}{A} \right] \geq \frac{1}{2N} \), so that our maximization problem is well specified. Appealing to the Envelope Theorem and Leibniz’ rule, we get the following

\[ \frac{\partial V_n}{\partial \alpha} = 0 = MA \frac{2N}{\alpha^2} - \alpha \Rightarrow \alpha_n^* = \left[ \frac{AM}{2N^2} \right]^\frac{1}{2} \]

\[ \frac{\partial V_n}{\partial I} = 0 = MA \frac{N}{I} - I \Rightarrow I_n^* = \frac{AM}{N} \]

Substituting the optimal investments into the condition for \( \hat{x} \) yields \( \left[ \frac{AM}{2N^2} \right]^\frac{1}{3} \left[ \frac{AM}{N} - \frac{c}{A} \right] \geq \frac{1}{2N} \) so that for small \( c > 0 \) we can consider by \( A \in (0, 1) \)

\[ \left[ \frac{AM}{2N^2} \right]^\frac{1}{3} \frac{AM}{N} > \frac{1}{2N} \iff (AM)^2 > \frac{1}{2N} \]

Thus, borrower mass \( M \) has to be sufficiently large relative to \( N \) in order for our problem to be well specified. However, for profits to be positive we require that

\[ V_n^* = V_n(\alpha_n^*, I_n^*) = \frac{1}{2} \left( \frac{AM}{N} \right)^2 - 3 \left[ \frac{AM}{2N^2} \right]^\frac{1}{2} - c \frac{M}{N} \geq 0 \]

Again, letting \( c \) be small, we find that this will be satisfied for \( N \) such that

\[ (AM)^2 > (3)^\frac{3}{2} \frac{1}{2} N \]

Hence, if this condition for positive profits is satisfied, so is the condition on \( \hat{x} \), so that we have indeed found the solution to the maximization problem. ■

**Proof of Proposition 3.** Let there be a fixed entry cost \( T \) together with a minimal scale requirement \( C(\alpha, I) = \frac{\alpha^2}{2} + \frac{I^2}{2} = \bar{C} \). The profit maximization problem becomes:

\[ \max_{\alpha, I} V_n = \max_{\alpha, I} \left\{ 2M \int_{0}^{\hat{x}} [A(2\phi(x) - 1) - c] \, dx - C(\alpha, I) - T \right\} \]
subject to $C(\alpha, I) = \frac{I^2}{2} + \frac{\alpha^2}{2} \geq \bar{C}$ where $\hat{x} = \min\left\{ \frac{1}{2N}, \frac{\alpha}{2} \left[ I - \frac{\alpha}{I} \right] \right\}$.

Integrating and forming the Lagrangian yields

$$L_n = 2M \left[ AI\hat{x} - \frac{A\hat{x}^2}{\alpha} - c\hat{x} \right] - \left( \frac{I^2}{2} + \frac{\alpha^2}{2} \right) - T - \lambda \left( \frac{I^2}{2} + \frac{\alpha^2}{2} - \bar{C} \right)$$

The first order conditions are for $\hat{x} = \frac{1}{2N}$

$$\frac{\partial L_n}{\partial \alpha} = 0 = \frac{MA}{2N^2\alpha^2} - \alpha(1 + \lambda)$$
$$\frac{\partial L_n}{\partial I} = 0 = \frac{MA}{N} - I(1 + \lambda)$$
$$\frac{\partial L_n}{\partial \lambda} = 0 = \frac{\alpha^2}{2} + \frac{I^2}{2} - \bar{C} \quad (8)$$

Eliminate $\lambda$ between the FOCs to find

$$2N\alpha^3 - I = 0 \quad (9)$$

which, along with the constraint, implicitly define the equilibrium values of $I$ and $\alpha$ given $N$.

Applying the Implicit Function Theorem now yields the first part of the proposition.

To show that there exists a free-entry equilibrium and to characterize it we add the zero-profit condition

$$\bar{C} + T \frac{MN^2}{M} - (AI - c) N + \frac{A}{2\alpha} = 0. \quad (10)$$

Equations (8) to (10) now implicitly define the free-entry equilibrium $\alpha_n^*, I_n^*$ and $N^*$ for the investment levels and number of active banks in terms of the cost parameters $T, \bar{C}$, and $c$. Letting

$$G(\bar{C}, T, c, A, \alpha, I, N) := \begin{cases} 2N\alpha^3 - I \\ \frac{\alpha^2}{2} + \frac{I^2}{2} - \bar{C} \\ \frac{\bar{C} + T}{M} N^2 - (AI - c) N + \frac{A}{2\alpha} \end{cases} \quad \text{for } (\alpha, I, N) \in \mathbb{R}^3$$

we must have in equilibrium $G(\bar{C}, T, c, A, \alpha^*, I^*, N^*) = 0$, where $(\alpha^*, I^*, N^*)$ is the solution to the free-entry problem. A necessary and sufficient condition for its existence is that the determinant of $\frac{\partial G}{\partial (\alpha, I, N)}$ does not vanish. But

$$\text{det} \left( \frac{\partial G}{\partial (\alpha, I, N)} \right) = \alpha (6I\alpha N + 1) \left( 2 \frac{\bar{C} + T}{M} N - AI + c \right) > 0$$

so that, by the Implicit Function Theorem, such a solution must exist and be differentiable. A further application of the theorem establishes the comparative statics results:

$$\frac{\partial I_n^*}{\partial T} = 2\alpha^3 N^2 \left[ (6I\alpha N + 1) \left( 2 \frac{\bar{C} + T}{M} N - AI + c \right) \right]^{-1} \leq 0$$
$$\frac{\partial I_n^*}{\partial T} = -2\alpha I N^2 \left[ (6I\alpha N + 1) \left( 2 \frac{\bar{C} + T}{M} N - AI + c \right) \right]^{-1} \geq 0$$
$$\frac{\partial I_n^*}{\partial T} = -N^2 \left( 2 \frac{\bar{C} + T}{M} N - AI + c \right)^{-1} \leq 0$$

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Note that, by the proof of Proposition 2, the zero-profit condition implies that \( \frac{\alpha^*}{2} [I_n^* - \frac{r}{A}] \geq \frac{1}{2n} \) holds in equilibrium so that the optimization problem is well defined. But then, constraint (8) implies \( I_n^* \in (0, 1) \) so that the credit worthiness signal is also well defined, i.e., \( \phi^*_{n} \in \left( \frac{1}{2}, 1 \right) \).
References


