Angels, Venture Capitalists, and Entrepreneurs: A Dynamic Model of Private Equity Financing

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Abstract

We consider a setting in which an entrepreneur chooses between angel and venture capital financing to fund his investment project. The entrepreneur may raise the required external financing over several rounds, though a certain minimum amount needs to be raised initially. There are four key ingredients driving the entrepreneur's choice between the above two sources of private equity financing in our model. First, venture capitalists are to able add value to some of the firms they finance, while angels are not able to add significant value. Second, the entrepreneur has private information regarding the nature of his own firm. Further, the extent of this private information evolves over time, since a financier who has financed the firm in prior rounds will know more about it than a new financier. Third, since the venture capitalist has to engage in privately costly effort to add value to the firm, the financial contract between the two has to provide him with the right incentives to maximize this value-addition. Finally, the entrepreneurs effort is also required to ensure project success. In the above setting, we derive: (i) The equilibrium financing path of the firm, including its choice between angel and venture capital financing over different financing rounds, and the amounts raised in these rounds; (ii) The equilibrium design of financial contracts between the entrepreneur and the angel or venture capitalist, with implications for the differences between angel and venture capital contracts; (iii) The dynamic evolution of venture capital contracts over financing rounds; (iv) The differences in the composition of projects financed by venture capitalists and angels and the structure of their holdings in these projects; (v) The effect of an announcement by any firm of a successful venture capital or angel financing upon other private equity investors assessment of its value.

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1 Introduction

It is well known that angel financing is an important source of financing for private firms in the United States. However, beyond the fact that the annual amount of angel financing market is much larger than that of venture capital financing, and that angels tend to be individuals who invest much smaller amounts than venture capital firms in individual firms, little is known about the important economic differences between venture capitalist and angel financing.[1] One of the objectives of this paper is to bridge this gap in the literature by developing a theoretical analysis of the different roles played by venture capitalists and angels in funding private firms, and to develop an understanding of the situations under which firms will make use of each type of financing.

The second objective of this paper is to develop an analysis of the dynamic features of financing contracts in the private equity market. The empirical evidences (as well as descriptions of individual cases) indicate that typically, firms undertake several rounds of private equity financing. Sometimes these different rounds of financing to a firm may come from the same source: for example, the same venture capital firm may provide multiple rounds of financing to a firm. In other situations, these different rounds of private equity financing may come from different sources: thus, a firm may be initially angel financed, and may later switch to venture capital financing; alternatively, a venture capitalist may provide funding initially, but may choose to sell his equity stake and leave the firm. The above situations lead us to ask several questions: First, are there any important differences between venture capitalist and angel financing contracts? Second, what motivates firms to switch from one form of private financing to another? Third, if firms make use of multiple rounds of financing from the same source, are there (and should there be) any systematic differences in the contracts between the entrepreneur and financier from one round to another (i.e., how do venture capital and angel financing contracts evolve over time?). Fourth, under what conditions are conflicts likely to arise between different kinds of financiers, or between the entrepreneur

[1] Frear et al (1996) estimate that around 250,000 angels invest between $10 billion and $20 billion in around 30,000 firms annually. This compares with around $6.6 billion committed in the venture sector of the organized private equity market in 1995, making the angel market several times larger. See Fenn, Liang and Prowse (1997), Prowse (1998), Wetzel (1983, 1987) for good descriptions of the private equity market.

and the financier from a previous round (for example, if a firm is initially angel financed, and the entrepreneur wishes to obtain a subsequent round of financing from a venture capitalist, the angel may object to the terms of this subsequent round of financing by the venture capitalist, and can potentially impose costs on the firm by preventing this financing from taking place).[2] We develop answers to many of these questions here.

Our analysis rests on a few assumptions based on certain stylized facts about the private equity market. First, we assume that in the early stages of a firm, the financier (venture capitalist or angel investor) may be able to add value to the firm, at least in some situations. Second, we assume that, while both the venture capitalist and the angel may be able to add value in this way, the venture capitalist is more capable of adding value (or equivalently, the venture capitalist can add value in more situations) than the angel. Third, we assume that, while the financier is capable of adding value, he has to engage in costly effort to add this value, so that he has to be given the appropriate incentives to put forth effort optimally on behalf of the firm. Fourth, we assume that, prior to the financier getting involved with the firm, the entrepreneur has private information regarding the nature of his own project (including the likelihood of the financier being able to add value to the project). However, if he does provide funding to the entrepreneurs project (thereby getting involved in its activities), the financier is able to learn more about the project over time, thus eliminating information asymmetry between the entrepreneur and the financier. Finally, we assume that the entrepreneurs effort is also required for the project to succeed, so that, the contract provided to the financier must be such that the entrepreneur also has the appropriate incentives to make the project a success.

In the above setting, we derive a variety of interesting predictions about entrepreneurs equilibrium choice of private equity financing and the structure of private equity financing contracts. First, we show why, in many situations, firms prefer venture capital financing over angel financing, even though venture capitalists are able to obtain a greater rate of return from their investment in the firm. Second, we characterize the conditions under which firms switch financing sources across financing rounds (angel to venture capitalist or venture capitalist to angel). Third, we characterize the equilibrium financing contracts between venture capitalists and entrepreneurs on the one hand, and angels and entrepreneurs on the other, thus allowing us to make predictions regarding

[2] An interesting case of such a financing conflict occurred in a financing involving Apex Investment Partners, described in Das and Lerner (1995).

the differences between the two kinds of contracts. Fourth, we make predictions regarding how the structure of venture capital contracts will evolve over time. Fifth, we develop implications for the composition of projects (early versus later stage) financed by venture capitalists and angels, and how this composition varies with changes in the scarcity of venture capital financing relative to angel financing. Sixth, we develop predictions regarding the announcement effects of various forms of financing, and the relationship between the dynamic path of firm financing and the quality of the firms projects. Finally, we characterize some of the situations under which financing conflicts between the entrepreneur and the early-stage financier over the terms of future rounds of financing can arise in equilibrium, giving rise to inefficient project implementation.

In the process of developing the above results, we offer a somewhat different rationale for the widely observed use of convertible features in venture capital contracts compared to those provided by the existing corporate finance and contracting literature. These rationales can be grouped into several categories. The first category deals with conflicts between stockholders and bondholders, and the related incentives of insiders to take on excessively risky projects (see, e.g., Green (1984)). A second category of papers deal with how the use of convertibles may be driven by asymmetric information between firm insiders and outsiders (e.g., Stein (1992), Constantitides and Grundy (1989), and Brennan and Kraus (1987)). Both of the above rationales apply to the use of convertibles by public as well as private firms. A third set of papers argue that the use of convertibles in venture capital contracts arises from the incompleteness of contracts between the between the venture capitalist and the entrepreneur, and the ability of different financial contracts to optimally switch control between the two (see, e.g., Hellmann (1998), Berglof (1994)).[3] [4] A fourth literature argues that convertible features arise from issues related to providing the right incentives to the entrepreneur in a setting of moral hazard (Cornelli and Yosha (1997), Repullo and Suarez (1998)).[5] [6] In contrast to this literature, in our paper the rationale for the use of convertible features emerges from the need to provide incentives to the venture capitalist to exert effort to add value to the firm. Further, to

[3] The incomplete contracting literature builds on the pioneering work of Grossman and Hart (1986). Three important papers in this literature are Aghion and Bolton (1992), Hart and Moore (1998), and Dewatripont and Tirole (1994). Many of the control theories of venture capital contracting make use of a modeling set-up similar to one or more of these papers.

[4] Marx (1993), who argues that when the venture capitalist is risk-averse, convertible preferred equity motivates the venture capitalist to intervene in the firm in response to poor performance. Gompers (1996) argues that venture capital convertible debt contracts are quite different from convertible debt in large public corporations. See also Bergemann and Hege (1998).

[5] The need to provide incentives to the entrepreneur to put forth optimal effort was argued in a number of early papers by Sahlman (see, e.g., Sahlman (1988, 1990)).

[6] See also Schmidt (1999) and Hellman (2001).

the best of our knowledge, none of the above papers have analyzed how the contract between the venture capitalist and the firm should evolve across multiple rounds of financing. Thus, our model predicts that, while convertible preferred equity or convertible debt will be used in both early and later stage financing, the relative magnitudes of the fixed income component and the upside (warrant) component will differ across financing rounds: while early stage financing with a venture capitalist will have more of a fixed income component and less upside, later rounds of financing with the same venture capitalist will feature a smaller fixed income component but a larger upside. Further, our analysis predicts that angel financing contracts are less likely to incorporate convertible features compared to venture capital contracts (see section 6 for details).[7]

Our paper is also related to other strands in the theoretical and empirical literature on private equity financing. Like our paper, Repullo and Suarez (1998) also study the moral hazard problem on the part of both the venture capitalist and entrepreneur. However, unlike in our paper, the driving factor in their paper is the allocation of the refinancing cost of the project across states. Because the later financier has to buy back financial contract from the initial financier, in their setting the optimal contract provides the initial financier a greater payoff when the state is high and a smaller payoff when the state is low, so that projects with smaller positive net present values can be financed. They also do not analyze the role of asymmetric information or the choice of firms between different kinds of financiers (angel or venture capitalist). Apart from this difference in deriving intuition, in their setting there is no asymmetric information between the entrepreneur and the outside financier; neither do they analyze the firm's choice between different kinds of financiers (angels and venture capitalists).[8] Admati and Pfleiderer (1994) study a setting in which a venture capitalist can observe the true state of a firm unlike the outside investors. They show that optimal investment decisions will be made by the firm in all states if and only if the venture capitalist is given a fixed-fraction equity contract, which eliminates his incentives to misrepresent the state to outside investors. Finally, our paper is related to the growing empirical literature providing detailed evidence on the structure of venture capital contracts in the U.S and other countries (prominent examples of this

[7] The practitioner literature indicates that both venture capital and angel contracts come in four basic forms: common stock, stock with warrants, convertible equity, and convertible notes (debt). Various additional provisions are added to these basic structures depending on the specifics of a given project or firm. See Bartlett (1995) for details.

[8] One paper which discusses the choice between different kinds of financiers is Leschinskii (1999). However, this paper is driven by the assumption that while the venture capitalist can fire the manager, business angels cannot, yielding the prediction that angel financing will be used only when replacing the manager is not optimal at any stage in the firm's life.

literature are Sahlman (1990), Gompers (1997), and Kaplan and Stromberg (2000, 2001)).[9]

The rest of the paper is structured as follows. Section 2 describes the basic model, while sections 3 and 4 characterize the equilibrium of the model and develop results. Section 5 develops an extension of the basic model to characterize the situations where conflicts may arise between the entrepreneur and the early stage financier over the terms of later financing rounds. Section 6 develops the empirical implications of our model. Section 7 concludes. The proofs of all propositions are confined to the appendix.

2 The Model

The model has three dates (t= 0, 1, 2) and three kinds of agents (entrepreneur, venture capitalist and angel), all of whom are risk neutral. The entrepreneur is endowed with a nondivisible project, which needs external financing I. Of the investment I, a minimum amount $\underline{I}_0$ is required at time 0; the entrepreneur may raise the remaining amount $(I - \underline{I}_0)$ either at time 0 or time 1. Thus if the entrepreneur has raised an amount $I_0 \geq \underline{I}_0$ at time 0, he will raise the remaining investment amount I_1 at time 1. We refer to the first period (time 0 to time 1) as the "earlier stage" of the firm's project, and the second period (time 1 to time 2) as the "later stage" of that project.[10]

The entrepreneur has two sources of external financing: The Venture Capitalist (VC, from now on) or the angel. There are two differences between the VC and the angel in our setting. First, the VC contributes not only capital but also effort, which helps in the successful implementation of the project. In contrast, the angel contributes only capital. Second, VC financing is scarce relative to angel financing.[11] At time 0 and time 1, the entrepreneur chooses between these two sources of financing depending on his private information and other relevant variables in the firm and the economy. We allow for the entrepreneur to refinance his project at time 1

[9] See also Cumming (2000) for Canadian evidence, Bascha and Walz (2001) for German evidence and Parhankangas and Smith (2000) for evidence from Finland.

[10] Private equity financing is often categorized into four stages. The "first round" refers to firms in the start-up, R&D, testing and market research stage. The "second round" refers to the prototype, further testing, and early expansion stage. The "third round" refers to full scale manufacturing and marketing. And the "fourth round" refers to the financing of firms which are profitable. In our model, early stage (time 0) financing can be thought of as corresponding to the first round in the above classification, where the VC's effort is less important relative to that of the entrepreneur's effort. On the other hand, late stage (time 1) financing corresponds to the second and the third round in the above classification, where the contribution of the VC's effort toward adding value is more significant. For example, the VC may help the firm hire technical as well as managerial talents, develope relationships with suppliers and potential clients etc.

[11] We will discuss the economic consequences of these two differences between VCs and Angels later on in this paper. In practice, this scarcity may arise from the fact that VCs commit not only financial capital but also human capital to firms that they are involved in, and the above human capital is limited.

(in case he decides to switch from an angel to a VC or vice versa). In other words, the amount raised from the time 1 financier can be more than the pure investment amount I_1 by the amount required to buy out the time 0 financier. The cash flows from the project are realized at time 2. We assume that there are only three possible outcomes for the project: "highly successful" (cash flow $\overline{X}$), "moderately successful" (cash flow $\underline{X}$), or "failure" (cash flow is 0), $0 < \underline{X} < I_0 < \overline{X}$.[12]

We assume that at time 0, the entrepreneur has private information about the likelihood of the VC being able to add value to his project. We model this private information in the following manner. The project will be in one of two possible states at time 1: state p or state n. If the project is in state p, then the VC 's effort will be productive with respect to the project. In other words, if the VC exerts effort in a firm in state p, he can increase the probability of the project being highly successful. If, on the other hand, the project is in state n at time 1, the VC's effort is not productive, so that the VC will have no impact on the probability of project success. At time 0, there are two kinds of projects: type G projects, with a probability λ_G of being in state p at time 1; and type B projects with probability λ_B, $0 < \lambda_B < \lambda_G$, of being in state p. We model the entrepreneur's private information by assuming that while entrepreneurs observe the type of their own projects, outsiders observe only the prior probability θ of a project being of type G.

We assume that the realization of the state (p or n) is observable by the entrepreneur and the firm's current financier, but not by outsiders. Thus, if a financier was involved with a firm from time 0 itself, he has the same information about the firm at time 1 as the entrepreneur. As a result, any further financing of the firm undertaken by that financier at time 1 would not suffer from asymmetric information. In contrast, if the firm switches financiers at time 1, the time 1 financing would suffer from information asymmetry, since the new financier would not observe the true time 1 state of the firm.[13] Since only the entrepreneur and the inside financiers observe the time 1 state, publicly enforceable contracts cannot be written on these states. Thus, we assume that all contracting is done on time 2 cash flow realizations. The sequence of events is summarized in

[12] The assumption that $\underline{I}_0 > \underline{X}$ ensures that the project cannot be financed through risk-free debt at time 0.

[13] We assume that there are a number of VCs, Angels and a number of projects of all stages, types and states in the economy. This implies that each VC or Angel is able to select both the stage (time 0, time 1) and the nature (type G or type B for a time 0 project and state-p or state-n for a time 1 project) of the project he wants to invest in, provided it is in the interest of the corresponding entrepreneur to select such financing. Conversely, each enterpreneur will also have available to him the financing (VC or Angel) of his choice, and the financing will proceed provided that it is in that financier's interest to invest in such a firm.

figure 1, and the project payoff and information structure is depicted in figure 2.

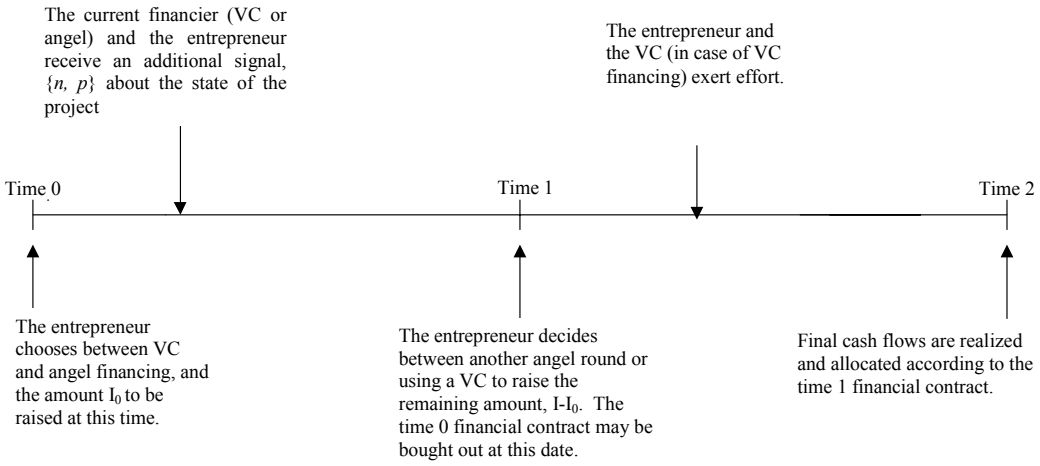
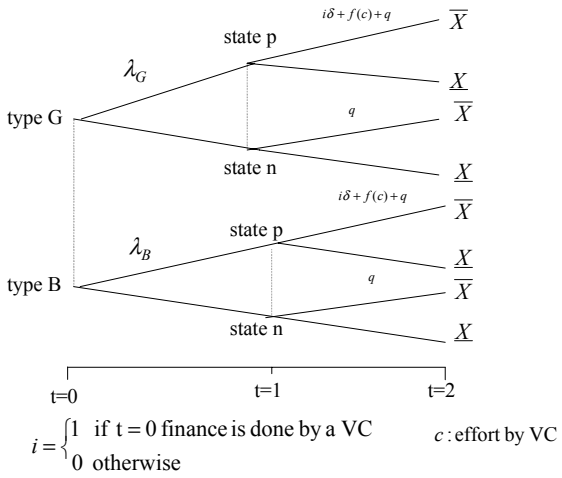


Figure 1: Sequence of Events

Figure 2: Payoff and Information Structure



2.1 The Entrepreneur

The project's success is affected by the entrepreneur's effort as well as (potentially) that of the VC. We model the entrepreneur's effort in the following way. The entrepreneur chooses to exert effort or not. If the entrepreneur exerts effort, he incurs a private cost k, but is able to ensure that the project is at least moderately successful (i.e., the project cash flow is greater than zero). If, on the other hand, he does not exert effort, he does not incur

any cost, but guarantees project failure (i.e., the project cash flow is 0 for sure). We assume that the above effort is exert in the second period (between time 1 and time 2). The above assumption captures the real-world notion that a minimum amount of effort by the entrepreneur is required for the success of the project. The entrepreneur's objective in choosing between angel and VC financing at time 0 and at time 1 is to maximize his time 2 expected payoff net of effort costs.

2.2 The Venture Capitalist

As discussed before, if the project is in state p, the VC can add value by exerting effort. We model the VC's effort and its impact on the project as follows. If the entrepreneur does not exert any effort, the project is guaranteed to fail regardless of the VC's effort and the state the project is in. If, however, the entrepreneur exerts effort, the VC can increase the probability that the project is highly successful (rather than moderately successful) by exerting effort (provided, of course, that the project is in state p). Thus , in the absence of the VC's effort, the probability of the project being highly successful (i.e., cash flow $\overline{X}$) is q. If the VC exerts effort, this probability increases to $q+ f(c)$, where c is the continuous effort level chosen by the VC. We assume that the VC incurs a private cost of effort.[14] The VC's effort cost is monotonically increasing in his effort level. We will use c to denote both the VC's effort level and the corresponding effort cost incurred by him. We assume that f is increasing and concave in c, with $f(0) = 0$ and $q+ f(\infty) < 1$. We assume that, like the entrepreneur, the VC's effort is also exerted in the second period (between time 1 and time 2).

Recall that, in state n, the VC's effort has no impact on the project success; we assume that probability of the project being highly successful (cash flow $\overline{X}$) in this case to be equal to q. VC may start financing a project either at time 0 (we refer to this as "earlier stage"financing) or at time 1 (we refer to this as "later stage" financing). Even though the VC exerts effort only in the second period, one advantage of having VC finance the project at an early stage is that the VC is able to accumulate knowledge about the firm's project in the first period, thus enabling him to be more efficient in adding value to the firm if its project turns out to be in state p. We capture this notion by assuming that for earlier-stage financing, the probability of the project being highly successful is given by $q + \delta + f(c)$ if the project ends up in state p at time 1. In other words, everything else equal, this

[14] As is standard in the moral hazard literature, we assume that the VC's effort is not observable. However, since the entrpreneur observes the VC's financial contract, he can infer the VC's effort choice in equilibrium, so that the bargaining between the entrepreneur and the VC will reflect this effort choice as well.

probability is greater than the probability of the project being highly successful in the case of later stage VC financing by δ.

The VC's ability to add value to the firm through his effort gives him considerable bargaining power as a financier. We model this by assuming that the VC can bargain with the entrepreneur and extract a fraction, $\rho < 1$, of the incremental project NPV created as a result of the VC's effort.[15] [16]

Since VC financing is scarce relative to angel financing, the VC requires a minimum (threshold) NPV, denoted by $R > 0$, from investing in a firm's project for the entire duration of the project. One can think of R as the NPV the VC can obtain from investing his capital in an alternative investment opportunity for two periods. For simplicity, we assume that the VC's alternative investment opportunity has symmetric cash flow net of investments across periods, so that if the VC invests in the opportunity for only one period, the NPV obtained will be $\frac{R}{2}$.[17] This contrasts with the angel, who only insists that the NPV from any investment he makes is positive. Since R reflects the current level of scarcity of VC financing in the economy, it may vary according to the extent of this scarcity. Thus, R will be high when VC financing is very scarce, and low when VC financing is less scarce.[18]

The objective of the VC in making his effort choice, as well as his investment decision, is to maximize his expected payoff net of his effort cost.

2.3 The Angel

As discussed before, the angel is a pure supplier of capital; he cannot affect the probability of project success through his effort. Thus, the angel has no bargaining power relative to the entrepreneur. Further, angel financing is abundant, so that the angel invests in all projects which yield him a positive NPV.

[15] For simplicity, we assume that the fraction of the surplus given to the VC is determined by Nash bargaining between the VC and the entrepreneur.

[16] It seems most natural to assume that the bargaining between the VC and the entrepreneur is only over the incremental value created by the VC. We thus implicitly assume that the VC's bargaining power is a function only of his ability to add value to the project. In practice, ρ can also be affected by the scarcity of the VC financing relative to Angel financing. For simplicity, we choose not to model this dependence of ρ on VC scarcity. We can show that explicitly accounting for the dependence of ρ on VC scarcity will not change the qualitative nature of our results.

[17] Throughout this paper we will refer to R as the VC's threshold NPV. It should, however, be understood that this threshold NPV is measured over two periods, so that if the VC's capital is tied up in a project for only one period, the minimum NPV required by the VC from that project will only be $\frac{R}{2}$.

[18] Scarcity of VC financing is a natural assumption to make in our setting, since all firms in the economy can benefit from the value added by the VC's effort. Further, as Lerner (1998) has pointed out, the supply of venture capitalists is quite inelastic, since the effective oversight of young companies is a highly specialized skill that can only be developed with years of experience. Since the hallmark of venture capital financing is value-addition, this means that venture capital firms cannot rapidly increase the supply of such financing by hiring new venture capitalists.

3 Equilibrium

Definition of equilibrium: The equilibrium concept we use here is Perfect Bayesian Equilibrium (PBE). An equilibrium consists of (i) the entrepreneur's time 0 and time 1 financing choices (between angel and VC), the contracts offered to these financiers, and the amounts raised; (ii) the entrepreneur's choice of effort in period 2; (iii) the VC's choice of effort in period 2, if VC financing is chosen by the entrepreneur and (iv) the financiers' decision to invest in the firm's project or not. Each of the above choice must be such that: (a) the choices of each party maximize his objective, given the equilibrium beliefs and choices of others; (b) in the case of a separating equilibrium, the equilibrium financing contract is that which maximizes the type G or the state-p firm's expected payoff; if there is more than one such contract, the contract which induces separation for the largest set of model parameters is defined as the equilibrium contract;[19] (c) in the case of a pooling equilibrium, the equilibrium financing contract is that which maximizes the type G or the state-p firm's expected payoff; (d) the belief of each party is consistent with the equilibrium choices of the others; further, along the equilibrium path, these beliefs are formed using Bayes rule. Any deviation from his equilibrium strategy by any party is met by beliefs by other parties which yield the deviating party lower expected payoff compared to that obtained in equilibrium.

To facilitate exposition, we present the equilibrium in reverse order: we first discuss the equilibrium behavior of various parties at time 1, for a given financing choice at time 0, and then go on to discuss the overall equilibrium.

3.1 The Entrepreneur's time 1 Financing Choice if the time 0 Financing was done by a VC

There are two kinds of project at time 1: those in state p and those in state n. If the firm is in state p, the VC's effort will be productive in the firm. Further, since the VC is the time 0 financier, he will clearly have the same information at time 1 as the entrepreneur (both observe the realized state at time 1). For both of these reasons, it is beneficial for a firm in state p to obtain another round of financing from the same VC who funded it at time 0. In this case, not only can the VC provide the requisite effort to add value to the firm, but it can also be ensured that the contract between the VC and the entrepreneur does not suffer from asymmetric information. This, in turn, means that the contract between the entrepreneur and the VC can provide the latter

[19] This feature of our equilibrium definition is somewhat similar to the "robustness" requirement for financial contracts imposed by Admati and Pfleiderer (1994).

with stronger incentives. Further, since the VC has accumulated knowledge about the firm in the first period, he is more efficient at adding value to the firm.

If, on the other hand, the firm is in state n, then the VC's effort is not productive, in the sense that the VC cannot add value to the firm. Further, such a firm cannot mimic a p-state firm, since as discussed above, a p-state firm would find it optimal to get its time 1 funding from the VC which financed it at time 0. As a consequence, it is not possible for a firm in state n to sell over-valued securities. This, in turn, implies that a firm in state n would be indifferent between raising funding from an angel and a VC.

From the VC's point of view, financing a firm in state p would be optimal, since the VC is able to add value to such a firm. Since the VC is able to extract a fraction ρ of the value added by him, funding p-state firm also ensures that he is able to meet his threshold NPV of R. If, however, the firm is in state n, the VC cannot add any value to the firm, and will therefore choose to leave the firm rather than continue funding the firm at time 1 (since he will be able to obtain a higher NPV by investing in his alternative investment opportunity). We summarize these insights in proposition 1.[20]

Proposition 1 ***(Choice between the angel and the VC)***
The equilibrium actions of the entrepreneur, the VC and the angel at time 1 can be characterized as follows :
(i)At time 1, a firm in state p will continue to use VC financing, with the contract specified in proposition 2.
(ii) A firm in state n will use angel financing.
(iii) The VC will not continue to provide funding to any firm in state n, but will instead leave the firm, selling his stake to an angel.

We now turn to the optimal design of financial contract between the entrepreneur whose firm is in state p, and the VC who is continuing to fund the firm at time 1. The objective of the contract design here is to ensure that both the VC and the entrepreneur put forth optimal effort. Tthe entrepreneur designs the contract to maximize his objective subject to: (i) the VC's incentive compatibility (IC) constraint, which incorporates the notion that the VC chooses his effort to maximize his objective; (ii) the entrepreneur's own incentive compatibility constraint, which ensures the notion that the entrepreneur will exert effort only if it is optimal for him to do so; (iii) the VC's individual rationality (IR) constraint, which ensures that the VC obtains adequate compensation for the investment amount he provides to the firm and his effort cost, and also receives his share ρ of the value added by

[20] The out-of-equilibrium belief supporting this equilibrium is that, if a firm is seeking outside financing at time 1, then the outsiders infer that it is in state n.

him to the firm; (iv) limited liability constraints.[21]

Let (a, b) specify the contract offered by the entrepreneur to the VC, where a is the share of the total cash flow of the project to the VC if $\overline{X}$ is realized (i.e., the project is highly successful) and b is the VC's share if $\underline{X}$ is realized (i.e., the project is moderately successful). By limited liability, $0 \leq a \leq 1$ and $0 \leq b \leq 1$. We will not worry about the VC's share when the project is a failure (i.e., the cash flow is zero), since, in this case, all parties, including the VC, get a zero cash flow. Denoted by V_H the value of VC's time 0 financial contract at time 1 when the firm is in state p; V_L is the value when the firm is in state n. We assume that, when the VC continues to fund the firm at time 1, the time 0 financial contract is swapped for a new contract at time 1 (in other words, the time 1 contract would also compensate the VC for the value of the time 0 contract, in addition to the variables in the VC's IR constraint, discussed above). Thus, the entrepreneur's problem can be characterized as:

$$\underset{a,b}{Max} \quad (1-a)(q+\delta+f(c))\overline{X} + (1-b)(1-q-\delta-f(c))\underline{X} - k., \tag{1}$$

$$s.t \quad c \in \arg\max\{a(q+\delta+f(c))\overline{X} + b(1-q-\delta-f(c))\underline{X}] - c\}, \tag{2}$$

$$0 \leq (1-a)(q+\delta+f(c))\overline{X} + (1-b)(1-q-\delta-f(c))\underline{X} - k, \tag{3}$$

$$a(q+f(c)+\delta)\overline{X} + b(1-q-\delta-f(c))\underline{X} \geq I_1 + \rho f(c)\Delta X + (1-\rho)c + V_H. \tag{4}$$

In the above, the constraint (2) is the VC's incentive compatibility constraint. The constraint (3) is the entrepreneur's incentive compatibility constraint (recall that k is the entrepreneur's cost of effort). The constraint (4) is the VC's individual rationality constraint.

It is clear that the VC's individual rationality constraint has to be binding at the optimum (since we are studying the case where the limited liability constraints are not binding). Using it to simplify the objective function, the problem is equivalent to maximizing $(1-\rho)(f(c)\Delta X - c) - k$ subject to the ICs and the IR. If the effort c is contractible, the first best effort level is given by the following first order condition:

$$f'(\widehat{c}) = \frac{1}{\Delta X}, \tag{5}$$

where $\Delta X \equiv \overline{X} - \underline{X}$. We assume that $\widehat{c} > 0$.

[21] We will focus here only the more interesting case where investment amount required by the project are such that the limited liability constraints on "a" and "b" are not binding at time 1. Details of the case where these constraints bind are available to interested readers upon request.

However, the VC's effort is determined by his IC, which yields the first order condition:

$$(a\overline{X} - b\underline{X})f'(c) = 1. \tag{6}$$

Rearranging, we get:

$$f'(c) = \frac{1}{a\overline{X} - b\underline{X}}. \tag{7}$$

(7) defines the VC's effort level c as a function of a and b. We now show that giving the VC equity alone cannot achieve the first best effort level. The reason is that by the entrepreneur's IC (3), if the equity is the contract given to the VC, then we must have $a = b < 1$. But this means that $a\overline{X} - b\underline{X} < \Delta X$. Comparing (5) with (7), we can see that equity always induces the VC to under-invest in effort. The intuition here is that equity limits the sensitivity of the VC's payoff to the value of the project, which leads the VC to under-invest in effort. We summarize the solution to the above contract design problem in the following proposition.

Proposition 2 ***(Equilibrium Financing Contract)*** *(i) The equilibrium financing contract between a firm in state p and the VC has the following features:*

(a) The financing contract at time 1 is:

$$a^* = \frac{1}{\overline{X}}\{V_H + I_1 + [1 - q - \delta - (1-\rho)f(\widehat{c})]\Delta X + (1-\rho)\widehat{c}\}; \tag{8}$$

$$b^* = \frac{1}{\underline{X}}\{V_H + I_1 - [q + \delta + (1-\rho)f(\widehat{c})]\Delta X + (1-\rho)\widehat{c}\}. \tag{9}$$

(b) $a^* > b^*$.

(c) Given the above contract, the first best effort level, $\widehat{c}$, is always achievable.

(d) Giving the VC equity alone ($a = b$) cannot implement the first best outcome.

(ii) A firm in state n will use angel financing. An entrepreneur in state n is indifferent to a variety of financing contracts to be given to the angel, as long as the value of the contract is $I_1 + V_L$.

Clearly, in order to induce the VC to put forth optimal effort, his payoff when the firm is highly successful (cash flow $\overline{X}$) has to be greater than when the firm is moderately successful (cash flow $\underline{X}$). This is ensured by setting $a^* > b^*$. Such a contract can be implemented by giving the VC convertible preferred equity (or equivalently preferred equity with warrants), convertible debt (or equivalently debt with warrants), or equity with warrants. Such a contract dominates any contract implementable by giving the VC equity alone ($a = b$), since contracts which set $a > b$ make the VC's payoff more sensitive to his effort, thus ensuring that the VC puts forth more effort compared to the case where $a = b$. Recall that the contracting here is between the entrepreneur and the VC

undertaking a second-round financing, so that there is no asymmetric information between the two contracting parties. This, in turn, allows the entrepreneur to provide very strong incentives to the VC, which (as we will see in the next section), will not be possible in the presence of asymmetric information between the contracting parties.

The equilibrium financing contract ensures that the entrepreneur also has enough incentive to exert effort. The entrepreneur is residual claimant here, receiving the cash flow left over after paying the VC.

We now come to the design of financial contract between a firm in state n and its financier. As discussed before, in this case, the entrepreneur chooses angel financing. The entrepreneur's problem is now to maximize his objective, subject to his own incentive compatibility constraint (which ensures that he exerts effort) and the angel's individual rationality constraint (which ensures that the angel is compensated for the investment amount he provides to the firm, as well as the amount he provides to the entrepreneur for buying out the VC who financed the firm at time 0).

The entrepreneur's problem is thus:

$$\underset{a,b}{Max} \quad (1-a)q\overline{X} + (1-b)(1-q)\underline{X} - k \tag{10}$$

$$s.t \quad 0 \leq (1-a)(q+\delta+f(c))\overline{X} + (1-b)(1-q-\delta-f(c))\underline{X} - k \tag{11}$$

$$aq\overline{X} + b(1-q)\underline{X} \geq I_1 + V_L \tag{12}$$

The solution to this contract design problem is given by part (ii) of proposition 2.

In this case, since the angel cannot add any value through his effort, the form of the contract is irrelevant, as long as the angel is compensated for the amount he provides to the firm. Thus, the entrepreneur is indifferent between providing the angel equity, convertible preferred equity, or convertible debt. In equilibrium, the angel buys out the financing contract of the VC who financed the firm at time 0 at their time 1 full-information value, V_L (we will discuss the determination of V_L and V_H in section 4, when we characterize the overall equilibrium of the model).

3.2 The Entrepreneur's time 1 Financing Choice if the time 0 Financing is done by an Angel

We now discuss the time 1 financing choice of a firm whose time 0 financing was undertaken by an angel. As in section 3.1, there are two kinds of projects at time 1: those in state p and those in state n. As in section 3.1, in this case also, it is beneficial for a firm in state p to seek VC financing at time 1, since the VC can add value to the firm through his effort. Here, however, any VC financing is undertaken under asymmetric information: since the time 0 financier was an angel, any VC would be new to the firm and would therefore not have observed the realization of the firm's state at time 1. As in section 3.1, the VC's effort is not productive if the firm is in state n. However, in contrast to the case discussed in section 3.1, a firm in state n has an incentive to mimic a firm in state p. Doing so allows the firm to sell overvalued securities to the VC who provides funding to the firm at time 1. Thus, an equilibrium which involves separation between the state p and state n firms has to satisfy the incentive compatibility conditions which ensure the firm in state n will not find it profitable to mimic a firm in state p and vice versa.

First, we present the incentive compatibility condition of an entrepreneur in state n. If this firm mimics the firm in state p, it will obtain VC financing so that its payoff is given by:[22]

$$(1-a)q\overline{X} + (1-b)(1-q)\underline{X} + V_H - k, \tag{13}$$

If, however, the firm in state n does not mimic the firm in state p, it will reveal himself to be an n-state firm and thus obtain only angel financing, its payoff is thus:

$$[q\overline{X} + (1-q)\underline{X}] - I_1 - k, \tag{14}$$

Combining the above two conditions, we get the incentive compatibility condition of the n-state firm as

$$[q\overline{X} + (1-q)\underline{X}] - I_1 - k \geq (1-a)q\overline{X} + (1-b)(1-q)\underline{X} + V_H - k, \tag{15}$$

We now consider the firm in state p. If the firm mimics the n-state firm, it will not able to get VC financing. If, however, he does not mimic the n-state firm, he will reveal himself as a p-state firm, and is thus able to obtain

[22] We assume that the entrepreneur always exerts effort here, we will give the conditions which guarantee it is the case later in the overall equilibrium.

VC financing. Therefore, the incentive compatibility condition for an entrepreneur of a firm in state p is given by:

$$[q\overline{X} + (1-q)\underline{X}] + (1-\rho)(f(c)\Delta X - c) - k - I_1 \geq [q\overline{X} + (1-q)\underline{X}] - I_1 - k \tag{16}$$

We will show in the appendix that the above incentive compatibility condition for the p-state firm will always hold. If, in addition, (15) also holds, we have the following separating equilibrium:

Proposition 3 ***(Choice between the angel and the VC)***
(i) At time 1, a firm in state p will obtain its second round financing from a VC, with the contract specified in proposition 4, and buy out the time 0 financier, the angel.
(ii) A firm in state n at time 1 will continue to use angel financing.

We now turn to the optimal design of contract between the entrepreneur and the VC (in the case of a p-state firm) and the entrepreneur and the angel (in the case of an n-state firm) in the above separating equilibrium. We first consider the contract between the p-state entrepreneur and the VC.

The entrepreneur designs the contract to maximize his objective subject to: (i) the VC's incentive compatibility constraint; (ii) the entrepreneur's own incentive compatibility constraint; (iii) the VC's individual rationality constraint; (iv) the firm's and the VC's limited liability constraints; and (v) the incentive compatibility conditions, (15) and (16), of the n-state and p-state firms respectively. Thus the entrepreneur's contract design problem is given by

$$\underset{a,b}{Max} \quad (1-a)(q+f(c))\overline{X} + (1-b)(1-q-f(c))\underline{X} - k., \tag{17}$$

$$s.t \quad c \in \arg\max\{a(q+f(c))\overline{X} + b(1-q-f(c))\underline{X}] - c\}, \tag{18}$$

$$0 \leq (1-a)(q+f(c))\overline{X} + (1-b)(1-q-f(c))\underline{X} - k, \tag{19}$$

$$a(q+f(c))\overline{X} + b(1-q-f(c))\underline{X} \geq I_1 + \rho f(c)\Delta X + (1-\rho)c + V_H, \tag{20}$$

$$(15) \text{ and } (16). \tag{21}$$

Recall that, unlike in section 3.1, the contracting here is done under asymmetric information between the VC and the entrepreneur, so that we need to impose incentive compatibility conditions (15) and 16), as well as the constraints (i)-(iv) imposed on the entrepreneur's maximization problem in section 3.1. In summary, the objective of the contract design here is to induce that both the VC and the entrepreneur put forth optimal effort, while ensuring separation between the n-state and p-state firms.

The n-state firm's incentive compatibility constraint can be simplified to:

$$\rho f(c(P)\Delta X + (1-\rho)c(P) - Pf(c(P)) \geq 0, \tag{22}$$

where $P \equiv a\overline{X} - b\underline{X}$. P can be thought of as measuring the "power" of the contract between the VC and the entrepreneur. It should be clear from the VC's first order condition (7) that the VC's effort level is a function only of P.

We summarize the solution to the above contract design problem in the following proposition:

Proposition 4 ***(Equilibrium Financing Contract)****(i) The equilibrium financing contract between a firm in state p and the VC has the following features:*

(a) The financing contract at time 1 is:

$$a^* = \frac{1}{\overline{X}}\{I_1 + V_H + (1-q)P^*\}, \tag{23}$$

$$b^* = \frac{1}{\underline{X}}\{I_1 + V_H - qP^*\}. \tag{24}$$

b) $a^* \geq b^*$ *if and only if*

$$\frac{\Delta X}{\underline{X}} \leq \frac{P^*}{I_1 + V_H - qP^*}, \tag{25}$$

where $P^* = a^*\overline{X} - b^*\underline{X}$ *is the maximum solution to (22) holding as equality and* c^**, the VC's equilibrium effort choice, is such that* $f'(c^*)P^* = 1$*.*

(c) The first best effort from the VC can never be achieved; furthermore, compared with the first best effort, the VC under-invests in effort even under the above optimal contract.

(ii) A firm in state n will continue to use angel financing. An entrepreneur in state n is indifferent to a variety of financing contracts to be given to the angel, as long as the value of the contract is $I_1 + V_L$*.*

Notice the above equilibrium contract provides weaker incentives for the VC to exert effort compared to the case in section 3.1, where the contracting proceeds under symmetric information between the entrepreneur and the VC. Asymmetric information prevents the provision of a stronger incentives to the VC. This is because the higher the incentive given to the VC, the higher the incentive of the n-state firm to mimic the p-state firm. This arises from the fact that the benefit to the n-state firm from mimicking the p-state firm is given by $(a\overline{X} - b\underline{X})f(c)$, which is an increasing function of $(a\overline{X} - b\underline{X})$. This, in turn, means that the power of the contract P, has to be smaller in this case than in section 3.1, thus leading the VC to under-invest in effort here. The above contract can be implemented by giving the VC convertible preferred equity (or equivalently, preferred equity with warrants), convertible debt (or equivalently, debt with warrants), or equity with warrants (though equity with warrants can

be used only when $a^* \geq b^*$). However, the value of the "upside" to the VC will be smaller here compare to the contract in section 3.1. Controlling for the total value of the securities issued to the VC, this implies the value of the conversion option (or warrant value when preferred equity or debt with warrants are issued) will be lower in this case compared to the contract in section 3.1. As in section 3.1, the equilibrium financing contract also ensures that the entrepreneur is motivated to exert effort in equilibrium. The entrepreneur is the residual claimant here also, receiving the cash flows left over after paying the VC.

The design of the financing contract between an entrepreneur with a firm in state n and the angel is very similar to that in section 3.1. We will therefore not discuss this contract design in detail here. The solution to this design problem is given by part (ii) of proposition 4.

4 The Overall Equilibrium

We now describe the overall equilibrium of the model. Depending upon the availability of the VC financing, one can think of three regimes in terms of the nature and the stage of projects financed by the VC. First, consider the case where VC financing is very scarce ("high scarcity"). In this case, the threshold NPV, R, required by the VC to invest in projects is high. Next, consider the other extreme where the VC financing is relatively freely available ("low scarcity"). In this case, the threshold NPV, R required by the VC will be rather low. Finally, consider the case where VC financing is moderately scarce: here the threshold NPV, R will lie between the two extremes discussed earlier.

We will demonstrate below that the nature and the stage of the projects financed by the VC will differ across these regimes. In particular, we will show that in the high-scarcity regime, the VC will only fund the later stage (time 1) projects in state p, leaving all earlier stage (time 0) projects and later projects in state n to the angel. In the moderate-scarcity regime, the VC will fund only type G earlier stage projects and later stage projects in state p; he will leave type B earlier stage projects and later stage projects in state n to the angel. Finally, in the low-scarcity regime, the VC will fund all earlier stage projects (both type G and type B) and later stage projects in state p, leaving later stage projects in state n to the angel.

The benefit to a VC from starting to finance a project at time 0 (earlier stage financing) is that he is able to add more value to the firm (and thereby gain a fraction ρ of this larger value added) for the two reasons discussed

before. However, the danger of starting to finance the firm early is that the firm may end up in state n, so that the VC may have to leave the firm at time 1 without adding any value, thus obtaining a low NPV from the capital employed. On the other hand, the benefit to the VC from starting to finance a firm only at time 1 (later stage financing) is that there is no uncertainty about the state of the firm (since the VC will only finance a firm in state p). The disadvantage of doing this, however, is that the VC is able to add only a smaller amount of value at time 1 (for the reasons discussed before). If the NPV to the VC from starting to finance a state-p project only at time 1 is greater than that from financing a type G project starting at time 0, the following inequality holds:

$$\rho\lambda_G(\delta\Delta X + f(\widehat{c})\Delta X - \widehat{c}) + (1-\lambda_G)\frac{R}{2} < 2\rho(f(c^*)\Delta X - c^*), \tag{26}$$

where the LHS is the NPV from funding a state-p project starting only at time 1, while the RHS gives the NPV from funding a type G project starting at time 0. If, on the other hand, the VC is better off starting to finance a project at time 0 rather than at time 1, then the above inequality is reversed. The above discussion of the nature and stage of the projects financed by a VC under different scarcity regimes assumes that inequality (21) holds; throughout the paper we will assume that this is the case (unless otherwise stated). We now formally characterize the equilibrium under different regimes of VC financing scarcity.

4.1 Moderate Scarcity of VC Financing

We will begin by describing the moderate-scarcity regime, where the equilibrium is separating at both time 0 and time 1. We define moderate-scarcity regime of VC financing as one where the following inequality holds:

$$\rho\lambda_B(\delta\Delta X + f(\widehat{c})\Delta X - \widehat{c}) + (1-\lambda_B)\frac{R}{2} \leq R \leq \rho\lambda_G(\delta\Delta X + f(\widehat{c})\Delta X - \widehat{c}) + (1-\lambda_G)\frac{R}{2}. \tag{27}$$

The above condition implies that the threshold NPV R of the VC is below that NPV obtained by him from financing a type G project, but above that from financing a type B project. If the above condition holds, the VC is only willing to fund type G projects at time 0, since the NPV from a type B project is below his threshold NPV, R. Further, by (26), the NPV from a p-state project at time 1 will also be above the VC's threshold NPV, so that he would be willing to fund all such projects as well.

At time 0, the VC suffers from asymmetric information, in that he cannot a priori distinguish between the type G and type B firms. The only difference between the two types of entrepreneur is their probability λ_G and

λ_B respectively of their firms of being in state p. Since both λ_G and λ_B are positive, both of these entrepreneurs can benefit from having the VC finance their firms if it ends up in state p at time 1. Recall that if the VC starts financing the firm at time 0 (rather than at time 1), he is able to add more value to the firm due to two reasons: more efficient contracting is possible between the VC and the entrepreneur if the firm is in state p at time 1, and further, the VC is able to accumulate knowledge about the firm if he enters the firm at time 0.

Thus the VC is a desirable financier than the angel for both types of firms. On the other hand, the VC financing is more expensive than the angel financing for both kinds of firms at time 0, since the pricing of the financial contract offered by the firm to the VC will incorporate the VC's requirement that he receives a fraction ρ of the value he add to the firm in state p. Further, since the probability λ_B of a type B being in state p is lower than the corresponding probability λ_G for a type G firm, the VC is more expensive for a type B firm than for a type G firm.

A separating equilibrium will exist if the benefit of VC financing at time 0 exceeds its cost for a type G firm but not for a type B, so that the former chooses VC financing while the later chooses angel financing. The incentive compatibility conditions of the type B and type G firm respectively, which ensure that this is the case, are given by (28) and (29) repectively.

$$\lambda_B[\overline{X}(q+f(c^*))+\underline{X}(1-q-f(c^*))]+(1-\lambda_B)W_L-I-\lambda_B[\rho f(c^*)\Delta X+(1-\rho)c^*]\geq \tag{28}$$
$$\lambda_B(W_H-V_H)+(1-\lambda_B)(W_L-V_L)-(I-I_0)-\lambda_B[\rho(f(\widehat{c})\Delta X-\widehat{c})+\widehat{c}].$$

In the above, W_H and W_L are the firm's time 1 value in state p and n respectively. $W_H \equiv \overline{X}(q+f(\widehat{c})+\delta)+\underline{X}(1-q-f(\widehat{c})-\delta)$, $W_L \equiv \overline{X}q+\underline{X}(1-q)$. Similarly, V_H and V_L are the value of the time 0 financier's (VC in this case) contract at time 1 in state p and n, respectively; we will define V_H and V_L precisely later. The LHS of (23) gives the payoff to the type B firm from not mimicking the type G firm, thereby revealing its type and obtaining only angel financing; the RHS gives the payoff to type B from mimicking the type G, and thus obtaining VC financing.

$$\lambda_G W_H+(1-\lambda_G)W_L-I-\lambda_G[\rho(f(\widehat{c})+\delta)\Delta X+(1-\rho)\widehat{c}]\geq \tag{29}$$
$$\lambda_G[\overline{X}q+\underline{X}(1-q)+(1-\rho)(f(c^*)\Delta X-c^*)]+(1-\lambda_G)W_L-I.$$

The type G's incentive compatibility condition (29) ensures that its payoff from not mimicking the type B and truly revealing its type (and obtaining VC financing) is greater than that obtained by mimicking the type B (and obtaining angel financing). If (28) and (29) are satisfied, we have a separating equilibrium at time 0. The following proposition characterizes this equilibrium.

Proposition 5 ***(Choice between the angel and the VC under Moderate Scarcity)***
(i) At time 0, the type G entrepreneur seeks (and receives) VC financing;
(ii) The type B entrepreneur seeks (and receives) angel financing.

We now turn to the optimal design of financial contract at time 0, and the optimal amount raised by the firm at time 0. We define a time 0 contract as $(\overline{a}, \overline{b}, \underline{a}, \underline{b})$, where $(\overline{a}, \overline{b})$ are the time 2 shares of the cash flows $\overline{X}$ and $\underline{X}$ respectively if the firm is in state p at time 1; $(\underline{a}, \underline{b})$ are the time 2 shares of the cash flows $\overline{X}$ and $\underline{X}$ respectively if the firm is in state n at time 1. Recall that the time 0 contracting is complicated by the fact that contract cannot be written on the time 1 states, but only on time 2 cash flows. Thus, a time 0 contract cannot distinguish between the case where the cash flow $\overline{X}$ is realized by a state-p or a state-n firm. We assume that the time 0 contract between the entrepreneur and the VC are two-period contract on $\overline{X}$ and $\underline{X}$, which are renegotiable at time 1. Renegotiation here takes the form of a buyout (or swap) at time 1, where the time 0 financier receives the equilibrium value of the financial contract given up by him at time 1.

Since contracting cannot be contingent on time 1 states, the entrepreneur has to ensure that the VC picks the right contract corresponding to the time 1 state, i.e., the sharing rule chosen by the VC in state p is $(\overline{a}, \overline{b})$ and in state n is $(\underline{a}, \underline{b})$. Thus any contract $(\overline{a}, \overline{b}, \underline{a}, \underline{b})$ has to satisfy the following incentive compatibility conditions:

$$\overline{a}(q + f(\widehat{c}) + \delta)\overline{X} + \overline{b}(1 - q - f(\widehat{c}) - \delta)\underline{X} \geq \underline{a}(q + f(\widehat{c}) + \delta)\overline{X} + \underline{b}(1 - q - f(\widehat{c}) - \delta)\underline{X} \tag{30}$$

$$\underline{a}q\overline{X} + \underline{b}(1 - q)\underline{X} \geq \overline{a}q\overline{X} + \overline{b}(1 - q)\underline{X}. \tag{31}$$

The objective of contract design by the type G entrepreneur at time 0 is to ensure separation between the two types of firms for the largest set of parameter values, subject to the above mentioned incentive compatibility conditions of the VC, and also the VC's individual rationality condition (32) (which ensures that he is compensated for the investment provided to the firm, and also obtains his share ρ of the incremental expected value added by him to the firm when he starts financing the firm at time 0).

$$I_0 + \rho\lambda_G\delta\Delta X \leq V_H\lambda_G + V_L(1 - \lambda_G). \tag{32}$$

In (32), V_H represents the time 1 value in state p of the contract provided to the VC at time 0, given by $V_H \equiv \overline{a}(q + f(\widehat{c}) + \delta)\overline{X} + \overline{b}(1 - q - f(\widehat{c}) - \delta)\underline{X}$; similarly V_L represents the time 1 value of the same contract in state n, given by $V_L \equiv \underline{a}q\overline{X} + \underline{b}(1 - q)\underline{X}$.

The above problem translates into minimizing the type B firm's incentive to mimic the type G (because we can show that the type G entrepreneur's IC is always satisfied) subject to the two ICs (30) and (31) and the VC's IR (32). This is equivalent to maximizing the cost minus the benefit to the type B of mimicking the type G, subject again to the above mentioned constraints. Thus, the type G entrepreneur's contract design problem becomes:

$$\underset{(\overline{a},\overline{b},\underline{a},\underline{b},I_0)}{Max} \rho\lambda_G\delta\Delta X - \lambda_B(1-\rho)[(f(\widehat{c})\Delta X - \widehat{c}) - (f(c^*)\Delta X - c^*)] \tag{33}$$

$$-\lambda_B\delta\Delta X - (V_H - V_L)(\lambda_G - \lambda_B) \tag{34}$$

$$s.t. \quad (30), (31), \text{ and } (32).$$

Note that the solution to the above program also yields the optimal investment amount I_0 to be raised by the type G firm. The solution to the above problem is summarized in the proposition 6. Finally, the contract given by the type B firm to the angel has to satisfy the angel's incentive compatibility constraints as well as his individual rationality conditions (these are analogous to (30), (31), and (32) respectively).

In equilibrium, the contract offered by the type G entrepreneur to the VC at time 0 should also not preclude the entrepreneur exerting effort at time 1. Recall that, even though the VC and the entrepreneur have the freedom to renegotiate their contract at time 1, the contract offered to the VC at time 0 has an impact on the outcome of the renegotiation. The following two parametric restrictions guarantee that the contract offered to the VC by the entrepreneur at time 0 satisfies the requirement that the entrepreneur also exert effort in equilibrium.

$$q\overline{X} + (1-q)\underline{X} - I - k - \rho\lambda_G\delta\Delta X \geq 0, \tag{35}$$

$$\begin{aligned}(1-\rho)[(f(\widehat{c})+\delta)\Delta X - \widehat{c}] \quad \geq \quad & Max[(1-\lambda_G)(f(\widehat{c})+\delta)\frac{I_0 + \rho\lambda_G\delta\Delta X - \underline{X}}{[q+\lambda_G(\delta+f(\widehat{c}))]}, \\ & (1-\lambda)(f(\widehat{c})+\delta)\frac{I_0 + \rho\lambda\delta\Delta X - \underline{X}}{[q+\lambda(\delta+f(\widehat{c}))]}],\end{aligned} \tag{36}$$

where $\lambda \equiv \theta\lambda_G + (1-\theta)\lambda$. Similarly, the time 0 contract offered by the type B entrepreneur to the angel also has to be such that it does not preclude the entrepreneur exerting effort at time 1. The following parametric

restriction,

$$(1-\rho)[f(c^*)\Delta X - c^*] \geq (1-\lambda_G)f(c^*)\frac{\underline{I}_0 - \underline{X}}{[q+\lambda f(c^*)]}, \tag{37}$$

and also (35) guarantee that the equilibrium financing contract offered by the type B entrepreneur to the angel at time 0 satisfies this requirement. In the following propositions, we assume that the above parametric restrictions (35), (36) and (37) are satisfied.

Proposition 6 ***(Equilibrium financing Contracts with Moderate Scarcity of VC Financing)***
(i)The equilibrium contract between the VC and the G type entrepreneur, $(\overline{a}, \overline{b}, \underline{a}, \underline{b})$*, is* $\overline{b} = \underline{b} = 1$ *and,*

$$\overline{a} = \underline{a} = \frac{1}{\overline{X}}\{\frac{\underline{I}_0 + \rho\lambda_G\delta\Delta X - \underline{X}}{[q+\lambda_G(\delta + f(\widehat{c}))]} + \underline{X}\}, \tag{38}$$

where $\widehat{c}$ *is the first best effort by the VC.*

(ii) The type G entrepreneur raises an amount I_0 *equal to the minimum amount of money required at time 0,* $\underline{I}_0$*. The type B entrepreneur is indifferent to raising any amount above* $\underline{I}_0$ *at time 0, i.e.,* $\underline{I}_0 \leq I_0 \leq I$*.*

(iii) At time 0, the angel is indifferent between a variety of contractual forms, as long as $(1-\rho)[f(c^*)\Delta X - c^*] \geq f(c^*)(V_H - V_L)$ *is satisfied.*

(iv) The time 1 equilibrium choice of financing, the equilibrium financing contract, and the VC's choice of effort are as specified in section 3.1 and 3.2 respectively, depending upon whether the time 0 financing is done by the VC or the angel. The entrepreneur will always choose to exert effort at time 1, regardless of the state the firm is in and the type of financing the firm receives at time 0.

The equilibrium financing contract involves setting $b = 1$, since this minimizes the payoff to the type B from mimicking the type G. This contract can be implemented by giving the VC convertible debt (or equivalently, debt with warrants), convertible preferred equity (or equivalently, preferred equity with warrants), but not equity with warrants (since $b > a$). Unlike in the case of time 1 contracting, the fixed income portion of the contract (i.e., face value of debt or preferred equity) is higher in this case than in later stage contracting (regardless of whether the existing VC is financing a second round or a new VC is entering the firm at time 1). This is because in earlier stage contracting, the focus is more on resolving the asymmetric information between the entrepreneur and the VC, while, in later stage contracting, the emphasis is also on motivating optimal effort by the VC (while ensuring that the entrepreneur also provides effort). The amount raised at time 0 is $\underline{I}_0$; since the amount is raised under asymmetric information, the type G entrepreneur minimizes this amount in order to minimize the incentive of the type B entrepreneur to mimic the type G firm.

4.2 High Scarcity of VC Financing

We now study the case where VC financing is very scarce ("high scarcity" of VC financing). We define the high-scarcity regime as one where the following inequality holds:

$$\rho\lambda_G(\delta\Delta X + f(\widehat{c})\Delta X - \widehat{c}) + (1-\lambda_G)\frac{R}{2} \leq R \leq 2\rho(f(c^*)\Delta X - c^*). \tag{39}$$

From the above inequality (and comparing with (26) and (27)), we can see that in a high-scarcity regime, the VC's threshold NPV is above that which can be obtained from financing either type G or type B projects at time 0, but below that from financing a state-p project at time 1. Thus, in equilibrium in the high-scarcity regime, the VC will fund only later-stage (time 1) projects in state p, and no early-stage project. Both types of early stage projects will now be financed be the angel, thus making the equilibrium at time 0 a pooling equilibrium.

We now briefly discuss the contract design in the high-scarcity regime. The contract design is very similar to the moderate-scarcity case, except that the objective of the contract design now is to maximize the type G entrepreneur's payoff, achieved by minimizing the extent of overpricing of the type B's financial contract sold to the angel at time 0. Since the angel finances both the type G and the type B firm at time 0, this optimization problem is constrained by the angel's ICs (A26) and (A27) and his individual rationality condition (A28) (given in the Appendix). The parametric restrictions (35), (36) and (37) ensure that the contracts offered at time 0 by the entrepreneur to the angel do not preclude the entrepreneur from exerting effort at time 1 (thus, we assume that these hold). We summarize the equilibrium financing contracts in the high-scarcity regime in the following proposition:

Proposition 7 ***(Equilibrium Financing Contract with High Scarcity of VC Financing)***
i) At time 0, both the type G and the type B entrepreneur seeks (and receives) angel financing.
(ii) The equilibrium financing contract at time 0 between the entrepreneur and the angel is $\overline{b} = \underline{b} = 1$ *and,*

$$\overline{a} = \underline{a} = \frac{1}{\overline{X}}\{\frac{\underline{I}_0 - \underline{X}}{[q + \lambda f(c^*)]} + \underline{X}\}, \tag{40}$$

where c^* *is the effort by the VC as specified in section 3.2.*
(iii) Both types of entrepreneur raise an amount I_0 *equal to the minimum amount of money required at time 0,* $\underline{I}_0$.
(iv) The time 1 equilibrium choice of financing, the equilibrium financing contract, and the VC's choice of effort are as specified in section 3.2. The entrepreneur will always choose to exert effort at time 1, regardless of the state the firm is in.

Notice that in the high-scarcity regime, the VC chooses to finance only later stage projects, since his threshold

NPV R is higher in this case. By financing only later-stage projects, the VC is guaranteed that his effort will be productive. By doing this, however, the VC is able to add only less value to the project, due to the reasons discussed before.

Since the angel finances both types of projects at time 0 in equilibrium, the contract offered to the angel by the type G entrepreneur sets $\overline{b} = \underline{b} = 1$, $\overline{a} = \underline{a} < 1$, thus promising the entire downside cash flow to the angel. By doing this, the type G entrepreneur is able to minimize the extent of overpricing of the type B firm's securities (contract), thus minimizing his subsidy to the type B entrepreneur. Such a contract can be implemented by giving the angel convertible debt or convertible preferred equity, but not equity alone ($a = b$ for equity) or equity plus options ($a > b$, in this case). As in the moderate scarcity regime, here also the type G entrepreneur raises only the minimum required for the project at time 0, since the financing is undertaken under asymmetric information, and doing this minimizes the type G's subsidy to the type B entrepreneur. The type B entrepreneur pools with the type G by raising the same amount $\underline{I}_0$ as well.

4.3 Low Scarcity of VC Financing

We now study the case where the extent of scarcity of VC financing is low ("low scarcity" of VC financing).[23] We define the low-scarcity regime as one where the following inequality holds:

$$0 < R \leq \rho\lambda(\delta\Delta X + f(\widehat{c})\Delta X - \widehat{c}) + (1-\lambda)\frac{R}{2}, \tag{41}$$

where $\lambda \equiv \theta\lambda_G + (1-\theta)\lambda$. From the above inequality, we can see that in a low-scarcity regime the VC's threshold NPV, R, is below the NPV to him from investing in either a type G or a type B project at time 0. Therefore, in the low-scarcity regime, the VC will fund both types of early stage (time 0) projects as well as later stage (time 1) projects in state p. Since all entrepreneurs with a positive probability of being in state p at time 1 prefer VC financing at time 0, both types of entrepreneur would prefer VC financing over angel financing at time 0. Thus, in equilibrium, the angel finances only later stage projects in state n in the low scarcity regime. Since the VC finances both types of projects at time 0, the equilibrium is pooling at time 0. Thus, as in the high-scarcity regime, here also, there are no incentive compatibility (truth-telling) conditions to be satisfied by either type of

[23] We characterize the equilibrium in the low-scarcity regime only for completeness. However, we believe that it is the high and moderate-scarcity regimes which reflect the relative scarcity of VC financing versus angel financing in the U.S. as well as in most other developed economies.

entrepreneur at time 0.

We now briefly discuss the contract design in the low-scarcity regime. The contract design here is similar to the moderate-scarcity regime, except that (as in the high-scarcity regime) the objective of contract design is to maximize the type G's payoff, achieved by minimizing the extent of overpricing of the type B's financial contracts, sold to the VC at time 0. Since the VC finances both the type G and the type B firm at time 0, this optimization problem is now constrained by the two ICs (30) and (31) of the VC, and his individual rationality constraint (this IR is different from that in the moderate-scarcity regime, and is given by (A30) in the appendix). As in the other two regimes, we assume that the parametric restrictions (35), (36) and (37) hold, thus ensuring that the contract offered at time 0 by the entrepreneur to the VC does not preclude the entrepreneur exerting effort at time 1. We summarize the equilibrium financing contract in the low-scarcity regime in the following proposition.

Proposition 8 ***(Equilibrium Financing Contract with Low Scarcity of VC Financing)***
(i) At time 0, both types of entrepreneur seek (and receive) VC financing.
(ii)The financing contract between the VC and the entrepreneur is $\overline{b} = \underline{b} = 1$ *and,*

$$\overline{a} = \underline{a} = \frac{1}{\overline{X}}\{\frac{\underline{I}_0 + \rho\lambda\delta\Delta X - \underline{X}}{[q + \lambda(\delta + f(\widehat{c}))]} + \underline{X}\}, \tag{42}$$

where $\widehat{c}$ *is the first best effort by the VC as specified in section 3.1.*
(iii) Both types of entrepreneur raise an amount I_0 *equal to the minimum amount of money required at time 0,* $\underline{I}_0$.
(iv) The time 1 equilibrium choice of financing, the equilibrium financing contract, and the VC's choice of effort are as specified in section 3.1. The entrepreneur will always choose to exert effort at time 1, regardless of the state the firm is in.

Thus, in the low-scarcity regime, VCs fund a greater proportion of projects compared to the other two regimes, though their expected return from investing in each project will be lower in the low-scarcity regime. Since the VC finances both types of projects at time 0, in equilibrium, the contract offered to the VC by the type G entrepreneur sets $\overline{b} = \underline{b} = 1$, $\overline{a} = \underline{a} < 1$, thus promising the entire downside cash flow to the VC. By doing this, the type G entrepreneur is able to maximize his own expected payoff by minimizing his subsidy to the type B entrepreneur. Such a contract can be implemented by giving the VC convertible debt or convertible preferred equity, but not equity alone (recall that $a = b$ for equity) or equity plus options ($a > b$ in this case).

As in the other two regimes, here also the type G entrepreneur raises the minimum amount required, $\underline{I}_0$ at time 0. Since the financing here also is undertaken under asymmetric information, doing this minimizes the

overpricing of the type B's securities (or equivalently, the type G's subsidy to the type B firm). As before, the type B pools with the type G by raising the same amount $\underline{I}_0$ at time 0 as well.

4.4 Composition of VC and Angel financing

We now characterize the proportion of early and later stage projects funded by VCs and angels under the moderate and high-scarcity regimes in the entire economy.

Proposition 9 *In periods of high or moderate scarcity of VC financing, the composition of private equity financing can be characterized as follows:*

(i) A greater proportion of earlier stage projects are funded by angels rather than VCs in both situations.

(ii) Under high-scarcity of VC financing, angels finance all earlier stage projects, and VCs finance only later stage projects.

As the scarcity of VC financing relative to angel financing goes up from low to high, the stage at which the VC first starts financing the firm goes from early to late. The moderate-scarcity regime reflects a situation between these two extremes, where the VC finances later stage projects as well as a small fraction of early stage projects. As discussed before, when the scarcity of VC financing is greater, the VCs maximize their return by focusing only on later stage projects, where their effort is guaranteed to be able to add value, thus earning a higher NPV on their investment (since the VC gets a share of the value added by him to the firm). In contrast, since angels supply only capital but not effort, this choice between earlier and later stage projects does not apply to them, so that angels finance a larger fraction of early-stage projects in equilibrium.

4.5 Announcement Effects of VC and Angel Financing

We now characterize the announcement effects of early as well as late stage financing of projects by VCs and angels.

Proposition 10 ***(Announcement Effects)***

(i) Under conditions of moderate-scarcity of VC financing, the announcement of a VC financing at time 0 conveys favorable information to outsiders, leading them to revise their expectation of the firm's value upward.

(ii) If a firm receives a new round of financing at a later stage from a VC, it conveys further positive information to outsiders, regardless of whether its earlier stage financing was undertaken by an angel or a VC.

(iii) If a VC exits a firm funded by him earlier, it conveys negative information to outsiders.

In the above proposition, by "announcement effect" we refer to the revision in firm valuation (upward or downward) by private equity investors (since the securities of the private firms we are analyzing are not traded). If a firm receives early stage financing from the VC, outsiders infer that there is a greater probability of the VC

adding value to that firm at a later stage, compared to the case of a firm which is angel financed at the early stage. As a result, outsiders revise their valuation of the firm upward. Similarly, if a firm receives later stage funding from a VC, it conveys further positive information to outsiders who infer that the VC will indeed be able to add value to the firm. In contrast, if a VC who financed an earlier round exits a firm at later stage, it clearly conveys to outsiders the VC's negative assessment of his ability to add value to that firm, leading them to revise the firm's value downward.

4.6 Numerical Illustration: The Dynamic Evolution of Financing Contracts

In this section we give a numerical example of what we studied before. We study only the moderate-scarcity regimeof VC financing. Let the parameter values be the following: $\theta = 0.05$, $\lambda_G = 0.9$, $\lambda_B = 0.1$, $\rho = 0.8$, $\delta = 0.1$, $I = 13.625$, $q = 0.4$, $k = 0.1$, $R = 0.5$ and $\underline{I}_0 = 12.625$. Further, $f(c) = \frac{c}{2(1+c)}$, and $\overline{X} = 20$ and $\underline{X} = 12$. It immediately follows that the first best effort from the VC is $\widehat{c} = 1$ and $f(\widehat{c}) = 0.25$.

Time 0 financing undertaken by the type G firm:

From propositions 5 and 6, we know that G type firm will choose VC financing in equilibrium and the financing contract is $\underline{a} = \overline{a} = 0.68$, $\underline{b} = \overline{b} = 1$. This contract can be naturally implemented by convertible preferred equity. The face value of the preferred is 12, the whole cash flow if $\underline{X}$ is realized; The preferred equity can be converted to 0.68 share of the firm at time 2. It is clear that the contract holder will not convert if $\underline{X}$ is realized. But if $\overline{X}$ is realized at time 2, the contract holder will convert for sure because 0.68 share of 20 is greater than 12. The financing raised is 12.625. This contract will take two values at time 1 depending on the state of the project. If the state is n, the value of this security is $V_L = 12.67$; if the state is p, the value is $V_H = 13.26$. The contract is worth 13.20 at time 0, which covers the VC's investment cost, 12.625 and VC's share of value created, 0.576.

Time 0 financing undertaken by the type B firm:

The type B firm will choose angel financing at time 0. The firm and the angel is indifferent to a wide range of securities and the amount being raised because the financial contract has no impact on the value created. We assume that the firm raises 12.625 at time 0, the same as the type G firm does.[24] The contract given to the angel is $b = 1$, and $a = 0.67$. The contract can also be implemented by convertible preferred equity for the same

[24] This is only for concreteness. Since the equilibrium in the moderate-scarcity regime is separating, the type B firm is indifferent to the amount raised (see proposition 6 (ii)).

reason as before. It should be clear that the share that the firm gives to the angel is less than that the type G firm gives to the VC. This contract will be worth $V_L = 12.59$ if state n is realized, $V_H = 12.94$ if state p is realized (in calculating V_H, we use the equilibrium effort of the VC, which will be calculated later)

To demonstrate why the VC's contract is of this form, we will show that equity will not be the equilibrium contract. If the type G firm gives equity to the VC, the share has to be given to the VC is 0.74. But let's look at the type B firm's incentive to mimic. The left hand side of condition (28) is -1.17 (it is 0.026 for the above convertible preferred), which means that the type B firm will have incentive to mimic if the contract is equity. The reason for using convertible preferred here is that it minimizes $(V_H - V_L)$. In other words, the potential overpricing of the type B firm's securities given to the VC is minimized.

Time 1 financing if the time 0 financing is done by a VC:

Case-1: the firm is in state p.

By proposition 1 and 2, we know that the firm will continue to use VC financing and the contract given to the VC is: $a = 0.85$, $b = 0.75$ ($a > b$ here always). This contract can be implemented by convertible preferred, convertible debt, or equity plus option, but not equity alone. The firm raises additional investment, $I_1 = 1$ from the VC. The VC swaps the old (time 0) contract for a new contract at time 1. The VC will exert effort $\widehat{c} = 1$ and the entrepreneur will choose to exert effort as well.

Case-2: the firm is in state n.

The VC will leave the firm at time 1. The firm therefore has to raise new financing from an angel. The angel will have to pay 13.67 to the firm. The firm will use 12.67 to buy out the VC and use the remainder to invest. This contract can be implemented using a variety of securities, including equity ($a = b = 0.9$), convertible preferred or convertible debt, as long as the value is 13.67. The entrepreneur will exert effort.

Time 1 financing if the time 0 financing is done by an angel:

Case-1: the firm is in state p.

By propositions 3 and 4, we know that the firm will switich to VC financing and buy out the angel. The contract given to the VC is: $a = 0.91$, $b = 0.92$ (from proposition 4, $a > b$ or $b > a$ depending on parameter values). This contract can be implemented by convertible preferred, but this time, the face value is $0.92 \times 12 = 11.04$. The firm raises 13.94 from the VC. The firm pays the angel 12.94 to buy out his contract and invests, $I_1 = 1$.

The VC will exert effort $c^* = 0.89$ and the entrepreneur will also choose to exert effort.

Case-2: the firm is in state n.

The firm will continue to use angel financing in this case. The firm raises additional investment, $I_1 = 1$ from the angel. The contract given to the angel can take a variety of forms, including equity ($a = b = 0.89$) as long as its value is 13.59. The entrepreneur will exert effort in equilibrium.

Finally, in this equilibrium, the angel finances 95% of the earlier stage firms. Among the late-stage firms financed by the VC, 68% are first-round financings, while 32% are second-round financings of firms financed at an earlier stage by the same VC.

5 An Extension to the Basic Model: Financing Conflicts Between the Entrepreneur and the Early Stage Financier

We now study an extension of the basic model, where there is a potential for conflicts between the entrepreneur and the early stage financier regarding the terms of later stage financing. In the basic model, renegotiation (buyout) of time 0 financial contract given to the early stage financier was always feasible, since there was no asymmetric information between the entrepreneur and the early stage financier about the future value of the firm. In practice however, asymmetric information between the entrepreneur and the early stage financier may arise due to a variety of reasons, so that in some situations, a successful buyout of the early stage financier's contract at a mutually agreed price may not be possible. This, in turn, means that the entrepreneur is unable to provide the later stage financier with the optimal incentives to add value to the firm. We characterize one such situation below.

For concreteness, we consider the case where the time 0 financier is an angel. Further, we assume that the asymmetric information between the entrepreneur and the angel at time 1 is about the VC's ability to add value to the firm. We model this asymmetric information as follows. There are two kinds of VCs: those with high ability ("high type VC") and those with low ability ("low type VC"). While the entrepreneur knows the VC's type, the angel observes only the prior probability ϕ of the VC being of the high type. We assume that the production function $f(c)$ of the high type VC takes the form of $h(c)$ while that of the low type VC takes the form of $j(c)$. We assume that $h'(c) > j'(c)$ and $h(0) = j(0)$, which guarantees that the probability of the project being

highly successful is greater under the high type VC than under the low type VC.

For analytical simplicity, we restrict the contracts between the entrepreneur and the financiers (VC or angel) to be equity.[25] Also, unlike in the basic model, we assume that there is no asymmetric information at time 1 between the entrepreneur and the new time 1 financier about the state of the firm, i.e., the time 1 state is public information.[26]

In our analysis below, we will focus only on the case where the firm is initially angel financed, but is in state p at time 1.[27] From our analysis in section 3.1, we know that it is optimal to give as much equity as possible to VC (subject to the entrepreneur's IC being satisfied) to ensure the VC exert as much effort as possible. However, the existence of asymmetric information between the angel and the entrepreneur about the type of the VC results in the angel not being aware of the true future value of the equity he holds. Thus, unlike in the basic model, where the time 1 buyout of the financial contracts held by the time 0 financier proceeds under symmetric information, here the buyout proceeds under asymmetric information.

We model the bargaining between the entrepreneur and the angel at time 1 as follows. The entrepreneur asks the angel to sell all the equity held by the him back to the entrepreneur, at a price to be named by the angel. The angel responds with a price for his equity. The entrepreneur accepts or rejects the angel's offer. If the entrepreneur accepts the angel's offer, the buyout takes place and the angel leaves the firm. If, however, the entrepreneur rejects the offer, the angel stays in the firm, the project continues, and the final (time 2) cash flows are shared according to the existing contracts.[28]

We solve the entrepreneur's problem at time 1 backwards. First consider the case where the entrepreneur rejects the angel's offer. Suppose, after the rejection, the entrepreneur attempts to bring in VC financing

[25] Unlike in the basic model, our focus here is on analyzing the possibility of conflicts between the entrepreneur and the early stage financier at time 1, and not on the optimal design of financial contracts. We therefore focus on equity for analytical simplicity. However, our results are qualitatively unchanged even when other financial contracts are allowed. We assume that the entrepreneur can raise more money than needed for investment.

[26] Allowing for asymmetric information about the time 1 state of the firm as well would add additional complexity without affecting our results qualitatively.

[27] This is the most interesting case. If a firm is initially Angel financed, but is in state n at time 1, we know from the basic model that it will continue Angel financing, and there is no potential for financial conflicts.

[28] For concreteness we adopt a specific bargaining procedure between the Angel and the entrepreneur. The price at which the Angel's existing financial contract are bought out (and thus, the sharing of surplus between the Angel and the entrepreneur) will depend on the nature of bargaining procedure adopted. However, the underlying insight that we are attempting to capture here, namely, that the buyout of the time 0 financier may not proceed, leading to inefficient financing of the firm at time 1, is robust to the specifics of the bargaining procedure employed (as long as the Angel has some bargaining power, enabling him to extract a portion of surplus created by the VC in the second period).

anyway (while the angel continues to hold equity in the firm). In this case, the entrepreneur solves the following optimization problem:

$$\underset{\pi_{vc}}{Max} \quad (\pi_e - \pi_{vc})([(q + f(c(\pi_{vc})))\overline{X} + (1 - q - f(c(\pi_{vc})))\underline{X}] + I_1 - (I - I_0) - k \tag{43}$$

$$\text{s.t.} \qquad f'(c) = \frac{1}{\pi_{vc}\Delta X} \tag{44}$$

$$(\pi_e - \pi_{vc})[(q + f(c(\pi_{vc})))\overline{X} + (1 - q - f(c(\pi_{vc})))\underline{X}] \quad \geq \quad k \tag{45}$$

$$\pi_{vc}[(q + f(c(\pi_{vc})))\overline{X} + (1 - q - f(c(\pi_{vc})))\underline{X}] - I_1 - \rho f(c)\Delta X - (1 - \rho)c \quad \geq \quad 0 \tag{46}$$

In the above, (43) is the entrepreneur's objective, which, as in the basic model, is to maximize his expected time 2 payoff, net of his effort. Notice that we now allow the entrepreneur to raise more money than needed. Here π_e is the entrepreneur's time 0 share of the firm's equity, out of which he will optimally give a fraction π_{vc} to the VC if he brings the VC in as a financier at time 1. The angel's share at time 0 (and at time 1 if the buyout does not proceed) is $\pi_a = (1 - \pi_e)$.[29] (44) is the VC's incentive compatibility condition ensuring that he puts forth optimal effort; (45) is the entrepreneur's incentive compatibility condition, ensuring that he exerts effort; and (46) is the VC's individual rationality constraint, which ensures that the VC is compensated for his investment and his effort cost, and also obtains a fraction ρ of the value created by him.

Simplifying (43) using the VC's IR (46), and using $\pi_e = (1 - \pi_a)$, the entrepreneur's objective becomes:

$$\pi_e[q\overline{X} + (1 - q)\underline{X}] + (1 - \pi_a - \rho)(f(c)\Delta X - c) - I_1 - k. \tag{47}$$

However, if the entrepreneur does not bring in VC financing but continues to use angel financing, his expected payoff would be $\pi_e[q\overline{X} + (1-q)\underline{X}] - I_1 - k$. Comparing the two payoffs, we can see that, if the time 0 share held by the angel, π_a, is greater than $(1 - \rho)$, the entrepreneur is worse off bringing in the VC compared to satisfying his time 1 financing needs by raising another round of angel financing. Therefore, if the bargaining between the entrepreneur and the angel breaks down (so that the buyout of the angel's time 0 equity holding in the firm does not take place) the entrepreneur will not raise VC financing. The intuition behind why the entrepreneur will not want to bring in the VC at all if the angel buyout does not proceed and if π_a is greater than $(1 - \rho)$ is the following. When the entrepreneur brings the VC to add value to the firm under those conditions, he has to bear

[29] For simplicity, we will take the time 0 financing as given in this section.

all the cost of doing so (since he gives up equity to the VC), but only gets a part of the benefit from doing so. The angel, on the other hand, bears no parts of the cost of bringing in the VC, but is able to benefit from the value gained from bringing in the VC. In other words, the angel is able to free-ride on the entrepreneur who bears all the cost of bringing in the VC. We will assume throughout this section that $\pi_a > (1-\rho)$.[30]

Now consider the case where the angel's offer is accepted by the entrepreneur. In this case, the entrepreneur would raise financing from the VC after buying out the angel. Since the entrepreneur owns the whole equity of the firm immediately after the buyout, he is now able to get $(1-\rho)$ fraction of the value added by the VC. The share of equity the entrepreneur gives to the VC in this case at time 1 is characterized by setting the entrepreneur's IC as equality (the VC is given as much equity as possible, as long as the entrepreneur is motivated to exert effort):

$$(1-\pi_{vc})[(q+f(c(\pi_{vc})))\overline{X}+(1-q-f(c(\pi_{vc})))\underline{X}]=k. \tag{48}$$

The following proposition characterizes the situations under which the buyout of the angel's time 0 equity stake by the entrepreneur goes through, and those under which the buyout does not occur.

Proposition 11 *Define D_h, D_j as follows,*

$$D_h \equiv h(c_h)\Delta X - c_h, \tag{49}$$
$$D_j \equiv h(c_j)\Delta X - c_j, \tag{50}$$

where c_i, $i = h$ or j refers to the equilibrium effort by the VC of high or low type, respectively. Then:

(i)if $\phi D_h > D_j$, and if the VC is truly of low ability, then the bargaining between the entrepreneur and the angel breaks down. In this case, the entrepreneur does not get time 1 financing from the VC, even when it is efficient to do so, and instead finances the project at time 1 with another round of angel financing.

(ii) If, however, $\phi D_h \leq D_j$, the entrepreneur is able to buy out the angel at time 1. He then obtains time 1 financing from the VC, giving him the fraction of equity characterized by (48).

In the above, $\pi_a[q\overline{X}+(1-q)\underline{X}]+D_j$ and $\pi_a[q\overline{X}+(1-q)\underline{X}]+D_h$ are the two possible price-demands that may be made by the angel to the entrepreneur. When the angel's prior ϕ about the VC being of high type is large, he will always demand the high price $\pi_a[q\overline{X}+(1-q)\underline{X}]+D_h$ for his equity, since he knows that the value of the firm will be high if a high-ability VC is involved. The entrepreneur will accept this demand only if the VC is indeed of high ability (i.e., with probability ϕ), rejecting it otherwise. It is value-maximizing for the angel to always demand the high price when ϕ is large, since the inefficiency arising from the loss in the potential value

[30] If, however, $\pi_a < 1-\rho$, it is optimal for the entrepreneur to bring in the VC at time 1, even when the bargaining between the Angel and the entrepreneur to buy out the Angel's equity stake breaks down. However, the effort put forth by the VC (and thus the value added by him) will be less than the case where the Angel's time 0 equity stake is successfully bought out, since the free-riding by the Angel continues to exist. We will, however, not study this case here.

added by the VC occurs only with small probability, $1-\phi$. If, in contrast, the angel's prior of the VC being of the high type ϕ is low, then the angel always demands the low price $\pi_a[q\overline{X}+(1-q)\underline{X}]+D_j$, for his equity, which will always accepted by the entrepreneur. The entrepreneur will then be able to obtain the firm's time 1 financing from the VC, as discussed in the basic model.

The above possibility of inefficient time 1 financing caused by financing conflicts between the angel and the entrepreneur means that, the firm would find it even more beneficial (compared to the basic model) to make use of the VC rather than the angel at time 0, since this would make it possible to avoid the time 1 value loss if the firm ends up in state p.

6 Empirical Implications

We highlight some of the empirical implications of our model below.

(i) Entrepreneurs' choice between venture capital and angel financing: Our model predicts that venture capitalists will finance firms in industries where the potential for adding value is the greatest, while angels will fund firms in industries where the potential for adding value is more limited. This means that venture capitalists will tend to finance firms in technologically sophisticated and knowledge-intensive areas, where they can create the most value. In contrast, angels will tend to finance projects in industries which are less technologically sophisticated and knowledge-intensive, with less opportunities for the financier to add value. By the same token, entrepreneurs who are technologically sophisticated themselves will tend to have self-financing or angel financing, since the incremental value that can be added by the financier will be smaller in such firms.

(ii) Differences between venture capital financing and angel financing contracts: Convertible features (convertible debt or convertible preferred equity) in venture capital financing contracts have two important roles in our setting. First, the fixed income portion, which offers downside protection to the financier, minimizes the pricing effects of the entrepreneurs' private information. Second, the "upside" of the convertible (i.e., the warrant component) motivates the venture capitalists to put forth optimal effort to add value to the firm. In contrast, convertible debt or convertible preferred, when used in angel contracts, only have the first role, namely, minimizing the pricing effects of the entrepreneurs' private information. Thus, our model predicts that angel financing contracts are less likely to have convertible features, and, when present, will have a smaller upside (*ceteris paribus*)

compared to venture capital financing contracts (recall that it is the upside of the convertible which motivates the venture capitalist to put forth optimal effort in our setting).[31]

(iii) The dynamic evolution of venture capital contracts: Our model has two predictions regarding the dynamic features of venture capital contracts. The first prediction is regarding the evolution of venture capital contracts across multiple rounds of financing between the entrepreneur and the venture capitalist. Our model predicts that, as the firm ages and engages in several rounds of venture capital financing, the form of the financing contract between the venture capitalists and the entrepreneur is altered. This is because the contract has to trade-off the need to minimize the impact of asymmetric information between the entrepreneur and the venture capitalist and the competing need to provide high-powered incentives to the venture capitalist to put forth effort to add value to the firm. Thus, in the earlier stages (when the extent of asymmetric information is greater), the contract gives much greater down-side protection (i.e., the fixed income component contract will be greater) to the venture capitalist. In later stages, when there is less asymmetric information between the venture capitalist and the entrepreneur, the need to provide incentives to the venture capitalist dominates, so that the fixed income component of the contract will be less, while the upside (warrant) component of the contract will be more.

Our second prediction is regarding the nature of venture capital contracts in two different situations. First, consider the contract between a firm and a venture capitalist who has been financing it for several rounds. Second, consider the contract between a firm (of a similar age and stage in its evolution as the previous firm, and which has been angel-financed in previous rounds) and a venture capitalist who is financing it for the first time. Our model predicts that the former contract, being characterized by less asymmetric information compared to the latter, will have a larger upside (warrant) component and a smaller fixed-income component compared to the latter. This is because the latter contracting situation is characterized by greater asymmetric information, so that the equilibrium contract has to focus more on dissipating the effects of this asymmetric information and less on motivating the venture capitalist to put forth optimal effort.

(iv) Differences in the composition of projects financed by venture capitalists and angels and the structure of

[31] Some preliminary evidence supporting this prediction is provided by Prowse (1998), who presents evidence on the angel market gathered from field research. He comments:"...unlike in the organized private equity market, many angels are content to take common stock." (Venture capitalists form part of the "organized private equity market" referred to here.) He further comments that, when angels do take convertible preferred, they are motivated by the objectives of minimizing downside investment risk and ensuring performance by management, which is also consistent with the rationale developed here.

their holdings in these projects: First, our model predicts that there will be systematic differences in the nature of projects financed by venture capitalists and angels (stage of financing as well as project quality). Thus, in periods of high scarcity of such financing, venture capital firms will fund only later stage projects, while angels will fund early stage projects. In periods of moderate scarcity of venture capital funds, venture capitalists will fund later stage projects and a small fraction of early stage projects, leaving the rest to angel financing. In summary, our model predicts that angels will fund the majority of early stage projects. In terms of quality, our prediction is that the average quality of projects funded by venture capitalists will be greater than those funded by angels. Further, the average quality of projects funded by both venture capitalists and angels will go up during periods of high scarcity of venture capital financing (as measured by lower flows into venture capital funds).

Second, our model predicts that venture capital firms will take concentrated positions in a few firms, while angels will invest significantly smaller amounts in a number of firms. Recall that the defining feature of venture capital financing here is value-addition, and if the venture capital firm makes an investment in a firm below a certain threshold value, it will not be incentive compatible for the firm to allocate any venture capitalist to that firm (thus, this effect is created partially because of the indivisibility of human capital). Since angels do not engage in significant value addition, they do not face such a minimum-investment constraint, thus enabling them to take small positions in a number of firms.

(v) The financing path of firms, firm qualities, and announcement effects: Our model makes the following predictions regarding the relationship between the financing path of firms and firm quality. Firms which are venture-financed at their early stages, and continue to attract further venture financing are of the highest quality (more likely to generate the greatest long-term cash flow). Firms which are angel-financed initially and attract subsequent rounds of venture financing are of lower quality. Finally, firms which are initially as well as subsequently angel-financed, or those which start out as venture-backed firms but attract only angel financing in subsequent rounds, will be of the lowest quality.

Consistent with this, our model predicts that if a firm successfully obtains venture capital financing in its early stages, it conveys favorable information to outside investors in private equity about that firm, who revise its value upward. Further rounds of venture capital financing serve as additional favorable signals to outsiders (as are later rounds of venture capital financing for a firm which is initially angel financed). Finally, exit by venture

capitalists from initially venture-backed firms serves as a negative signal to outsiders.

(vi) The potential for conflicts between entrepreneurs and angels over future rounds of financing: Our extension to the basic model provides predictions regarding the potential for such conflicts and the resulting inefficiencies. The first prediction is that in firms which are initially angel financed, difficulties in providing appropriate financial incentives to venture capitalists in later rounds are likely to be the greatest when the angel's uncertainty about the incoming venture capitalist's quality is the greatest. Thus, when the incoming venture capitalist's reputation is very high, such conflicts between the entrepreneur and the angel are less likely to arise, since in this case, the extent of asymmetric information between the entrepreneur and the angel about the venture capitalist's quality is likely to be low. Second, such conflicts and inefficiencies are more likely to arise when the angel is less sophisticated, and therefore less likely to correctly estimate the incoming venture capitalist's contribution to the firm.

7 Conclusion

We have developed a dynamic model of private equity financing. We considered a setting in which an entrepreneur chooses between angel and venture capital financing to fund his investment project. The entrepreneur could raise the required external financing over several rounds, though a certain minimum amount needed to be raised initially. There were four key ingredients driving the entrepreneur's choice between the above two sources of private equity financing in our model. First, venture capitalists are to able add value to some of the firms they finance, while angels are not able to add significant value. Second, the entrepreneur has private information regarding the nature of his own firm. Further, the extent of this private information evolves over time, since a financier who has financed the firm in prior rounds will know more about it than a new financier. Third, since the venture capitalist has to engage in privately costly effort to add value to the firm, the financial contract between the two has to provide him with the right incentives to maximize this value-addition. Finally, the entrepreneurs effort is also required to ensure project success. In the above setting, we derived: (i) The equilibrium financing path of the firm, including its choice between angel and venture capital financing over different financing rounds, and the amounts raised in these rounds; (ii) The equilibrium design of financial contracts between the entrepreneur and the angel or venture capitalist, with implications for the differences between angel and venture capital contracts; (iii) The

dynamic evolution of venture capital contracts over financing rounds; (iv) The differences in the composition of projects financed by venture capitalists and angels and the structure of their holdings in these projects; (v) The effect of an announcement by any firm of a successful venture capital or angel financing upon other private equity investors assessment of its value.

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Appendix A: Proofs of Propositions

Proof of proposition 1. Because the out of equilibrium belief is such that if a firm seeks outside financing, it is in state n. So, the type p firm will not deviate by seeking outside finance because (i) the firm can not issue any overvalued security by this belief; ii) the type p can not improve the VC's effort by deviating because the VC's first best effort has already been achieved.

The firm in state n can not do better by continuing using VC financing because it can not benefit from VC's effort. Further it can not sell over-valued security because it can not raise any outside fund without reveal it's state. Thus a firm in state n can not benefit anyway by hiding its state. Once its state is revealed, he is just indifferent to any financing contracts of the same value.

If a VC finds out that his firm is in state n, if he stays, he will be just like an angel, his net payoff is 0. But if he leaves the firm, he can get $\frac{R}{2} > 0$. So, it is strictly better off for a VC to sell his stake in a firm which is in state n.

Proof of proposition 2: For part (i): simplified the objective function using the VC's IR, the problem becomes maximize $f(c)\Delta X - c$. So the optimal effort is thus the first best effort $\widehat{c}$. By (5) and (7), we know that $(a\overline{X} - b\underline{X}) = \Delta X$. $a > b$ follows because otherwise $a\overline{X} - b\underline{X} \leq b\Delta X \leq \Delta X$, equality requires $a = b = 1$, but the entrepreneur's IC will not be satisfied. a and b are determined by $(a\overline{X} - b\underline{X}) = \Delta X$ and VC's IR (4). Solving the two equations gives us part (i). We assume that the VC's IR is binding here, it is the case because we assume that the limited liability constraints are not binding here. So if the VC's IR is not binding, the entrepreneur can reduce a and b such that $(a\overline{X} - b\underline{X})$ unchanged, it is clear that the new contract is better for the entrepreneur.

For part (ii), It is clear that the angel's IR has to be binding. But if that is the case, the entrepreneur's objective is thus $q\overline{X} + (1-q)\underline{X} - k - V_L - I_0$, which doesn't depends on the contract anymore. .

Proof of proposition 3: It follows by our construction. Type n firm will not have incentive to obtain VC financing and type p firm will benefit from obtaining VC financing.

Proof of proposition 4: Just as in the proof of proposition 2, it is easy to see that the entrepreneur's objective is to maximize $f(c)\Delta X - c$. The proof is done by showing the following results.

Result 1: The first best $\widehat{c}$ can not be achieved and (22) has to be binding.

The first best $\widehat{c}$ requires $f'(\widehat{c})\Delta X = 1$, or $P = \Delta X$. Substitute this into the LHS of (22), we get

$$(1-\rho)(\widehat{c} - f(\widehat{c})\Delta X) \tag{A1}$$

But

$$\begin{aligned} \widehat{c} - f(\widehat{c})\Delta X &= \widehat{c} - \int_0^{\widehat{c}} f'(u)\Delta X du \\ &< \widehat{c} - \widehat{c} = 0 \end{aligned} \tag{A2}$$

The inequality follows because $f'(u)\Delta X > 1$ for $u < \widehat{c}$, from the concavity of f. So the first best violates (22). Comparing the problem here with the problem in section 3.1, if (22) is not binding, then the two problems have the same first best solution (the p-state firm's IC is satisfied as long as $f(c)\Delta X - c \geq 0$, but $c = \widehat{c}$ clearly satisfies it). But it is clearly not the case because we just showed that $\widehat{c}$ would violate (22). So (22) has to be binding. As a result, the optimal P is determined by (22).

Result 2: There is always underinvestment of effort, and, the p-state firm's IC is always satisfied. To do this, we prove the first part by showing that overinvestment is never optimal for our problem. We know that the VC chooses c such that

$$f'(c^*)P = 1. \tag{A3}$$

So if $c^* > \widehat{c}$, we must have,

$$P > \Delta X. \tag{A4}$$

Differentiating the LHS of (22) with respect to P, we get:

$$\begin{aligned} (1-\rho)\frac{dc}{dp} - (P - \rho\Delta X)f'(c^*)\frac{dc}{dp} - f(c) &= (1-\rho)\frac{dc}{dp} - (1 - \rho\Delta X f'(c^*))\frac{dc}{dp} - f(c^*) \\ &< (1-\rho)\frac{dc}{dp} - (1 - \rho P f'(c^*))\frac{dc}{dp} - f(c^*) \\ &= -f(c^*) < 0. \end{aligned} \tag{A5}$$

This means that we can reduce P without violating (22). This will result in improved objective function. Together with result 1, the only possibility is underinvestment.

Because the p-state firm's IC is satisfied as long as $f(c^*) - c^* > 0$, but this is true because $c^* < \widehat{c}$ and $f(c)$ is concave.

Result 3: By result 1, the optimal contract is determined by (22) holding as an equality and the VC's IR, (4) holding as equality (here again the VC's IR has to be binding because we only focus on the case where the limited liability constraints are not binding). Let P^* be the maximum solution to $\rho f(c(P)\Delta X + (1-\rho)c(P) - Pf(c(P)) = 0$. We can see that it is optimal because we know from rsult 2 that the VC always underinvests his effort, thus the largest P minimizes the underinvestment.

Notice that the maximum solution exists. The reason is the following: let P^{**} be the supreme of all the solutions to the equation. By result 2 we know $P^{**} \leq \Delta X$. There exists a sequence P_n such that each P_n is a solution to the above equation and $P_n \longrightarrow P^{**}$. But because the left hand side of the equation is continuous in P, P^{**} is also a solution to the equation. Thus the maximum solution exists.

Thus the optimal contract is the solution to

$$\begin{cases} a^*\overline{X} - b^*\underline{X} = P^* \\ a^*(q + f(c^*))\overline{X} + b^*(1 - q - f(c^*))\underline{X} = I_1 + \rho f(c^*)\Delta X + (1-\rho)c^* + V_H \end{cases} \tag{A6}$$

where c^* is such that $f'(c^*)P^* = 1$

With some simple algebra, the solution is thus

$$b^* = \frac{1}{\underline{X}}\{I_1 + \rho f(c^*)\Delta X + (1-\rho)c^* + V_H - (q + f(c^*))P^*\}$$

$$a^* = \frac{1}{\overline{X}}\{I_1 + \rho f(c^*)\Delta X + (1-\rho)c^* + V_H + [1 - (q + f(c^*))]P^*\} \tag{A7}$$

$a^* \leq b^*$ iff

$$\frac{\Delta X}{\underline{X}} \geq \frac{P^*}{I_1 + \rho f(c^*)\Delta X + (1-\rho)c^* + V_H - (q + f(c^*))P^*}. \tag{A8}$$

Otherwise, $a^* > b^*$. Using the binding (22) to simplify the above condition, we get our results. Thus we proved part (i). The proof of part (ii) is the same as the proof of part (ii) in proposition 2.

Proof of proposition 5: The results follow by construction.

Proof of proposition 6: The proposition is done by proving the following results:.

Result 1: (30) is binding.

If not, increase $\underline{a}$ and $\underline{b}$ (increase V_L) and decrease $\overline{a}$ and $\overline{b}$ (decrease V_H) such that the VC's IR (32) and (30) are satisfied. Clearly (31) is satisfied. But we have smaller objective function. Contradicts.

Result 2: $\overline{a} \geq \underline{a}$, and $\overline{b} \leq \underline{b}$.

(30) and (31) can be simplified to

$$(\overline{b} - \underline{b})\underline{X} + [(\overline{a} - \underline{a})\overline{X} - (\overline{b} - \underline{b})\underline{X}](q + f(\widehat{c}) + \delta) \geq 0, \tag{A9}$$

$$(\overline{b} - \underline{b})\underline{X} + [(\overline{a} - \underline{a})\overline{X} - (\overline{b} - \underline{b})\underline{X}]q \leq 0. \tag{A10}$$

Substracting (A10) from (A9), we have

$$(\overline{a} - \underline{a})\overline{X} - (\overline{b} - \underline{b})\underline{X} \geq 0. \tag{A11}$$

Together with (A10), it implies $\overline{b} - \underline{b} \leq 0$. Finally, (A9)) gives us $\overline{a} - \underline{a} \geq 0$.

Result 3: (31) is satisfied if (30) is binding..

The fact that (30) is binding gives us

$$(\overline{a} - \underline{a})(q + f(\widehat{c}) + \delta)\overline{X} + (\overline{b} - \underline{b})(1 - q - f(\widehat{c}) - \delta)\underline{X} = 0. \tag{A12}$$

By result 2, we have

$$(\overline{a} - \underline{a})(q + f(\widehat{c}) + \delta)\overline{X} \geq (\overline{a} - \underline{a})q\overline{X}, \tag{A13}$$

$$(\overline{b} - \underline{b})(1 - q - f(\widehat{c}) - \delta)\underline{X} \geq (\overline{b} - \underline{b})(1 - q)\underline{X}. \tag{A14}$$

Adding them up, using (A12), we have

$$(\overline{a} - \underline{a})q\overline{X} + (\overline{b} - \underline{b})(1 - q)\underline{X} \leq 0, \tag{A15}$$

which is (A10), equivalent to (31). Thus we have proved result 3.

By result 1 and 3, the optimization problem becomes

$$\underset{(\overline{a},\overline{b},\underline{a},\underline{b})}{Min} \ (V_H - V_L) \tag{A16}$$

$$s.t. \quad (\overline{a} - \underline{a})(q + f(\widehat{c}) + \delta)\overline{X} + (\overline{b} - \underline{b})(1 - q - f(\widehat{c}) - \delta)\underline{X} = 0 \tag{A17}$$

$$I_0 + \rho\lambda_G\delta\Delta X = V_H\lambda_G + V_L(1 - \lambda_G). \tag{A18}$$

(A12) implies that $V_H = \underline{a}(q + f(\widehat{c}) + \delta)\overline{X} + \underline{b}(1 - q - f(\widehat{c}) - \delta)\underline{X}$, as a result, the objective function

$$\begin{aligned} V_H - V_L &= \underline{a}(q + f(\widehat{c}) + \delta)\overline{X} + \underline{b}(1 - q - f(\widehat{c}) - \delta)\underline{X} - [\underline{a}q\overline{X} + \underline{b}(1 - q)\underline{X}] \\ &= (f(\widehat{c}) + \delta)(\underline{a}\overline{X} - \underline{bX}). \end{aligned} \tag{A19}$$

Result 4: The t=0 contract is monotonic, i.e., $\underline{a}\overline{X} \geq \underline{bX}$, $\overline{a}\overline{X} \geq \overline{b}\underline{X}$ and $V_H \geq V_L$.

If $\underline{a}\overline{X} < \underline{bX}$, by (A19)

$$V_H < V_L < \underline{bX}. \tag{A20}$$

This implies,

$$V_H\lambda_G + V_L(1 - \lambda_G) < \underline{bX} \leq \underline{X}. \tag{A21}$$

But we assume that $\underline{X} < \underline{I}_0$, as a result, the VC's IR, (A18) can not be satisfied. Contradicts.

So we have $\underline{a}\overline{X} \geq \underline{bX}$, this in turn implies $V_H \geq V_L$. $\overline{a}\overline{X} \geq \overline{b}\underline{X}$ follows from the fact that $\overline{a} \geq \underline{a}$, $\overline{b} \leq \underline{b}$.

Result 5: $\underline{b} = 1$.

Suppose the optimal contract is such that $\underline{b} < 1$, we can find another contract which is strictly better. Let the original contract be $(\overline{a}', \overline{b}', \underline{a}', \underline{b}')$ and $\underline{b}' < 1$. By (A12), we have

$$V_H' = \underline{a}'(q + f(\widehat{c}) + \delta)\overline{X} + \underline{b}'(1 - q - f(\widehat{c}) - \delta)\underline{X}. \tag{A22}$$

So we can rewrite the VC's IR, (A18), as

$$\begin{aligned} I_0 + \rho\lambda_G\delta\Delta X &= [\underline{a}'(q + f(\widehat{c}) + \delta)\overline{X} + \underline{b}'(1 - q - f(\widehat{c}) - \delta)\underline{X}]\lambda_G + [\underline{a}'q\overline{X} + \underline{b}'(1 - q)\underline{X}](1 - \lambda_G) \\ &= \underline{a}'\overline{X}[(q + f(\widehat{c}) + \delta)\lambda_G + q(1 - \lambda_G)] + \underline{b}'\underline{X}[(1 - q - f(\widehat{c}) - \delta)\lambda_G + (1 - q)(1 - \lambda_G)] \end{aligned} \tag{A23}$$

We can increase $\underline{b}'$to b and decrease $\underline{a}'$ to a and keep the above equation satisfied. Now we claim that the contract (a, b, a, b) is a strictly better solution than $(\overline{a}', \overline{b}', \underline{a}', \underline{b}')$. It satisfies VC's IR constraint by construction. It obviously satisfies (A17). And

$$V_H' - V_L' = (f(\widehat{c}) + \delta)(\underline{a}'\overline{X} - \underline{b}'\underline{X}) > (f(\widehat{c}) + \delta)(a\overline{X} - \underline{bX}) = V_H - V_L, \tag{A24}$$

the objective function is strictly better.

Result 6: Any optimal contract $(\overline{a}, \overline{b}, \underline{a}, \underline{b})$ is equivalent to $(\underline{a}, \underline{b}, \underline{a}, \underline{b})$.

Because the objective function is only function of $(\underline{a}, \underline{b})$ by (A19), the two contracts have the same objective function value. It is clear that $(\underline{a}, \underline{b}, \underline{a}, \underline{b})$ satisfies all the constraints. So, without loss of generality, the optimal incentive compatible contract is a single contract for both state p and n.

Result 7: The optimal amount of fund raised at t=0 is $\underline{I}_0$.

The optimal contract has $\underline{b} = 1$. By VC's IR,

$$I_0 + \rho\lambda_G\delta\Delta X = \underline{a}\overline{X}[(q + f(\widehat{c}) + \delta)\lambda_G + q(1 - \lambda_G)] + \underline{X}[(1 - q - f(\widehat{c}) - \delta)\lambda_G + (1 - q)(1 - \lambda_G)], \tag{A25}$$

it is clear that $\underline{a}$ increases in I_0. Together with (A19), this implies that the objective function increases in I_0. So higher I_0 leads to worse objective function.

Result 8: The optimal contract is $\overline{b} = \underline{b} = 1$ and $\overline{a} = \underline{a} = \frac{1}{X}\{\frac{I_0 + \rho\lambda_G\delta\Delta X - \underline{X}}{[q + \lambda_G(\delta + f(\widehat{c}))]} + \underline{X}\}$.

By result 6 and 7, we have $\overline{b} = \underline{b} = 1$. The later part follows by solving $\overline{a} = \underline{a}$ in the VC's t=0 IR..

Result 9: Entrepreneur will always exert effort. If the time 0 is VC financing, the equilibrium contracts and condition (35) and (36) guarantee that the entrepreneur will choose to exert effort, because doing so he will get positive payoff. If the time 0 financier is angel, condition (35) and $(1 - \rho)[f(c^*)\Delta X - c^*] \geq f(c^*)(V_H - V_L)$ (condition (37) guarantees that there exists at least one contract satisfied this condition, this contract minimizes $(V_H - V_L)$) ensures that the entrepreneur will exert effort because he will get positive payoff if he exerts effort and zero otherwise. Part (iii) follows because the type B entrepreneur gets his full information payoff if he chooses angel financing as long as angel financing doesn't preclude him exerting effort. Thus he is indifferent to contracts which dosn't preclude him exerting effort.

Proof of proposition 7: part (i) is by construction, the out of equilibrium belief is that the firm is of B type. For the rest part, the proof is very similar to the proof of proposition 6. Let the contract be $(\overline{a}, \overline{b}, \underline{a}, \underline{b})$. Again, the contract has to be incentive compatible. In pooling equilibrium, the angel's IC is

$$\overline{a}(q + f(c^*))\overline{X} + \overline{b}(1 - q - f(c^*))\underline{X} \geq \underline{a}(q + f(c^*))\overline{X} + \underline{b}(1 - q - f(c^*))\underline{X} \tag{A26}$$

$$\underline{a}q\overline{X} + \underline{b}(1 - q)\underline{X} \geq \overline{a}q\overline{X} + \overline{b}(1 - q)\underline{X}; \tag{A27}$$

and the angel's IR is

$$I_0 = V_L + \lambda(V_H - V_L) \tag{A28}$$

where $V_L = q\underline{a}\overline{X} + (1-q)\underline{b}\underline{X}$, and $V_H = (q + f(c^*))\overline{a}\overline{X} + (1 - q - f(c^*))\overline{b}\underline{X}$. c^* is the VC's optimal effort as specified in section 3.2. $\lambda = p\lambda_G + (1-p)\lambda_B$. The payoff to the type G entrepreneur in the pooling equilibrium is

$$\lambda_G(W_H - V_H) + (1-\lambda_G)(W_L - V_L) - (I - I_0) - \lambda_G[\rho(f(c^*)\Delta X - c^*) + c^*].$$

where $W_H = (q + f(c^*))\overline{X} + (1 - q - f(c^*))\underline{X}$ and $W_L = q\overline{X} + (1-q)\underline{X}$. Substituting angel's IR (A28) into the above expression, we have

$$\lambda_G W_H + (1-\lambda_G)W_L - I - \lambda_G[\rho(f(c^*)\Delta X - c^*) + c^*] - (\lambda_G - \lambda)(V_H - V_L)$$

The last term is the subsidy the type G entrepreneur gives to the type B entrepreneur by pooling with him. To maximize type G's payoff is equivalent to

$$\begin{aligned} &\underset{(\overline{a},\overline{b},\underline{a},\underline{b})}{Min}\ (V_H - V_L) \\ &s.t. \quad \text{(A26), (A27) and (A28).} \end{aligned} \tag{A29}$$

The rest of the proof is the same as the proof of proposition 6.

Proof of proposition 8: The VC's IR is

$$I_0 + \rho\lambda\delta\Delta X = V_H\lambda + V_L(1-\lambda) \tag{A30}$$

The rest of the proof almost the same as the proof of proposition 7.

Proof of proposition 9: Follows directly from proposition 5, 6, 7.

Proof of proposition 10: Follows from proposition 5,6,7,8.

Appendix B: Derivation of Angel's Optimal Bidding Strategy in Section 5

First we show that $h(c_h)\Delta X - c_h > j(c_j)\Delta X - c_j$.

Because $(1 - \pi^j_{vc})[(q + j(c(\pi^j_{vc})))\overline{X} + (1 - q - j(c(\pi^j_{vc})))\underline{X}] = k$. If $\pi^h_{vc} < \pi^j_{vc}$, we can give the VC $\pi^h_{vc} = \pi^j_{vc}$, by the foc of the VC, $j'(c(\pi^j_{vc})) = \frac{1}{\pi^j_{vc}\Delta X} = h'(c(\pi^h_{vc}))$. By our assumption, $j'(c) < h'(c), \forall c$, and concavity

of the functions, we know that $c(\pi_{vc}^j) < c(\pi_{vc}^h)$. This implies $j(c(\pi_{vc}^j)) < h(c(\pi_{vc}^h))$. This in turn implies $(1-\pi_{vc}^h)[(q+h(c(\pi_{vc}^h)))\overline{X}+(1-q-h(c(\pi_{vc}^h)))\underline{X}] > k$. So the entrepreneur can do better by giving the VC more share $\pi_{vc}^h = \pi_{vc}^j$, thus makes the VC to work harder and at the same time keep the entrepreneur's IC satisfied. Contradiction. So we have $\pi_{vc}^h \geq \pi_{vc}^j$. Because $1 \geq \pi_{vc}^h \geq \pi_{vc}^j$, the VC's effort is always less than the optimal effort $\widehat{c}$, thus we have $j(c(\pi_{vc}^j))\Delta X - c(\pi_{vc}^j) \leq j(c(\pi_{vc}^h))\Delta X - c(\pi_{vc}^h) < h(c(\pi_{vc}^h))\Delta X - c(\pi_{vc}^h)$.

Had the angel know the VC's type, he would offer

$$O_j \equiv (1-\pi_a)[q\overline{X}+(1-q)\underline{X}] + (1-\rho)(j(c_j)\Delta X - c_j), \tag{A31}$$

or

$$O_h \equiv (1-\pi_a)[q\overline{X}+(1-q)\underline{X}] + (1-\rho)(h(c_h)\Delta X - c_h), \tag{A32}$$

However, the angel does not know the VC's type, the payoff to the angel from offering O is

$$P(O) = \begin{cases} (1-\pi_a)[q\overline{X}+(1-q)\underline{X}] \ \text{ if } O > O_h \\ O \text{ if VC is of high type; } (1-\pi_a)[q\overline{X}+(1-q)\underline{X}] \ \text{ if the VC is of low type } \ \text{ if } O_j < O \leq O_h \\ O \ \text{ if } O \leq O_j \end{cases} \tag{A33}$$

from the above, we can see that the only possibility for the angel to offer is O_h or O_j. The angel will offer O_h iff,

$$\phi O_h + (1-\phi)(1-\pi_a)[q\overline{X}+(1-q)\underline{X}] \geq O_j. \tag{A34}$$

That is,

$$\phi(h(c_h)\Delta X - c_h) \geq j(c_j)\Delta X - c_j. \tag{A35}$$

or

$$\phi D_h \geq D_j \tag{A36}$$