Primary Market Design:
Direct Mechanisms And Markets

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Abstract

We examine the interaction of two processes for gathering information required for pricing unseasoned securities: i) mechanisms for eliciting information directly from investors, and ii) information revelation through when-issued trading, i.e. forward trading in not-yet-issued securities. We find that the structure of the optimal direct mechanism changes if when-issued trading becomes possible. Such trading can interfere with the process of eliciting information directly from investors. But, unless information is closely held and the when-issued market is unable to attract broad participation, permitting such trading will increase the expected proceeds of a securities issue, thus benefiting issuers.

Key words:
primary security markets; mechanism design; when-issued trading
JEL classification: G32
1 Introduction

Different primary markets employ different procedures in order to price securities and allocate the securities to investors. In Treasury auctions, investors submit bids for securities that are priced and allocated according to explicit rules. In many IPO markets underwriters employ bookbuilding methods to solicit indications of interest from investors. Like auctions, bookbuilding procedures can be interpreted as mechanisms designed to obtain information directly from investors, albeit with implicit rather than explicit pricing and allocation rules.\(^1\)

Primary markets vary further in that some markets allow when-issued trading of unseasoned securities, while others do not.\(^2\) In the U.S. there is an active market for when-issued trading of Treasury securities, but no such market exists for IPO shares, most likely due to regulatory restrictions. In other countries, when-issued trading of IPO shares does take place. For example, most German IPOs feature a very active when-issued market; offer prices for most IPOs are set following both when-issued trading and bookbuilding.\(^3\)

Our objective in this paper is to understand how when-issued trading of unseasoned securities affects the primary market pricing of these securities. We are particularly interested in the seller’s ability to obtain information that is needed in order to set an offer price. At first cut, it is expected that when-issued trading will enhance the information discovery process, by enabling the release of information through trading. For example, Nyborg and Sundaresan (1996) argue that when-issued trading of Treasuries plays a price discovery role. Aussenegg, Pichler and Stomper (2003) find that when-issued trading of IPO shares reveals information that is of use in setting the offer prices. In both cases, however, issuers also make use of mechanisms, such as auctions or bookbuilding procedures, for obtaining information from investors. These observations leave us with the following questions: Is it necessary to use a direct mechanism to gather information, even in the presence of when-issued trading? If so, how does when-issued trading affect the mechanism? And finally, does when-issued trading benefit the issuer? These questions cannot be addressed with available data. In this

\(^1\)For example, Back and Zender (1993) and Biais, Bossaerts and Rochet (2002) use mechanism design to analyze Treasury auctions and bookbuilding procedures, respectively.

\(^2\)When-issued trading takes place before the securities are issued. Trades are cleared when, and if, the securities are issued.

\(^3\)The German IPO market is described in Aussenegg, Pichler and Stomper (2003), Löfler, Panther and Theissen (2002) and Dorn (2002).
paper we develop a model that builds on the existing empirical evidence and answers these questions.

Our analysis consists of three parts. We first characterize the optimal direct mechanism for eliciting information from investors, both with and without when-issued trading. We find that the presence of a when-issued market significantly changes the nature of the optimal mechanism. This occurs because when-issued trading offers outside opportunities to all parties involved, i.e. to the investors and the issuer of unseasoned securities. For polled investors, a when-issued market provides an opportunity to trade on information they hold back. For the issuer, when-issued trading reveals information that can be compared to information obtained directly from investors. Thus, the issuer (or her representative) can avoid errors in pricing the issue, and identify investors who may have misrepresented their private information when they were polled. As a result, when-issued trading affects in two ways the pricing and allocation policies in securities issues. First, informational rents need to be paid to investors who provide information, irrespective of whether they report positive or negative information about the value of the securities. (Without when-issued trading, either no rents must be paid, or rents are paid only to those investors who report positive information.) Second, polled investors receive larger allocations if their reports are consistent with information that is later revealed through when-issued trading, irrespective of whether the information is positive or negative. (Without when-issued trading, larger allocations are given to investors who report more positive information.)

A direct implication of the above results is that when-issued trading can increase the cost of eliciting information from informed investors by means of a direct mechanism. However, a when-issued market may reveal sufficient information for pricing unseasoned securities, rendering superfluous information gathering through a direct mechanism. In the second part of our analysis we address the question of whether when-issued trading can supplant a direct mechanism as a source of information for pricing unseasoned securities. We show that this may not be possible, due to problems of market failure. As in Glosten and Milgrom (1985), when-issued trading may fail to open if asymmetries in information across potential traders are too large. To jump-start the when-issued market, a direct mechanism may be used:
trading starts once traders observe information that has been obtained from a few informed investors polled under the mechanism. This indeed seems to be what happens in European when-issued markets for IPO shares. According to information from one of the largest brokers in these markets, when-issued trading never starts before the underwriters announce price ranges, giving an indication of the price at which shares will be offered. These price ranges reveal information that underwriters have obtained directly from investors.

In the third part of our analysis, we investigate whether when-issued trading is in the interest of issuers of unseasoned securities. In doing so, we take into account the result that information generated by a direct mechanism may be necessary in order to “jump-start” the when-issued market. In this case, the probability that when-issued trading opens is endogenous. This probability decreases if a polled investor misrepresents her information about the value of the issue. This is because misrepresentation makes it more likely that investors participating in the mechanism contradict each other, thus lowering the informativeness of the mechanism. The implication of this is two-fold: First, lying may be self-defeating. While investors have incentives to lie, due to trading opportunities, lying also lowers the expected value of such trading opportunities. Second, if information gathered through the mechanism is necessary in order for the when-issued market to open, then for a wide range of parameter values, when-issued trading will decrease, rather than increase, the cost of eliciting information from informed investors by means of a direct mechanism.

We propose a mechanism design for directly eliciting information from investors, assuming that when-issued trading is permitted. This mechanism differs from the typical definition of an auction in that the pricing and allocations depend not just on information submitted by the participants in the mechanism, but also on information learned from when-issued trading that takes place after the mechanism. This mechanism need not take the form of bookbuilding as currently practiced. An intermediary, such as an investment bank, may be needed in order to identify informed investors to participate in the mechanism. Beyond that, however, the mechanism can be set up so that information is submitted in a verifiable form, and pricing and allocations are a function of this information. In the case that when-issued trading cannot open without the revelation of information from such a mechanism, then

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4We thank Gary Beechener of Tullet & Tokyo Liberty (securities) Ltd. for providing this information.
the mechanism design is similar to a standard auction in the sense that an investor who reports positive information about the security value will receive a more favorable allocation than an investor who reports negative information. This last characteristic does not hold if the functioning of the when-issued market is independent of information obtained from the direct mechanism.

Our analysis provides a foundation for a re-assessment of regulations which prevent when-issued trading of IPO shares in the U.S. Our results suggest that allowing when-issued trading will typically be in the best interest of issuers, since this trading constitutes a source of free information for the pricing of unseasoned securities. This is not to say that when-issued trading is never detrimental to an issuer. When-issued trading can increase the cost of pricing unseasoned securities by means of a direct mechanism. We find, however, that this occurs only in two cases: i) either the issue is priced by means of a direct mechanism, even though the when-issued market alone could reveal the right price, or ii) private information is held by a very small number of investors. If information is not closely held, so that the latter concern is not relevant, then any rationale for ruling out when-issued trading must hinge on the notion that a direct mechanism is useful for the pricing of unseasoned securities, for reasons outside of our analysis. In the absence of such reasons, our results suggest that the expected proceeds of many issues would increase if when-issued trading were permitted.

Our results are consistent with a number of stylized facts. While direct mechanisms, such as bookbuilding, have been recognized as sources of information for setting prices of unseasoned securities, practitioners seem to also view the when-issued market as an indicator for how such securities should be priced. To quote one of the largest market makers in the German when-issued market: “By observing when-issued trading, the underwriter can gauge the market’s interest in an IPO.”

Aussenegg, Pichler and Stomper (2003) analyze the German IPO market and find evidence consistent with this quote. For when-issued trading of Treasury securities, Nyborg and Sundaresan (1996) argue that the when-issued market reveals information about the expected depth of a Treasury auction and the diversity of the

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5This quote was taken from the website of Schnigge AG, http://www.schnigge.de/info/service/pre-ipo-trading.html. The original quote was in German: “Der Emissionsführer kann auf Grund der Handelstätigkeit im Handel per Erscheinen das Interesse des Marktes an der Neuemission messen.”
auction participants.

Our paper extends the existing literature on the design of mechanisms for pricing unseasoned securities. The paper is most closely related to recent contributions by Biais, Bossaerts and Rochet (2002) and Maksimovic and Pichler (2002) in that we examine optimal mechanisms for eliciting information directly from informed investors. Our paper differs in that we determine the form of such mechanisms in the presence of when-issued trading of the securities. In this regard our work is related to analyses of auctions for Treasury securities. Back and Zender (1993) and Viswanathan and Wang (2000) model Treasury auctions. The latter paper also allows for when-issued trading. Bikchandani and Huang (1993) and Nyborg and Sundaresan (1996) examine the when-issued market for U.S. Treasury securities.

In addition, our paper is related to the literature on when-issued trading of IPO shares. Löffler, Panther and Theissen (2002) and Aussenegg, Pichler and Stomper (2003) examine the when-issued market for German IPOs. The first paper finds that the final prices in this market are unbiased predictors of opening prices in the secondary market. The latter paper provides evidence that bookbuilding is not used to gather information once when-issued trading commences, but may be used as a source of information prior to the trading. Dorn (2002) examines this same when-issued market to investigate whether sentiment drives retail participation. Cornelli, Goldreich and Ljungqvist (2004) analyze the relation between when-issued trading and short-run and long-run returns of IPOs, with a focus on investor sentiment. Ezzel, Miles and Mulherin (2002) examine when-issued trading of shares of publicly traded subsidiaries prior to full divestiture.

The paper is organized as follows. In the next section, we provide a brief description of some existing primary markets. In the third section, we present the basic model for pricing unseasoned securities. In Section 4 we determine the optimal direct mechanism for eliciting information from investors, both with and without when-issued trading. In Section 5 we examine the opening of the when-issued market. In Section 6 we redo the second part of Section 4, taking into account the results of Section 5. This final analysis then enables us to

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6Busaba and Chang (2002) examine bookbuilding and aftermarket trading. When-issued trading differs from aftermarket trading in that when-issued prices are revealed prior to the setting of the primary market offer price and the allocation of securities.
answer the question: Is when-issued trading beneficial for issuers?

2  A selective survey of institutional features of primary markets

In this section, we briefly survey the structure of some primary markets. This survey highlights a difference between the institutional framework of U.S. Treasury markets and that of U.S. IPOs: the existence of a market for when-issued (forward) trading of Treasuries but not for IPO shares. We point out that some European markets feature when-issued trading of IPOs, and we describe one notable example, the German Neuer Markt.

When-issued trading of Treasury securities: Bikhchandani and Huang (1993) and Nyborg and Sundaresan (1996) provide detailed descriptions of institutional features of the primary market for U.S. Treasury securities, including the market for when-issued trading of Treasuries. The when-issued market is a forward market for trading in not yet issued securities. Trading starts on the date of the announcement of a Treasury auction and continues after the auction takes place (up until the issue date). The forward contracts represent commitments to trade when, and if, the security is issued. The contracts specify physical delivery of the underlying security on the date at which this security is issued.

Initial public offerings of shares: Ritter and Welch (2002), and Ritter (2002) provide recent surveys of the institutional structure of IPO markets and the extensive literature on IPOs. Ljungqvist, Jenkinson and Wilhelm (2003) point out that the U.S. method of IPO pricing through bookbuilding has become increasingly popular outside the U.S. Price discovery through bookbuilding differs from price discovery through when-issued trading in that, in bookbuilding information is gathered directly from investors. According to Benveniste and Spindt (1989), in order to provide incentives for investors to truthfully reveal positive information about an issue, underwriters only partially adjust the IPO prices in response to such information. The underwriters then allocate underpriced shares to those investors who provided the positive information. Hence, investors who hold positive information earn informational rents.
When-issued trading of IPO shares: In the United States there is no market for when-issued trading of IPO shares. Such when-issued trading is restricted by securities laws.\(^7\) The stated reason for the restriction is: “Such short sales could result in a lower offering price and reduce an issuer’s proceeds.”\(^8\)

In contrast, IPO markets in many European countries feature when-issued trading. Since many of these markets also employ bookbuilding methods to price IPOs, this implies that there are potentially two sources of information for IPO pricing. Price discovery may take place both through the market for when-issued trading (by analogy to Nyborg and Sundaresan (1996)), and through a direct mechanism in bookbuilding (as suggested by Benveniste and Spindt (1989)).

Aussenegg et al. (2003) investigate one notable example of an IPO market with bookbuilding and when-issued trading of IPO shares, the German Neuer Markt IPO market. They provide a detailed description of the institutional framework of this market that can be summarized by the timeline in Figure 1. The timeline has three stages: the period before the opening of when-issued trading (Stage 1), the period during which when-issued trading occurs (Stage 2), and the period after when-issued trading (Stage 3). During Stage 1, the underwriters may gather information to use in setting price ranges. When-issued trading

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\(^7\) Regulation M, Rule 105 prohibits the covering of short positions in IPO shares that were created within the last five days before pricing, with allocations received in the IPO. In addition to this rule, there are also restrictions on trading in unregistered shares.

opens at time $t_W$, after these ranges are set and publicly posted. Selling in this market is, by definition, short selling because the shares have not yet been issued. In the German when-issued market for IPOs the market makers place restrictions on such trading, typically only allowing institutional traders to sell short. It is quite likely that much of the net selling activity in this market can be attributed to investors who expect to receive allocations at the IPO offer price. When-issued trading continues beyond the time $t_P$ at which the underwriters set the IPO offer prices, up to the evening before the first day of trading in the secondary market.

3 The Basic Model and the Value of Information

In this section we present the basic model and some preliminary results that demonstrate the benefit of gathering information before setting the offer price. A list of variables is given at the beginning of the Appendix.

3.1 The basic model

Information about the value of the securities: We take as exogenous the number of securities that are sold; only the offer price is endogenous. We also assume that the offering is uniform price; that is, all investors at the offer pay the same price. The symbol $\hat{V}$ represents the unknown secondary market value of the total offering: per security value times the number of securities sold. An investor who purchases securities obtains a fraction of this value. $\hat{V}$ is given by:

$$\hat{V} = v_0 + s w,$$ (1)

where $v_0$ is the prior expected value of $\hat{V}$ and $w$ is a positive parameter that is strictly smaller than $v_0$. $s$ is a random variable that can take on the realizations 1 or −1; each outcome has a prior probability of $\pi_0 = 1/2$.

A number of “informed” investors have observed noisy signals of $s$. The signal of investor $i$ is a random variable $\hat{c}_i$ which can take on one of two realizations, $\hat{c}_i \in \{-1, 1\}$. Conditional on the realization of $s$, the signals $\hat{c}_i$ and $\hat{c}_j$ of any two informed investors $i$ and $j$ are
independent of each other and identically distributed. With probability \( q > 1/2 \), any given informed investor has correctly observed the realization of \( \tilde{s} \). For an investor who sees a positive signal, the probability that \( s = 1 \) is \( q \) and the probability that \( s = -1 \) is \( 1 - q \), so that the expected value of \( \tilde{s} \) is given by \( q - (1 - q) = 2q - 1 > 0 \). For an investor who sees a negative signal, the expected value of \( \tilde{s} \) is \( 1 - 2q < 0 \). We will assume that a fraction \( \alpha (0 < \alpha < 1) \) of all potential investors are informed. On average, a fraction \( q\alpha \) of investors will have correctly observed the realization of \( \tilde{s} \) and \( (1 - q)\alpha \) will have observed \(-\tilde{s}\). The model of equation (1) and the parameters \( v_0, w, \alpha \) and \( q \) are all common knowledge.

3.2 Pricing the unseasoned securities: preliminary results

All players are risk neutral. Thus, the basic objective of the issuer is to maximize expected offering proceeds. We will assume that in order to ensure a successful offering, uninformed investors must be willing to buy securities in the primary market.\(^9\) However, these investors will face adverse selection risk if they participate in the offering. The reason is as follows: Suppose that the seller prices the securities at the prior expected value of \( v_0 \). Because \( q > 1/2 \), fewer informed investors will place orders when the issue is overpriced (\( s = -1 \)) than when it is underpriced (\( s = 1 \)). Uninformed investors will thus receive a larger allocation if the issue is overpriced than if it is underpriced. This means that, if the offering is priced at \( v_0 \), they will achieve a negative expected return. In order to induce uninformed investors to participate, the offering must be priced so that their expected return is nonnegative. Hence, the following constraint must be satisfied:

\[
p_I \leq E[\tilde{V}] - u_{AS},
\]

where \( u_{AS} > 0 \) is the a priori expected underpricing due to adverse selection risk.

Our model for expected underpricing due to adverse selection risk is a simplified version of the model in Rock (1986). Investors arrive randomly in the primary market and decide whether to buy a small fraction of the issue. Allocations are given on a first-come first-served basis until the entire issue is sold. We will say that an investor “participates in the offering”

\(^9\)Note, if \( q \) is close enough to one, then the only way to ensure sufficient informed participation is to set the offer price equal to the lowest possible value of \( \tilde{V} \): \( v_0 - w \). Ensuring uninformed participation avoids this outcome.
if the investor joins the queue for an allocation. As described above, we assume that a fraction $\alpha$ of all investors have observed a private signal of the value of the securities. If the realized value of $\tilde{s}$ is 1, then on average a fraction $q$ of the informed investors will participate; if the realized value of $\tilde{s}$ is -1, then on average a fraction $1 - q$ of the informed investors will participate. Uninformed investors will participate only if the security is sufficiently underpriced to make it a fair bet for them, despite the adverse selection risk. The following required level of underpricing is derived in the Appendix:

$$u_{AS} = \frac{q - 2\pi_T(1 - \pi_T) - (2\pi_T - 1)^2 q}{1 - ((2\pi_T - 1)q + 1 - \pi_T)\alpha} \omega$$

where $\pi_T$ is the probability that $s = 1$, given all of the information that is available at the time of setting the offer price. If no information is gathered prior to pricing the offering, then $\pi_T = \pi_0 = 1/2$, and

$$u_{AS} = \frac{(2q - 1)\omega}{2 - \alpha} .$$  

Figure 2 illustrates the relation between expected underpricing and information gathering. If $\pi_T = \pi_0 = 1/2$, then no information has been obtained prior to setting the offer price. As more information is learned, $\pi_T$ moves toward either zero or one. The following lemma summarizes the relation between information and expected underpricing. Lemma 1 is stated in a technically precise manner; an intuitive explanation follows.

**Lemma 1.** $u_{AS}$ and information.

a) For all $\pi_T \leq 1/2$, $u_{AS}$ is strictly increasing in $\pi_T$.

b) For all $\pi_T \geq q$, $u_{AS}$ is strictly decreasing in $\pi_T$, and $u_{AS}(\pi_T) < u_{AS}(\pi_0)$. 

Figure 2: Expected underpricing 

$w = 1.5$, $q = .75$, $\alpha = .3$
On the basis of just one informed investor’s information, $\pi$ with be equal to either $q$ (which is $> 1/2$) or $1 - q$ (which is $< 1/2$). Thus, Lemma 1 states that if the issue is priced after obtaining information that is at least as good as that of one informed investor, then underpricing due to adverse selection risk will be lower as a result of having gathered information. This result is intuitive. If the issue is priced based on more information, then there remains less asymmetry of information between informed and uninformed investors. Thus, the risk of adverse selection, faced by uninformed investors, is mitigated.

A direct implication of Lemma 1 is that gathering information prior to pricing the issue increases the expected issue proceeds, and is thus valuable for the issuer. We have up to this point, however, ignored any costs of information gathering. If information is obtained from investors through a direct mechanism, then this may in itself lead to expected underpricing. In the ensuing analysis we will model the direct mechanism and we will determine sufficient conditions such that issuers will expect to benefit from when-issued trading.

4 Direct Mechanisms with and without When-Issued Trading

The purpose of this section is to answer the question: Will allowing for when-issued trading change the nature of the optimal direct mechanism that induces informed investors to truthfully report their information about the issue? We begin by determining the optimal mechanism in the absence of when-issued trading.

4.1 The direct mechanism without when-issued trading

To keep the model simple we assume that information is solicited from only two “polled” investors. Shares may be allocated, however, to a large number of other investors, as well as to these two informed investors. There are three possible outcomes for the polled investors’ reports: either both report positive information, both report negative information, or one reports positive information and the other negative. We represent these outcomes with the pair $(a, b) \in \{(+, +), (+, -), (-, -)\}$, where $(++)$ indicates that both polled investors reported positive information.
Information gathered through the mechanism can also be represented with a simple sufficient statistic: the number of reported positive signals minus the number of reported negative signals. We denote this difference with the parameter $z$. $z = 2$ corresponds to $(++)$ and $z = -2$ to $(--)$. If investors’ reports are the only sources of information for pricing the issue, then $\pi_T = \pi(z)$:\(^{10}\)

$$\pi(z)\bigg{|}_{z \geq 0} = \frac{q^2}{q^2 + (1 - q)^2}$$
$$\pi(z)\bigg{|}_{z \leq 0} = \frac{(1 - q)^{|z|}}{q^{|z|} + (1 - q)^{|z|}}$$

(5)

A zero value of $z$ indicates that either no information was gathered, or the investors’ reports are contradictory ((+-) or (-+)) and, hence, overall uninformative: $\pi(0) = \pi_0 = 1/2$. If information is learned, then the probability that $\hat{s} = 1$ moves away from 1/2: $\pi(2) > 1/2 > \pi(-2)$. Also, $\pi(2) = 1 - \pi(-2)$.

consistent with securities regulations in many markets, we will assume that the issue is uniform price. Since the issue size is exogenously given, we can write the seller’s objective as simply minimizing expected underpricing. We invoke the Revelation Principle so that we need only consider mechanisms that induce truthtelling. The seller will thus solve the following problem:

$$\min \ E[u_{ab}] \equiv \left(\frac{q^2}{2} + \frac{(1 - q)^2}{2}\right) u^{++} + 2q(1 - q)u^{+-} + \left(\frac{q^2}{2} + \frac{(1 - q)^2}{2}\right) u^{--}$$

(6)

where $u_{ab}$ is the expected level of underpricing, conditioned on a given outcome $(a, b)$, and the weights on the $u_{ab}$’s are the probabilities of each outcome. The objective function may also be written as:

$$\min \ E[u_{ab}] \equiv Er^R + Er^+ + Er^-$$

(7)

where

$Er^+$ = expected return to a polled investor who sees and reports +

$$= \left(\frac{q^2}{2} + \frac{(1 - q)^2}{2}\right) u^{++} + 2q(1 - q)u^{+-} + \left(\frac{q^2}{2} + \frac{(1 - q)^2}{2}\right) u^{--}$$

(8)

$Er^-$ = expected return to a polled investor who sees and reports −

$$= \left(\frac{q^2}{2} + \frac{(1 - q)^2}{2}\right) u^{--} - 2q(1 - q)u^{+-}$$

(9)

$Er^R$ = expected return to retail investors (nonpolled investors)

$$= \left(\frac{q^2}{2} + \frac{(1 - q)^2}{2}\right) u^{++}(1 - 2h^{++}) + \left(\frac{q^2}{2} + \frac{(1 - q)^2}{2}\right) u^{--}(1 - 2h^{--})$$

See the Appendix for the derivation of $\pi(z)$.

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and where $h_{ab}$ is the fraction of the offering that is allocated to a polled investor who reports $a$ while the other reports $b$. Writing the objective function in this way emphasizes the fact that underpricing represents wealth that is transferred from the seller to the buyer of the securities. The objective function is minimized subject to following constraints.

**Participation constraints:** Investors who participate in the mechanism must earn positive expected returns:\[11\]

$$u_{ab}^e \geq 0 \quad \forall u_{ab} \in \{u^+, u^-, u^{--}\} \quad (PC-I)$$

**Incentive compatibility constraints:** The expected value of the issue, given the reports of the polled investors is

$$E[\tilde{V} | z] = v_0 + \frac{(z/2)(2q - 1)w}{q^2 + (1-q)^2} ,$$

$z \in \{2, 0, -2\}$. An investor who lies increases or decreases the perceived value of $z$ by 2. An investor who sees positive information, but reports a negative signal, thus causes the expected value of $\tilde{V}$ to be lower by an amount $w_L$:

$$w_L \equiv \text{expected impact of lie} = \frac{(2q - 1)w}{q^2 + (1-q)^2}$$

Similarly, an investor who sees negative information and reports positive information will cause the seller's expected value of $\tilde{V}$ to be higher by an amount $w_L$. Investors will truthfully report their information, as long as the following incentive compatibility constraints are satisfied:

$$Er^+ \geq \left( q^2 + (1-q)^2 \right) (u^{+-} + w_L) h^+ + 2q(1-q) \left( u^{--} + w_L \right) h^-$$

$$Er^- \geq \left( q^2 + (1-q)^2 \right) (u^{--} - w_L) h^- + 2q(1-q) \left( u^{+-} - w_L \right) h^+$$

where $Er^+$ and $Er^-$ are as given in (8) and (9). The constraint (IC$^-$) is written based on the notion that an investor cannot refuse an allocation, without revealing that she has lied.

\[11\]Satisfying this constraint means that the seller does not overprice the securities in expected value, conditioned on the information that is learned through the direct mechanism. Ex post realized overpricing may occur, if there remains uncertainty about $\tilde{V}$. We could also introduce an a priori strictly positive participation constraint: $(Er^+ + Er^-)/2 \geq \gamma > 0$, but adding this extra complexity would not alter our qualitative results.
For this reason, providing false positive information is generally not beneficial to investors, and \((IC^-)\) will typically be nonbinding. This is one aspect of the mechanism design problem that will change when we model the mechanism in the presence of when-issued trading.

**Allocation constraints:** The seller may allocate securities to the two investors who participate in the mechanism, as well as to other investors. We will refer to the latter investors as retail investors, even though there may some institutional investors among them. The significant differences between “retail” allocations and allocations awarded to polled investors are that: i) individual retail allocations are very small and ii) they are awarded anonymously and randomly on a first-come first-served basis. As a result, the expected allocation for any individual investor who bids for part of the retail allotment will be very small. We will thus assume that, even though the polled investors are not prohibited from bidding for part of the retail allotment, this will have no significant effect on the polled investors’ incentives.

We assume that the seller must allocate a fraction \(h_R\) of the issue for retail allocations. The purpose of such a constraint is typically to ensure that at least some investors who do not have special relationships with the issuer, or intermediary, are able to participate in the offering. In many offerings there is also often a specific requirement that shares be set aside for retail investors.

\[
\begin{align*}
h_R^a &\geq 0 \\
h^- \cdot h^+ &\leq \frac{1 - h_R}{2} \\
h^- + h^- &\leq 1 - h_R
\end{align*}
\]

where \(h_R > 0\) is an exogenously given constant.

**Participation of retail investors:** We have assumed that a fraction \(\alpha\) of all investors are privately informed. Consistent with the analysis of Section 3, in order to ensure that the retail allocation will be purchased, uninformed investors must be willing to bid for securities. Thus, their expected return must be nonnegative, despite adverse selection risk:

\[
\begin{align*}
&u^{++} \geq u_{AS}(2), \quad u^{+-} \geq u_{AS}(0), \quad u^{-+} \geq u_{AS}(-2) \\
&\quad (PC - R)
\end{align*}
\]

\(^{12}\)Investors who participate in the mechanism do not face an adverse selection risk, because their allocations are based only on the reported information, not on any information that may still reside with other investors. For this reason, the constraint \((PC - I)\), which requires only nonnegative underpricing, is sufficient to ensure the participation of the polled investors.
$u_{AS}$, introduced in Section 3, is expected underpricing due to residual adverse selection risk. We use the notation $u_{AS}(z)$ to indicate that this expected underpricing is a function of the information obtained through the mechanism.

The optimal direct mechanism: It is shown in the Appendix that, without when-issued trading: 1) Positive expected underpricing is required in each state, due to residual adverse selection risk, and 2) no further underpricing is needed in order to induce truthful reporting on the part of the polled investors.\textsuperscript{13} Thus,

\[
E[u^{ab}]_{\text{no when-issued trading}} = E[u_{AS}(z)] = 
\left(\frac{q^2 + (1-q)^2}{2}\right) u_{AS}(-2) + 2q(1-q)u_{AS}(0) + \left(\frac{q^2 + (1-q)^2}{2}\right) u_{AS}(2) \quad (12)
\]

The allocation policy in the optimal direct mechanism calls for a polled investor to receive the maximum possible allocation when she reports positive information, and no allocation otherwise. This mechanism is similar to an auction in that the pricing and allocations depend only on information received from polled investors in the mechanism. Also, as in a standard auction, more securities are allocated to the investors who report more positive information about the security value. The constraints $(PC - R)$ are strictly binding; the incentive compatibility constraints are not binding. However, satisfying the $(PC - R)$ constraints results in strictly positive expected underpricing in every state, and so a polled investor expects to earn a strictly positive return when she has observed positive information, and a zero return otherwise. We will refer to the mechanism described here as the benchmark mechanism.

4.2 The direct mechanism in the presence of when-issued trading

When-issued trading changes the mechanism design problem in two ways. i) Polled investors have an extra incentive to lie, because of profitable trading opportunities in the when-issued market. ii) When-issued trading reveals information that can be compared to information

\textsuperscript{13}This follows directly from Maksimovic and Pichler (2002). They model a mechanism without when-issued trading and show that, in the absence of allocation constraints requiring that shares be allocated to polled investors, underpricing is not needed to induce truthhtelling.
obtained from investors who are polled under the mechanism. Thus, errors in the primary market pricing of the issue can be avoided. Also, the expected impact of a polled investor’s report on the offer price is reduced. This latter effect reduces an investor’s incentives to falsely report overly negative information, but also reduces the penalty for falsely reporting overly positive information. (Without when-issued trading, a polled investor who has observed negative information does not want to report positive information, because this will lead to an allocation of overpriced shares.)

In order to capture the above effects we develop a model of the direct mechanism and when-issued trading that is consistent with the institutional details described in Section 2. For simplicity, we assume that truthful reporting deprives the polled investors of any profits they could otherwise earn by trading in the when-issued market. This is the case if when-issued trading opens only after information that has been obtained through prior mechanism-based information gathering is publicly released.\(^{14}\) We will also allow the pricing and allocation of securities to depend not only on information provided in the mechanism, but also on information obtained from when-issued trading. In this sense, the mechanism will not satisfy the strict definition of an auction.

In the presence of when-issued trading, the following incentive compatibility constraints must be satisfied:

\[
Er^+ \geq \left( q^2 + (1 - q)^2 \right) ((u^{+-} + w_{LT})h^{+-} + \psi_L^{+-}) + 2q(1 - q)((u^{--} + w_{LT})h^{--} + \psi_L^{--}) \quad (IC_T^+)
\]

\[
Er^- \geq \left( q^2 + (1 - q)^2 \right) ((u^{+-} - w_{LT})h^{+-} - \psi_L^{+-}) + 2q(1 - q)((u^{--} - w_{LT})h^{--} + \psi_L^{--}) \quad (IC_T^-)
\]

As in \((IC^+)\) and \((IC^-)\), \(Er^+\) and \(Er^-\) are as given in (8) and (9). These constraints differ from \((IC^+)\) and \((IC^-)\) in two ways. First, the expected impact of a lie on the offer price is lower because when-issued trading will reveal information that, with probability \(q > 1/2\), will counteract the effect of the lie: thus, \(w_{LT} < w_L\). Second, the right-hand side of each constraint includes the expected profit that informed investors can earn by trading on their information. \(\psi_L^{+-}\) denotes the expected trading profit for an investor who sees + but reports −, while the other polled investor reports \(b\), and \(\psi_L^{-b}\) is the expected trading profit for an

\(^{14}\) In the German IPO market, there seems to be an informational spillover from bookbuilding to the when-issued market when underwriters set price ranges. When-issued trading starts immediately after the price ranges are set.
The optimal direct mechanism: The benchmark mechanism described above will not satisfy the new incentive compatibility constraints. To see this, consider an informed investor with negative information about the value of the issue. Under the benchmark mechanism without when-issued trading, the investor would never falsely report positive information. This is because such a report would result in the investor receiving an allocation of securities that are overpriced. If when-issued trading takes place, this is no longer the case. Information revelation in the when-issued market will “correct” any pricing error that would result from an overly positive report. If the issuer employs the benchmark mechanism, the investor will expect to receive an allocation (as a reward for reporting positive information), and because of when-issued trading, the allocation will not be overpriced. In addition, false reporting gives the investor an informational advantage relative to other traders in the when-issued market. An investor who falsely reports positive information can profitably sell her allocation by taking a short position in the when-issued market. Thus, in the presence of when-issued trading, the benchmark mechanism does not satisfy the incentive compatibility constraints.

When-issued trading further changes the mechanism design problem in that the issuer can use information from the when-issued market in order to detect whether a polled investor has lied. Thus, one way in which the optimal mechanism differs from the benchmark mechanism is that the seller will allocate nothing to a polled investor whose report is contradicted by information released in the when-issued market. Underpriced shares are allocated to each polled investor who is not contradicted by when-issued trading, regardless of whether the investor’s report was positive or negative. The following proposition summarizes how when-issued trading changes the optimal direct mechanism.

Proposition 1.

1. The optimal direct mechanism in the presence of when-issued trading is

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15 The participation constraints will also change in that the right-hand-side will be strictly positive, instead of zero. $E\rho^+$ and $E\rho^-$ must both be at least as large as expected trading profits, given that the investor doesn’t participate. The new participation constraints will be satisfied as long as the new incentive compatibility constraints are satisfied.

16 In order to realize a profit by doing this, the investor would need to trade before information has been fully revealed. If there are many informed investors who could potentially trade ahead of the polled investor, then there is no guarantee that the investor will realize a strictly positive profit. But, the a priori expected value of the trading profit will be strictly positive.
qualitatively different from the optimal mechanism without when-issued trading. The optimal mechanism in the presence of when-issued trading has the following characteristics:

(a) The incentive compatibility constraints are strictly binding.

(b) Securities are allocated to polled investors whose reports are consistent with information revealed by the when-issued market, regardless of whether the information is positive or negative.

(c) Rents must be paid in order to induce truthful reporting, both to polled investors with positive and with negative information.

2. Allowing when-issued trading can make direct information gathering more expensive.

This mechanism differs from a standard auction in two ways: i) The price and allocations depend on information received from polled investors in the mechanism and on information obtained from when-issued trading. ii) Securities are not necessarily allocated to the investor who reports a more positive valuation.\(^{17}\)

The results in Proposition 1 imply that when-issued trading can increase the cost of gathering information by means of a direct mechanism. This is because, in the presence of when-issued trading, informational rents must be paid in order to induce truth telling. The extent of these rents depends on the trading profits that a polled investor expects to earn after misreporting her signal.

Even though when-issued trading imposes a cost to inducing truth telling, this does not imply that such trading is always detrimental for the issuer. The when-issued market can reduce the expected underpricing of the issue since it reveals information that results in a lower residual adverse selection risk. Depending on the magnitude of these two effects, expected underpricing may be lower or higher than without when-issued trading.

In the analysis up to this point, we have taken it for granted that a direct mechanism is used to gather information prior to the pricing of the issue. However, mechanism-based information gathering may not be necessary if when-issued trading is suited to provide

\(^{17}\)If allocations are determined prior to the onset of when-issued trading, then the optimal mechanism will allocate nothing to the polled investors when they disagree with each other, and give them underpriced allocations when they agree with each other. (Collusion on the part of polled investors is ruled out; the seller can prevent collusion simply by not informing one polled investor of the identity of the other.) The polled investors will receive underpriced allocations both when they agree on positive information and when they agree on negative information.
sufficient information for pricing unseasoned securities. This is the case since information from when-issued trading is freely available whereas rents may have to be paid in order to obtain information directly from the investors. In the next section, we will consider whether when-issued trading can indeed serve as the sole source of information for pricing the issue. We will show that this may not be the case since the when-issued market may fail to open.

5 When-issued trading

In this section, we will determine conditions such that a when-issued market can function in the absence of information gathering from any other source, such as a direct mechanism. The results of this section will also provide the necessary groundwork for exploring the interaction between when-issued trading and a direct mechanism, in the case that these conditions are not satisfied.

5.1 The Model of When-issued Trading

Our model of when-issued trading is similar to the model of Glosten and Milgrom (1985). This is a model of a dealer market, such as the when-issued market for IPO shares in Germany. The model illustrates, in a very simple manner, the problem that a market may not function if informational asymmetries are too great. And, even if the market does function, spreads may be so wide that an investor who is trading on private information cannot make a large profit. The details of the model are described below.

Players: The players are the same as we have modeled so far, with the following exceptions: i) the seller/intermediary does not participate in when-issued trading;\[^{18}\] ii) the market is facilitated by purely competitive, risk-neutral market makers. As before, a fraction $\alpha$ of the investors (traders) are informed, with their information structure being the same as described in Section 3. The market makers are uninformed. All of the players are risk neutral. The market makers have no inventory costs, or costs of trading.

\[^{18}\]This assumption is consistent with common practice on the German Neuer Markt. See Aussenegg et al. (2003).
**Time-line:** Before the opening of the market, the market makers post competitive bid and ask prices. Traders arrive sequentially. Each arrival either buys one unit at the ask price, or sells one unit at the bid price. Market makers update their bid and ask prices after a trade takes place. All bid, ask and transaction prices are publicly observed. Because the market makers have identical information and no inventory costs, all of them post the same quotes at any given time. In what follows, we will thus refer to a single bid and a single ask price at a certain point in time.

**Trading:** The trader who arrives at time \( t \) values the traded asset at \( Y_t = E[\tilde{V} | F_t] + \rho_t \), where \( F_t \) is the time \( t \) trader’s information set and \( \rho_t \) is a private value component which can be interpreted as a trader’s liquidity need. The size of a unit trade is an exogenously given fraction of the total offering size, \( \eta << 1 \). Because the unit size is exogenous, we can ignore this parameter for now. \( Y_t \) thus represents trader \( t \)’s valuation of the total offering. (The parameter \( \eta \) will be reintroduced in the following section.) We assume that \( \rho_t = 0 \) for all informed traders, and for the market makers. For uninformed traders, \( \rho_t \in \{ -\rho, \rho \} \), \( \rho \geq 0 \). An uninformed trader with a valuation parameter of \( \rho \) is thus a potential buyer, while an uninformed trader with a valuation parameter of \( -\rho \) is a potential seller.\(^{19}\)

The trading rule is the same as in Glosten and Milgrom (1985): A trader will buy if \( Y_t > A_t = \text{the ask price} \), and sell if \( Y_t < B_t = \text{the bid price} \). If all \( Y_t \in [B_t, A_t] \), then no trade occurs. Because the market makers are competitive and risk neutral they post “no regret” bid and ask prices:

\[
A_t = \text{ask price} = E[\tilde{V} | H_t, \text{ time } t \text{ arrival is a buyer}]
\]

\[
B_t = \text{bid price} = E[\tilde{V} | H_t, \text{ time } t \text{ arrival is a seller}]
\]

where \( H_t \) is the public information set, just before the \( t \)th arrival. \( H_t \) includes all past bid, ask and transaction prices, as well as any information that has been revealed prior to the

\(^{19}\)A difference between our model and that of Glosten and Milgrom (1985) is that our valuation parameter is additive, rather than multiplicative. A multiplicative valuation parameter has the effect that liquidity increases when the spread decreases and/or when the expected value of the asset increases. This means that obtaining positive information through bookbuilding will always increase the liquidity of the when-issued market, but obtaining negative information may worsen the liquidity because it decreases the expected value. While this may be realistic, it adds a level of complexity to our analysis that is not central to our work. For this reason we choose an additive form for our valuation parameter. The effect of this additive form is that liquidity depends only on the absolute value of the spread.
start of when-issued trading. An informed trader’s information set \( F_t \) includes \( H_t \) and the trader’s signal of \( \tilde{s} \).

5.2 The Possibility of Market Failure

We first consider the case such that no information, apart from equation (1) and the prior distribution on \( \tilde{s} \), has been revealed prior to when-issued trading. If uninformed traders do not participate in when-issued trading, then \( A_1 = v_0 + (2q - 1)w \) and \( B_1 = v_0 - (2q - 1)w \). At these quotes no informed traders have incentives to buy or sell, since the quotes are equal to their valuations. Hence, no trade occurs and the market breaks down at the open.

If uninformed traders do participate, then the probability that an informed trader arrives equals \( \alpha \). The opening quotes, given that no information has been gathered are:

\[
\hat{A}_1 = v_0 + E[\tilde{s} | Y_1 > A_1]w = v_0 + \alpha(2q - 1)w, \tag{13}
\]

\[
\hat{B}_1 = v_0 + E[\tilde{s} | Y_1 < B_1]w = v_0 - \alpha(2q - 1)w. \tag{14}
\]

Uninformed traders will participate at the open if and only if \((\hat{A}_1 - \hat{B}_1)/2 < \rho\). Thus, a necessary and sufficient condition for the when-issued market to open, with no prior information gathering is:

\[
\alpha(2q - 1)w < \rho. \tag{15}
\]

Resolving problems of market failure: We next show that problems of market failure can be resolved if traders receive some information about the value of the issue. This analysis is inspired by a stylized fact about European when-issued markets for IPO shares: the market never opens before the underwriter posts a price range indicating possible offer prices for the issue. Consistent with this stylized fact, we will consider the possibility that a direct mechanism is used to generate information that is then publicly revealed prior to when-issued trading. As in the last section, let \( z \) denote the number of positive reports minus the number of negative reports. We can thus generalize equations (13) and (14):

\[
A_1(z) = v_0 + \alpha^+(z)E[\tilde{s}|z + 1]w + (1 - \alpha^+(z))E[\tilde{s}|z]w \tag{16}
\]

\[
B_1(z) = v_0 + \alpha^-(z)E[\tilde{s}|z - 1]w + (1 - \alpha^-(z))E[\tilde{s}|z]w \tag{17}
\]
where $\alpha^+(z)$ ($\alpha^-(z)$) is the probability that the first trader in the market is informed, given that a buyer (seller) has arrived at the open. ($\alpha^+(0) = \alpha^-(0) = \alpha$.) The expressions for $\alpha^+(z)$ and $\alpha^-(z)$ are derived in the Appendix where it is shown that for $z \geq 1$, $\alpha^+(z) = \alpha^-(z) > \alpha > \alpha^-(z) = \alpha^+(z)$.

The parameter range, such that the when-issued market will open, will increase if the spread ($S_1(z) \equiv A_1(z) - B_1(z)$) decreases. The size of the opening spread is independent of the sign of $z$; it depends only on $|z|$. The precise equation for the opening spread, given that $|z| \geq 1$, is given in the proof of Proposition 2 in the Appendix. Our general results concerning the opening of when-issued trading are summarized in the following proposition.

**Proposition 2.** When-issued trading will open if and only if:

$$
(\alpha(2q - 1) - r(z, q))w < \rho
$$

where $r(z, q)|_{z=0} = 0$ and $r(z, q)|_{|z|\geq1} > 0$. Thus, prior information gathering enlarges the parameter range such that when-issued trading can commence.

If none of the private information is revealed prior to the opening of the when-issued market, then condition (18) reverts to (15). Information gathering, using for example the mechanism modeled in the previous section, will be both necessary and potentially sufficient for the opening of the when-issued market if (15) is not satisfied, but (18) is satisfied for $z \in \{-2, 2\}$. The likelihood that prior information gathering will be both necessary and sufficient for the opening of the when-issued market depends on the accuracy of the information held by informed investors, as measured by $q$. From equation (15) it can be seen that a larger value of $q$ decreases the parameter range such that when-issued trading will open without information gathering. We show in the Appendix that $r(z, q)$ is increasing in $q$. Moreover, as $q$ increases, the probability of contradictory reports decreases, and so the mechanism is less likely to result in a zero value of $z$. We thus have the following Corollary to Proposition 2.

**Corollary 1.** Prior information gathering is both more necessary and more beneficial for enabling when-issued trading when the information held by informed investors is more accurate (higher value of $q$).
6 The Costs and Benefits of When-Issued Trading

Up to this point our results appear to be mixed. By itself when-issued trading is beneficial for an issuer, because such trading freely reveals information that, in expected value, leads to lower underpricing and thus higher issue proceeds. In contrast to this, in Section 4, we presented results showing that when-issued trading can lower issue proceeds by increasing the cost of gathering information through a direct mechanism. However, this result was obtained under the implicit assumption that when-issued trading can open, i.e. that condition (15) is satisfied. This is arguably not the right case to consider in order to judge the benefit of when-issued trading. Only if the when-issued market fails without a direct mechanism, is there a clear rationale for using such a mechanism to obtain investors’ private information.

In this section, we will determine the optimal direct mechanism assuming that condition (15) is not satisfied. In this case, when-issued trading can open only if the mechanism is informative, that is only if non-contradictory reports are elicited from the polled investors: \( z \neq 0 \). Market failure thus occurs with an endogenous probability; this probability increases if a polled investor misrepresents her information about the value of the issue. This is because misrepresentation makes it more likely that the mechanism generates contradictory information. As a consequence, it is to a certain extent self-defeating for polled investors to misrepresent their information: by undermining the resolution of market failure, they lower the expected value of opportunities to trade on their information.\(^{20}\)

**Incentive compatibility constraints:** As pointed out in Section 4, the when-issued market affects the incentive compatibility constraints in two ways: i) polled investors have an opportunity to make trading profits; and ii) because when-issued trading occurs prior to the setting of the offer price, this market can correct misinformation that was provided through the mechanism. However, a polled investor can expect to profit from these two effects only

\(^{20}\)In this analysis, we continue to assume that two investors are polled under the mechanism. With a larger number of polled investors, a single investor would only affect the opening of the when-issued market if that investor had valuable, independent information. This is not unreasonable in reality since there are typically many sources of uncertainty about the value of an issue and not just one, as in our model. In addition, even if the when-issued market does not fail completely, the dealers in the market will set wider spreads if the information obtained prior to the opening of the market is less accurate. This will also lower the expected profit for a polled investor who misrepresents her information.
if the when-issued market opens. Since the probability of market failure is endogenous, the incentive compatibility constraints differ from those stated in Section 4:

\[ Er^+ \geq \left( q^2 + (1 - q)^2 \right) (w_L + u^+ - w^-)h^+ + 2q(1 - q)((w'_{LT} + u^-)h^- + \psi^-) \quad (IC_T^+^-) \]

\[ Er^- \geq \left( q^2 + (1 - q)^2 \right) (u^- - w_L)h^+ + 2q(1 - q)((u^+ - w'_{LT})h^+ + \psi^+) \quad (IC_T^-^+) \]

The above constraints are a cross between the constraints \((IC_T^+^-)\) and \((IC_T^-^+)\) (when-issued trading opens regardless), and those without when-issued trading, \((IC^+^-)\) and \((IC^-^+)\). \(Er^+\) and \(Er^-\) are the same in all three sets of constraints. The differences between the above constraints and \((IC_T^+^-)\) and \((IC_T^-^+)\) are: i) upon sending false reports, the polled investors’ expected trading profits are smaller since it is possible that the when-issued market will not open, and ii) the expected impact of a false report is larger because information aggregation in the market may not correct a resulting pricing error. If the polled investors disagree, then the market does not open, and the impact of a false report on pricing is the same as without when-issued trading: \(w_L > w_{LT}\). If the polled investors agree, even after a false report, then the when-issued market opens and a pricing error may be avoided. In this case, the expected pricing impact of a false report will be \(w'_{LT}\), where \(w_L > w'_{LT} \geq w_{LT}\).\(^{21}\) This change in the consequences of a false report is particularly significant for an investor who has observed negative information. Such an investor will be loath to falsely report positive information, given that such misinformation may not be corrected by the market, and the investor may thus receive an allocation of overpriced securities.

**Expected trading profit for a polled investor who submits a false report:** Before we can characterize the optimal direct mechanism, we must first model the expected profit for a polled investor who trades in the when-issued market after falsely reporting in the mechanism. We will not attempt to fully determine this expected profit, as this would require placing additional structure on the model, beyond what is needed to solve the mechanism design problem. We can, however, using only the given structure, derive an upper bound on the expected trading profits of a polled investor who sends a false report. This will be sufficient to characterize the optimal direct mechanism.

\(^{21}\)The second inequality is due to the fact that when-issued trading may break down after the opening.
If the when-issued market opens following a false report by a polled investor, and if that investor trades at the open, then the expected profit from this trade is:

\[(B_1|z=2 - v_0)\eta = (v_0 - A_1|z=-2)\eta \equiv \psi_0(w, \alpha, q) \times \eta.\]  \hspace{1cm} (19)

\(B_1|z=2\) is the dealers’ opening bid price, given that both polled investors reported positive information; \(A_1|z=-2\) is their opening ask price, given that the polled investors reported negative information. \(\eta\) is the fraction of the issue that can be traded in a single trade, \(0 < \eta << 1\). Following directly from the results of Section 5,

\[\psi_0(w, \alpha, q) = \left(1 - 2q(1 - q)\alpha^- (2)\right) w_L,\]  \hspace{1cm} (20)

where \(w_L\) is given by equation (11) and \(\alpha^- (2)\), defined in Section 5, is the probability of an informed arrival at the open, given that both polled investors reported positive information and the trader at the open was a seller.\(^{22}\)

A polled investor who has misrepresented her information may trade later than at the open, and more than once, but her expected profit from trading should be directly related to (20). This expected profit is decreasing in \(\alpha\), the fraction of investors who are privately informed. This occurs for three reasons: i) The market makers set a wider spread if there is a higher probability of trading with an informed trader. ii) A higher value of \(\alpha\) means that an informed trader faces more competition to trade on private information. (Traders arrive sequentially in the market. If many informed traders attempt to trade at the open, then the odds are low that the polled investor will be able to trade ahead of other informed investors.) iii) Following from the first two points, a higher value of \(\alpha\) means that the dealers will update their quotes more in response to trading activity. Thus, the higher the value of \(\alpha\), the less likely it is that a polled investor who has misrepresented her information can trade before much of her information is incorporated into the dealers’ quotes. We will thus represent the expected trading profit in equations (\(IC_{T}^+\)) and (\(IC_{T}^-\)) as

\[\psi_L^{ab} = x\psi_0(w, \alpha, q),\]  \hspace{1cm} (21)

where \(x\) is almost certainly much less than one, and \(x\) is decreasing in \(\alpha\).

\(^{22}\)It was demonstrated in Section 5 that \(\alpha^- (2) = \alpha^+ (−2)\).
6.1 The optimal direct mechanism, with when-issued trading that requires information gathering prior to opening

Before we characterize the mechanism we present a technical result indicating that when-issued trading can lower the cost of obtaining information through a direct mechanism if prior revelation of information is requisite for trading to open.

**Lemma 2.** *If prior information gathering is requisite for when-issued trading to open, then

1. \[(1 - h_R) \times u_{AS}(0) \geq \psi_L^+ - \psi_L^-\] (22)

is sufficient such that i) as is the case with no when-issued trading, the incentive compatibility constraints are nonbinding; and ii) the expected cost of eliciting information through a direct mechanism is lower with when-issued trading than without.

2. The left-hand side of (22) is increasing in \(\alpha\), the fraction of privately informed investors; the right-hand side of (22) is decreasing in \(\alpha\).*

The right-hand side of (22) is the expected trading profit of a polled investor who gives a false report. The left-hand side of (22) is the maximum allocation that can be given to a polled investor, \((1 - h_R)\), times the expected underpricing due to adverse selection risk when no information is obtained through the mechanism. This is the reward that a polled investor receives for reporting positive information while the other polled investor reports negative information, and when-issued trading fails to open. If (22) holds, then the incentive compatibility constraints will be nonbinding. If these constraints are nonbinding, then when-issued trading is strictly beneficial for the issuer: Such trading will in expected value lead to additional information revelation, and consistent with Lemma 1, this leads to lower expected underpricing. If condition (22) is satisfied, then this benefit is obtained at no cost to the issuer.

Condition (22) is merely a sufficient condition, not a necessary condition, such that when-issued trading is strictly beneficial to the issuer. As discussed above, the expected trading profit is decreasing in \(\alpha\), the fraction of potential investors who hold private information. \(u_{AS}(0)\) is increasing in \(\alpha\). As such, for small enough values of \(\alpha\), condition (22) will not be satisfied.
The optimal direct mechanism has characteristics of each of the previous two mechanisms: the optimal mechanism without when-issued trading and the optimal mechanism in the presence of when-issued trading that can open without prior information gathering. If the polled investors report conflicting information, then when-issued trading does not open and the seller optimally follows an allocation policy that is the same as in the benchmark mechanism (without when-issued trading). The investor who has reported a positive signal receives the maximum possible allocation; the other investor receives no allocation. If the polled investors agree with each other, then they will receive allocations only if the when-issued market confirms the accuracy of their information. If condition (22) is satisfied, then: i) no informational rents are paid when polled investors agree; ii) underpricing is determined entirely by the residual adverse selection risk, not by the need to induce truthtelling; and iii) investors who have observed negative information expect zero rents. If condition (22) is satisfied, then the mechanism is very similar to the benchmark mechanism. Even if condition (22) is not satisfied, investors who possess positive information expect strictly higher rents than those who have negative information.

**Proposition 3.** *If prior information gathering is requisite for the opening of when-issued trading, then the optimal direct mechanism has the following characteristics:*

1. *As in the case with no when-issued trading, a polled investor who has observed negative information expects lower rents for her information than a polled investor who has observed positive information.*

2. *If (22) holds, then allowing for when-issued trading lowers the expected underpricing.*

3. *Allowing for when-issued trading can increase the a priori expected underpricing only if both conditions (22) and (15) do not hold.*

As in Section 4, the optimal direct mechanism in the presence of when-issued trading does not satisfy the strict definition of an auction in that information other than what is provided by participants in the mechanism may affect the pricing and allocations. However, the mechanism proposed here does share in common with a standard auction the characteristic that an investor who reports a more positive valuation is favored over an investor who reports...
negative information. In fact, if condition (22) is satisfied, then as is the case with no when-issued trading, an investor with negative information will expect to receive no rents for her information.

The second part of Proposition 3 simply restates results of Lemma 2. As per the discussion following the lemma, if there are many informed investors, then allowing for when-issued trading is strictly beneficial for issuers. In the last part of the proposition we bring together results presented in this section and our earlier arguments that if when-issued trading can open without any prior information gathering, then allowing for such trading benefits issuers because the trading provides information for free. Thus, there are two conditions that are necessary, but not sufficient, for when-issued trading to be detrimental for issuers: (22) does not hold and when-issued trading cannot open on its own ((15) does not hold). As discussed above, for the first of these to be satisfied, the fraction of informed investors, $\alpha$, must not be too large. The second condition (market failure without prior information gathering) is satisfied either if i) spreads are too wide, because the market makers are quite concerned about trading with informed investors, or ii) uninformed investors’ liquidity needs are too small ($\rho$ is small). The first reason for market failure occurs either due to the presence of too many informed investors ($\alpha$ large), and/or because informed investors hold very valuable private information ($q$ and $w$ large). Thus, it is only when there is a small number of informed investors, and the when-issued market is illiquid (in the sense that liquidity traders do not want to trade) or the informed investors have very valuable information, that the existence of a when-issued market may increase the cost of gathering information that is needed to set the offer price. Even if the cost of information gathering increases due to when-issued trading, such trading may be beneficial for the issuer. This is because when-issued trading enables the revelation of additional information prior to setting the offer price. In the following section we present some numerical analysis that illustrates our results and highlights conditions such that when-issued trading is either beneficial or detrimental to issuers.
Expected underpricing is graphed vs $q$, the probability that any informed investor has correct information. Information is solicited using a direct mechanism. When-issued trading, if allowed, can open only if some information has first been obtained.

Solid curve: $Eu$, given no when-issued trading; Dashed curve: $Eu$, given when-issued trading

$$w = 1.5, \quad x = .1, \quad h_R = 0.05$$

### 6.2 Numerical examples

The case such that when-issued trading cannot open without prior information gathering:

Figure 3 presents graphs of expected underpricing versus $q$, the probability that any given informed investor has accurate information about the security value. Expected underpricing, given that a direct mechanism is used and there is no when-issued trading, is represented by the solid curve. This value is determined by equation (12). Expected underpricing, given that a direct mechanism is used and there is when-issued trading that opens only if information is learned through the direct mechanism, is represented by the dashed curve. This value is derived at the end of the Appendix.

In Figure 3a 30% of the investors participating in the primary market and in the when-issued market are expected to be informed; in 3b this number is only 15%. Consistent with Lemma 2 and Proposition 3, expected underpricing is lower with when-issued trading for the higher value of $\alpha$, and higher for the lower value of $\alpha$. In both graphs, the parameter $x$ in equation (21) is set equal to 0.1. This means that, if the polled investors concur after one of them has lied, then the expected trading profit of the investor who lied is equal to the amount that the investor would expect to profit if she could trade 10% of the entire issue at
Figure 4: Expected underpricing

Expected underpricing is graphed vs \( q \), the probability that any informed investor has correct information.

Information is solicited using a direct mechanism.

Solid curve: \( E_u \), given no when-issued trading

Small dashes: \( E_u \), given when-issued trading that opens only if mechanism is informative

Large dashes: \( E_u \), given when-issued trading that opens regardless

\( w = 1.5, \alpha = .3, h_R = 0.05 \)

The opening when-issued quotes.\(^{24}\)

The case such that when-issued trading can open without prior information gathering

The bottom two curves of Figure 4a are the same as the curves in Figure 3a; the scale of the graph has been changed. The top curve, with the wider dashes, represents expected underpricing, given a direct mechanism and when-issued trading that can open without the mechanism. Expected underpricing is higher in this case than if information gathering is a prerequisite for when-issued trading to open. The reason is simply that, in the former case, trading opportunities, and thus potential profits from misrepresenting information, are greater. As a result, it is more costly to elicit information through a direct mechanism. Figure 4b is the same as 4a, except that the parameter \( x \) takes on a much smaller value; expected trading profits are only one-fourth as large in 4b as compared to 4a. The solid curve, expected underpricing with no when-issued trading is the same in both graphs of Figure 4 (the scales

\(^{24}\)30% is not an overly high fraction for informed participation. This number means that with probability .3 any given order has originated from an investor with some nonpublic information. It is expected that such informed investors are more active in IPOs, than uninformed investors. If a securities offering attracts very wide interest, then \( \alpha \) may be smaller. But, in this case we would also expect \( x \) to be smaller due to greater competition to trade. The pattern observed in Figure 3 holds for much smaller values of \( \alpha \) if \( x \) is also smaller.
of the graphs are different). The expected underpricing represented by the dashed curves is lower in 4b than in 4a. In particular, we see that there do exist parameter values such that expected underpricing decreases in the presence of when-issued trading, even if this trading can open without any prior information gathering.

7 Conclusion

We examine in this paper an alternative method for obtaining information to use in pricing unseasoned securities. This method combines direct information gathering from prospective investors, such as in bookbuilding, with information gathering from when-issued trading. We refer to this method as “alternative” because even though this method is commonly used in European IPO markets, trading in shares prior to their public issuance is not legal in the United States. It is our understanding that the rationale behind this ruling is a concern that when-issued trading may interfere with a firm’s ability to raise capital. The results of this paper call into question the soundness of that concern.

Our results indicate that, the existence of a when-issued market can interfere with a seller’s ability to gather pricing-relevant information directly from investors, thus leading to higher expected underpricing and lower proceeds. However, this occurs either if direct information gathering is unnecessary, or if private information is held by a very small number of investors. In the first case, our results would provide a rationale for ruling out when-issued trading only if a direct mechanism is used for reasons outside of our analysis. In the absence of such reasons, information should be obtained for free from the when-issued market. We find that it is only in the case that direct information gathering is an essential part of the pricing process, and private information is held by a very small number of investors, and either the information is very valuable or the when-issued market is illiquid, that allowing when-issued trading may increase expected underpricing, and thus decrease the expected issue proceeds. Apart from these special cases, when-issued trading is strictly beneficial to issuers in that it leads to greater expected issue proceeds.

Our results thus call into question the concern that when-issued trading will be detrimental for issuers. We cannot claim, however, based solely on the work here that when-issued
trading of IPO shares should be permitted. For this market to be beneficial to issuers it is necessary that both informed and uninformed investors are willing to participate in the when-issued market. Such participation is not generally beneficial to uninformed investors.\textsuperscript{25}

\textsuperscript{25}See Dorn (2002) for evidence that retail traders have consistently lost money in the German when-issued markets.
Appendix

Notation:
Random variables:
\[ \tilde{V} = \text{secondary market value} \in \{ v_0 + w, v_0 - w \} \]
\[ \tilde{s} \equiv \frac{\tilde{V} - v_0}{w} \in \{ 1, -1 \} \]
\[ \zeta_i = \text{informed trader } i's \text{ signal of } \tilde{s}. \]
Exogenous parameters: (The exogenous parameters are all common knowledge)
\[ v_0 = \text{prior expected value of } V \]
\[ w = \text{constant (See above for } \tilde{V} ) \]
\[ \pi_0 = 1/2 = \text{prior probability that } s = 1 \]
\[ q = \text{probability that } \zeta_i = s, \text{ i.e., that any given informed investor has correct information} \]
\[ \alpha = \text{fraction of traders who are informed. } 0 < \alpha < 2/3 \]
\[ h_R = \text{minimum fraction of the offering that must be allocated to retail investors} \]
\[ \rho = \text{valuation parameter for liquidity traders } \geq 0; \text{ buyers have parameter } \rho_t = \rho; \]
\[ \text{sellers have parameter } \rho_t = -\rho \]
\[ \eta = \text{size of a single trade unit, relative to the total issue size} \]
Other variables:
\[ p_I = \text{offer price} \]
\[ \pi_T = \text{probability that } s = 1, \text{ conditioned on all information known by the seller} \]
\[ \text{just before setting the offer price} \]
\[ u_{AS} = \text{expected underpricing due to adverse selection risk} \]
\[ z = \text{sum of signals reported by informed investors } = \# \text{ of positive } - \# \text{ of negative reports} \]
\[ \pi(z) = \text{probability that } s = 1, \text{ given } z \]
\[ u^{ab} = \text{expected underpricing when one polled investor reports } a \text{ and the other reports } b \]
\[ h^{ab} = \text{fraction of offering allocated to polled investor who reports } a \text{ when other reports } b \]
\[ w_L = \text{impact of a lie on the expected value, without when-issued trading} \]
\[ w_{LT} = \text{impact of a lie on the expected value, with when-issued trading} \]
\[ \psi_L^{+b} (\psi_L^{-b}) = \text{expected trading profit for an investor who sees } (+) \text{ but reports } (-), \]
\[ \text{while the other polled investor reports } b \]
\[ S_t \equiv A_t - B_t = \text{market makers’ time } t \text{ quoted spread } = \text{ask price } - \text{bid price} \]
\[ \alpha^+(z) = \text{probability of an informed arrival at the open, given } z \text{ and given a buyer arrived} \]
\[ \alpha^-(z) = \text{probability of an informed arrival at the open, given } z \text{ and given a seller arrived} \]

Underpricing due to adverse selection risk:
\[ E[\tilde{V}|\pi_T] = v_0 + (2\pi_T - 1)w \quad (23) \]
An informed investor sees a signal of \( \tilde{s} \): \( \zeta_i \in \{-1, 1\} \).
\[ \text{prob}\{s = 1|\pi_T, \zeta_i = 1\} = \frac{q\pi_T}{q\pi_T + (1-q)(1-\pi_T)}, \]
Table 1: Expected Value and Allocations

<table>
<thead>
<tr>
<th>Realization of $s$</th>
<th>$s = -1$</th>
<th>$s = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of this realization</td>
<td>$1 - \pi_T$</td>
<td>$\pi_T$</td>
</tr>
<tr>
<td>Expected secondary market value $\tilde{V}$</td>
<td>$v_0 - w + \tilde{d}_T$</td>
<td>$v_0 + w + \tilde{d}_T$</td>
</tr>
<tr>
<td>Allocation to informed investors</td>
<td>$(1 - q)\alpha$</td>
<td>$q\alpha$</td>
</tr>
<tr>
<td>Allocation to uninformed investors</td>
<td>$\frac{1 - \alpha}{1 - q\alpha}$</td>
<td>$\frac{1 - \alpha}{1 - (1 - q)\alpha}$</td>
</tr>
</tbody>
</table>

Given these probabilities, an informed investor values the issue as follows:

$$\text{prob}\{s = 1 | \pi_T, \varsigma_i = -1\} = \frac{(1 - q)\pi_T}{(1 - q)\pi_T + q(1 - \pi_T)}.$$

Given these probabilities, an informed investor values the issue as follows:

$$E[\tilde{V} | \pi_T, \varsigma_i = 1] = v_0 + \frac{q\pi_T - (1 - q)(1 - \pi_T)}{q\pi_T + (1 - q)(1 - \pi_T)}w + \tilde{d}_T > E[\tilde{V} | \pi_T]$$

$$E[\tilde{V} | \pi_T, \varsigma_i = -1] = v_0 + \frac{(1 - q)\pi_T - q(1 - \pi_T)}{(1 - q)\pi_T + q(1 - \pi_T)}w + \tilde{d}_T < E[\tilde{V} | \pi_T]$$

Table 1 presents the expected value and the expected relative allocations to each group of investors (informed and uninformed), for each realization of $\tilde{s}$. The table is written assuming that $p_I > E[\tilde{V} | \pi_T, \varsigma_i = -1]$, so that investors who have observed negative signals do not participate; this is checked below.\(^{26}\) Because $q > 1/2$, the uninformed will on average receive more securities if the value of these securities is low ($s = -1$).

Uninformed investors will participate in the offering only if their expected return is non-negative. When underpricing is minimized, this expected return is zero:

$$0 = (1 - \pi_T) \left( v_0 - w + \tilde{d}_T - p_I \right) \frac{1 - \alpha}{1 - q\alpha} + \pi_T \left( v_0 + w + \tilde{d}_T - p_I \right) \frac{1 - \alpha}{1 - (1 - q)\alpha},$$

Solving equation (26) for $p_I$ yields:

$$p_I = v_0 + \tilde{d}_T + \left( \frac{2\pi_T - 1 - (\pi_T + q - 1)\alpha}{1 - ((2\pi_T - 1)q + 1 - \pi_T)\alpha} \right) w.$$  

The above expression is $> E[\tilde{V} | \pi_T, \varsigma_i = -1]$, so those who have observed negative signals do not participate. The expected underpricing due to adverse selection risk is:

$$u_{AS}(\pi_T) = E[\tilde{V} | \pi_T] - p_I = \frac{q - 2\pi_T(1 - \pi_T) - (2\pi_T - 1)^2q}{1 - ((2\pi_T - 1)q + 1 - \pi_T)\alpha} \alpha w.$$  

\(^{26}\)The informed participation given in Table 1 is the participation, conditioned on the realization of $\tilde{s}$. In everything that follows, we will assume that the number of investors who reveal their information, either through a direct mechanism or when-issued trading, is small relative to the total number of informed investors who may participate in the offering. Thus, the fraction of informed investors is not affected by information gathering.
Proof of Lemma 1. We define the following variable:

\[ \bar{u}_{AS} \equiv u_{AS}(\pi) = \frac{q - 2\pi(1 - \pi) - (2\pi - 1)^2 q}{1 - ((2\pi - 1)q + 1 - \pi)\alpha} \]

\[
\frac{\partial \bar{u}_{AS}}{\partial \pi} = (2q - 1) \left( \frac{2(1 - 2\pi)(1 - (1 - q)\alpha - (2q - 1)\pi\alpha) + \alpha(q - 2\pi(1 - \pi) - (2\pi - 1)^2 q)}{(1 - ((2\pi - 1)q + 1 - \pi)\alpha)^2} \right) = (2q - 1) \left( \frac{2(1 - 2\pi) - 2(1 - \pi)^2 \alpha + (1 - 2\pi)^2 q\alpha + q\alpha}{(1 - ((2\pi - 1)q + 1 - \pi)\alpha)^2} \right) \tag{29} \]

The numerator of (29) is strictly decreasing in \( \pi \). (29) is strictly positive if \( \pi = 1/2 \) and strictly negative if \( \pi = (1 - \alpha)/2 + \alpha q \). (The last result can be obtained by setting \( q = 1/2 + \varepsilon \), where \( 0 < \varepsilon < 1/2 \).) Thus, \( \partial u_{AS}/\partial \pi \) is positive \( \forall \pi \leq 1/2 \) and negative \( \forall \pi \geq q \). In addition,

\[ \bar{u}_{AS} \bigg|_{z=0} = \frac{2q - 1}{2 - \alpha} > \bar{u}_{AS} \bigg|_{z=1} = \frac{(2q - 1)2q(1 - q)}{1 - ((2q - 1)q + 1 - q)\alpha} \]

Derivation of \( \pi(z) \): We know that

\[ \pi(0) = 1/2 \quad \pi(1) = q \quad \pi(-1) = 1 - q = 1 - \pi(1) \]

We can define \( \pi(z) \) as a function of all signals obtained, except \( i \)'s signal, together with \( i \)'s signal \( \varsigma_i \). If \( \varsigma_i = 1 \):

\[
\pi(z) = \frac{\text{prob}\{\varsigma_i = 1|s = 1\} \pi(z - \varsigma_i)}{\text{prob}\{\varsigma_i = 1|s = 1\} \pi(z - \varsigma_i) + \text{prob}\{\varsigma_i = 1|s = -1\} (1 - \pi(z - \varsigma_i))} = \frac{q\pi(z - 1)}{q\pi(z - 1) + (1 - q)(1 - \pi(z - 1))} \tag{30} \]

If \( \varsigma_i = -1 \):

\[
\pi(z) = \frac{(1 - q)\pi(z + 1)}{(1 - q)\pi(z + 1) + q(1 - \pi(z + 1))} \tag{31} \]

Using these equations and the above values for \( \pi(1) \) and \( \pi(-1) \), we obtain

\[ \pi(2) = \frac{q^2}{q^2 + (1 - q)^2} \quad \pi(-2) = 1 - \pi(2) \]

Repeating the above we obtain:

\[ \pi(z) \bigg|_{z\geq0} = \frac{q^z}{q^z + (1 - q)^z} \tag{32} \]

\[ \pi(z) \bigg|_{z\leq0} = \frac{(1 - q)^{|z|}}{q^{|z|} + (1 - q)^{|z|}} \tag{33} \]

Because \( q > 1/2 \), for \( z \geq 0 \), \( \pi(z) \) is increasing in \( z \); for \( z < 0 \), \( \pi(z) \) is decreasing in \( |z| \).
Underpricing due to residual adverse selection risk. \( u_{AS}(\pi(z)) \) is written as \( u_{AS}(z) \).

From equations (3) and (5):

\[
\frac{u_{AS}(0)}{w\alpha} = \frac{2q - 1}{2 - \alpha} \quad (34)
\]

\[
\frac{\pi(2)}{w\alpha} = \frac{q^2}{q^2 + (1 - q)^2} = 1 - \pi(-2)
\]

\[
\frac{u_{AS}(2)}{w\alpha} = \frac{q - \frac{2q^2(1 - q)^2}{(q^2 + (1 - q)^2)\pi} - \left(\frac{2q - 1}{q^2 + (1 - q)^2}\right)^2 q}{1 - \left(\frac{2q - 1}{q^2 + (1 - q)^2}\right)^2 q} - \frac{q(2q - 1)2q^2(1 - q)^2}{(q^2 + (1 - q)^2)\pi q\alpha - \left(\frac{q^2}{q^2 + (1 - q)^2}\right)\alpha}
\]

\[
= \frac{(q^2 + (1 - q)^2)(q^2 + (1 - q)^2 - \alpha(1 - 3q(1 - q)))}{(q^2 + (1 - q)^2)(q^2 + (1 - q)^2 - \alpha q(1 - q))} \quad (35)
\]

\[
\frac{u_{AS}(-2)}{w\alpha} = \frac{q - \frac{2q^2(1 - q)^2}{(q^2 + (1 - q)^2)\pi} - \left(\frac{2q - 1}{q^2 + (1 - q)^2}\right)^2 q}{1 + \left(\frac{2q - 1}{q^2 + (1 - q)^2}\right)\alpha q - \left(\frac{q^2}{q^2 + (1 - q)^2}\right)\alpha}
\]

\[
= \frac{(q^2 + (1 - q)^2)(q^2 + (1 - q)^2 - \alpha q(1 - q))}{(q^2 + (1 - q)^2)(q^2 + (1 - q)^2 - \alpha q(1 - q))} \quad (36)
\]

\( q > 1/2 \implies 1 - 3q(1 - q) > q(1 - q) \implies u_{AS}(2) > u_{AS}(-2) \).

\( u_{AS}(0) \) is strictly increasing in \( q \). When \( q \) is close to 1/2, \( \partial u_{AS}(2)/\partial q \) and \( \partial u_{AS}(-2)/\partial q \) are positive; when \( q \) is close to one, \( \partial u_{AS}(2)/\partial q \) and \( \partial u_{AS}(-2)/\partial q \) are negative.

Optimal direct mechanism, without when-issued trading. Rearranging the incentive compatibility constraints:

\[
\left(q^2 + (1 - q)^2\right)\left(u^+h^+ - (w_L + u^-)h^-ight) + 2q(1 - q)\left(u^-h^- - (w_L + u^-)h^-ight) \geq 0 \quad (37)
\]

\[
\left(q^2 + (1 - q)^2\right)\left(u^-h^- - (w_L + u^-)h^+\right) + 2q(1 - q)\left(u^+h^+ - (w_L + u^+)h^+\right) \geq 0 \quad (38)
\]

(37) is satisfied by setting \( h^+ = h^- = 0 \).

Constraints (\( PC-R \)) require that \( u^- \geq u_{AS}(0) \) and \( u^+ \geq u_{AS}(2) \). (38) is satisfied because:

i) \( u_{AS}(0) < w_L \). This follows directly from comparing equations (11) and (34). And

ii) \( u_{AS}(2) < u_{AS}(0) \).

The expected underpricing is thus given by (12).

Proof of Proposition 1. We assume that the when-issued market is expected to be fully informative. Thus, \( w_{LT} \rightarrow 0 \) and residual adverse selection risk is expected to be zero.

Allowing allocations to be conditioned on when-issued trading, the constraints (\( IC_T^+ \)) and (\( IC_T^- \)) can be rewritten as:

\[
q^2\left(u^+h^+_c - u^-h^+_w\right) + (1 - q)^2\left(u^+h^+_w - u^-h^+_c\right) + q(1 - q)\left(u^-h^+_c - u^-h^+_w\right) - u^+_w - u^-h^+_w \geq 0
\]

\[
\left(q^2 + (1 - q)^2\right)\psi^+_L + 2q(1 - q)\psi^-_L \quad (39)
\]
\[ q^2 (u_c^- h_c^- - u^+ h_w^+) + (1 - q)^2 (u_w^- h_w^- - u^+ h_c^+) + q(1 - q) (u^+ (h_c^+ + h_w^+) - u_c^+ h_c^+ - u_w^+ h_w^+) \geq \left( q^2 + (1 - q)^2 \right) \psi_L^- + 2q(1 - q)\psi_L^- \]  

(40)

where the subscript \(c\) (\(w\)) represents the state such that when-issued trading indicates that the investor’s report was correct (wrong). We will assume that \(\psi_L^+ = \psi_L^-\) and \(\psi^+ = \psi^-\). (This is supported by the proof of Proposition 2.) Thus, because \(q > 1 - q\), the optimal mechanism calls for zero rents to be paid if the when-issued market indicates that an investor’s report was wrong: \(u_{ab}h_{w}^{ab} = 0\), \(\forall\) pairs \((a, b)\). Also, because the right-hand sides of (39) and (40) are strictly positive, and because residual adverse selection risk goes to zero, (39) and (40) are strictly binding:

\[
q \left( q u_c^+ h_c^+ + (1 - q) u^+ h_c^- \right) - (1 - q) \left( q u_c^- h_c^- + (1 - q) u^+ h_c^- \right) = \left( q^2 + (1 - q)^2 \right) \psi_L^+ + 2q(1 - q)\psi_L^- \]  

(41)

\[
q \left( q u_c^- h_c^- + (1 - q) u^+ h_c^- \right) - (1 - q) \left( q u_c^+ h_c^+ + (1 - q) u^+ h_c^- \right) = \left( q^2 + (1 - q)^2 \right) \psi_L^- + 2q(1 - q)\psi_L^+ \]  

(42)

Thus the solution requires that:

\[
qu_c^+ h_c^+ + (1 - q) u^+ h_c^- = qu_c^- h_c^- + (1 - q) u^+ h_c^+ = \frac{\left( q^2 + (1 - q)^2 \right) \psi_L^+ + 2q(1 - q)\psi_L^- }{(2q - 1)} \]  

(43)

An investor who reports negative information receives the same expected rents as an investor who reports positive information.

In the optimal mechanism \(h_c^+ = h_c^- = (1 - h_R)/2\). \(h_c^+ = h_c^- = 1 - h_R\). Combining (43) and (6), the expected underpricing following bookbuilding and when-issued trading is

\[
\frac{2q}{(2q - 1)(1 - h_R)} \left( \left( q^2 + (1 - q)^2 \right) \psi_L^+ + 2q(1 - q)\psi_L^- \right) \]  

(44)

---

**Proof of Proposition 2.** When-issued trading and updating of beliefs.

\(\alpha^+(z)\) = probability that trader at the open is informed, given that the trader is a buyer.

\(\alpha^-(z)\) = probability that trader at the open is informed, given that the trader is a seller.

\[
\alpha^+(z) = \frac{\Pr\{\text{informed buyer at open}\}}{\Pr\{\text{buyer at open}\}} = \frac{q\pi(z) + (1 - q)(1 - \pi(z))}{(1 - \alpha)/2 + \alpha q\pi(z) + \alpha(1 - q)(1 - \pi(z))} 
\]

\[
\alpha^-(z) = \frac{\Pr\{\text{informed seller at open}\}}{\Pr\{\text{seller at open}\}} = \frac{(1 - q)\pi(z) + q(1 - \pi(z))}{(1 - \alpha)/2 + \alpha(1 - q)\pi(z) + \alpha q(1 - \pi(z))} 
\]

Applying equation (32), for \(z \geq 1:\)

\[
\frac{\alpha^+(z)}{\alpha} = \frac{q^{z+1} + (1 - q)^{z+1}}{(1 - \alpha)(q^z + (1 - q)^z)/2 + \alpha(q^{z+1} + (1 - q)^{z+1})} 
\]
but with \( z \) replaced by \( |z| \). Thus, for \( z \geq 1 \):

\[
\alpha^+(z) = \alpha^-(z) > \alpha > \alpha^-(z) = \alpha^+(z)
\]

(47)

**Conditions for opening.** The case in which \( z = 0 \) has already been proved. We need to show that \( S_1 \) is smaller for \( |z| \geq 1 \) than for \( z = 0 \). If \( z \geq 1 \):

\[
E[\bar{V}|z] = v_0 + (2\pi(z) - 1)w = v_0 + \frac{q^z - (1-q)^z}{q^z + (1-q)^z}w.
\]

From the above equations, and (32), (33) and (47), we know that the opening spread is the same regardless of whether \( z \) is positive or negative. We can thus replace \( z \) with \( |z| \), and the half spread is:

\[
\left. \frac{S_1}{2} \right|_{|z| \geq 1} = \frac{A_1}{2} - \frac{B_1}{2}
\]

\[
= \alpha^+(z)\left(\pi(z+1) - \pi(z)\right)w + \alpha^-(z)\left(\pi(z) - \pi(z+1)\right)w
\]

(50)

(\( \pi(z) - \pi(z-1) < \pi(1) - \pi(0), \forall z \geq 2 \). Thus:

\[
\left. \frac{S_1}{2} \right|_{|z| \geq 1} < (\alpha^+(z) + \alpha^-(z))\left(\pi(1) - \pi(0)\right)w
\]
\( \Delta \alpha^+ \) in equation (45) is strictly less than \( \Delta \alpha^- \) in (46). Thus, \( \alpha^+(z) + \alpha^-(z) < 2\alpha \). Therefore, \( S_1|z| \geq 1 < S_1|z=0 \).

**Bound on informed trading profits.** From the proof of Proposition 2:

\[ A_1|z=-2 = v_0 - (2q - 1)w\alpha^+(-2) - \frac{2q - 1}{q^2 + (1 - q)^2} w(1 - \alpha^+(-2)) \]

\[ B_1|z=2 = v_0 + (2q - 1)w\alpha^-(-2) + \frac{2q - 1}{q^2 + (1 - q)^2} w(1 - \alpha^-(-2)) \]

\( \alpha^- (2) = \alpha^+(-2) \), thus

\[ (B_1|z=2 - v_0)\eta = (v_0 - A_1|z=-2)\eta \]

\[ = (2q - 1) \left( \frac{1 - 2q(1 - q)\alpha^-(-2)}{q^2 + (1 - q)^2} \right) w\eta \]

\[ = \left( 1 - 2q(1 - q)\alpha^-(-2) \right) w_L\eta \]

where \( w_L \) is given by equation (11). \( \psi_0 \equiv B_1|z=2 - v_0 \). For shorthand we’ll let \( \alpha' \equiv \alpha^-(-2) \).

\[ \frac{\partial \psi_0}{\partial \alpha'} = -2q(1 - q)w_L \quad \text{and} \quad \frac{\partial \alpha'}{\partial \alpha} > 0 \quad \Rightarrow \quad \frac{\partial \psi_0}{\partial \alpha} < 0 \]

\[ \frac{\partial \psi_0}{\partial q} = 2(2q - 1)\alpha'w_L + (1 - 2q(1 - q)\alpha') \frac{\partial w_L}{\partial q} - 2q(1 - q)w_L \frac{\partial \alpha'}{\partial q} \]

\[ \frac{\partial w_L}{\partial q} = \frac{4q(1 - q)w}{(1 - 2q(1 - q))^2} > 0 \]

\[ \frac{\partial \alpha'}{\partial q} = \frac{-\alpha(1 - \alpha)(2q - 1)/2}{((1 - \alpha)(q^2 + (1 - q)^2)/2 + \alpha q(1 - q))^2} < 0 \quad \Rightarrow \quad \frac{\partial \psi_0}{\partial q} > 0 \]

**Proof of Lemma 2.** The participation constraint for informed investors will change, but it will continue to be nonbinding. The reason is that there are more than two investors who can potentially reveal private information. In order to be willing to participate, an informed investor must expect to receive as much as she can profit, in expected value, from trading on her private information, after two other investors have revealed their information. This amount is not nearly as high as the expected profit from trading after lying. Thus, satisfying the incentive compatibility constraints will leave the participation constraint nonbinding. For this reason, we focus only on the incentive compatibility constraints.

If the investors agree with each other following a lie by one investor, then with probability 1/2 the market will break down after it opens (with probability 1/2 the reported information was wrong and is thus contradicted by the trading activity). If the market does not break down, then it confirms that the lie was actually correct, in which case \( w_{LT} \rightarrow 0 \). Allowing the allocations to be conditioned on when-issued trading, the constraints \( (IC_T^+) \) and \( (IC_T^-) \) can be rewritten as:

\[ q^2 u^{++}_c h^{++} + (1 - q)^2 u^{++}_b h^{++} - \left( q^2 + (1 - q)^2 \right) (w_L + u^{++})h^{++} + 2q(1 - q)u^{--}h^{--} \]
\[-q(1-q)\left(u_c^- h_c^- + (u_b^- + w'_L h_b^-)\right) \geq 2q(1-q)\psi_L^+\]  
\[q^2 u_c^- h_c^- + (1-q)^2 u_b^- h_b^- + \left(2(1-q)u^+ - (1-h_R)\right) \geq 2q(1-q)u^+ - h^+\]  
\[-q(1-q)\left(u_c^+ h_c^+ + (u_b^+ - w'_L h_b^+)\right) \geq 2q(1-q)\psi_L^+\]  

The subscript \(c\) indicates that the when-issued market confirmed the investor’s reports; \(b\) indicates market breakdown. The participation constraint for retail participation requires that: \(u^+, u_b^+, u_b^- \geq u_{AS}(0)\). From equations (11) and (34):

\[u_{AS}(0) = \left(q^2 + (1-q)^2\right)\left(\frac{\alpha}{2 - \alpha}\right) w_L\]  

We have assumed that \(\alpha < 2/3\). Thus, \(u_{AS}(0) < w_L/2\). As in Section 4, we assume that \(\psi_L^+ = \psi_L^-\). We use the notation \(\psi_A^+ = \psi_L^-\) to indicate that the polled investors agree after a lie. The optimal mechanism is thus similar to that without when-issued trading in that it calls for: \(h^+ = 0\) and \(h^+ - h_R\). It is also optimal to set \(h_b^+ = h_b^- = 0\).

The constraints are thus:

\[q^2 u_c^+ h_c^+ - q(1-q)u_c^- h_c^- + 2q(1-q)u^+ h^+ (1-h_R) \geq 2q(1-q)\psi_L^-\]  
\[q^2 u_c^- h_c^- - q(1-q)u_c^+ h_c^+ + \left(2(1-q)u^+ - (1-h_R)\right) \geq 2q(1-q)\psi_L^+\]  

If the when-issued market is fully informative (given that it doesn’t break down), then the underpricing needed in each state to just satisfy \((PC - R)\) is:

\[u_c^+ = u_c^- = 0 \quad \text{and} \quad u^+ = u_b^+ = u_b^- = u_{AS}(0)\]  

We have shown already that \(w_L - u_{AS}(0) > u_{AS}(0)\). Also, \(q^2 + (1-q)^2 > 2q(1-q)\). Thus,

\[u_{AS}(0)(1-h_R) \geq \psi_A^+\]  

is a sufficient condition such that (60) will satisfy (58) and (59). In this case rents are paid only if the when-issued market doesn’t open and only to the polled investor who reported positive information.

If condition (61) is not satisfied, then \(u_c^+ h_c^+ > 0\). Because \(q^2 + (1-q)^2 (w_L - u^+) > 2q(1-q)u^+\), (59) is easier to satisfy than (58). Thus, \(u_c^+ h_c^+ > u_c^- h_c^-\); higher rents are paid to investors who report positive than negative information.

If (61) is satisfied, then the binding constraints are \((PC - R)\), not the IC constraints. This mechanism differs from the mechanism without when-issued trading only in that, for some states adverse selection risk will be eliminated. Thus, the a priori expected value of underpricing is lower with when-issued trading, than without.

**Proof of Proposition 3.** See the proof of Lemma 2 and the derivation of (15).

**Expected underpricing, given a direct mechanism and when-issued trading.**
If when-issued trading opens only after information gathering (Lemma 3): If condition (61) is satisfied, then there is no underpricing if the when-issued market opens and confirms the polled investors’ reports. Otherwise underpricing $= u_{AS}(0)$. Thus, if condition (61) is satisfied, $Eu = (1 - q^2)u_{AS}(0)$.

If (61) is not satisfied, but the weaker condition

$$u_{AS}(0)(1 - h_R)(1/\alpha - q) \geq \psi^A_L$$

is satisfied, then (58) is binding, but (59) is not, and

$$Eu = 2q(1 - q)\frac{\psi^A_L}{1 - h_R} + (1 - q)^2u_{AS}(0)$$

If (62) is not satisfied, then both (58) and (59) are binding, and

$$Eu = \frac{4q^2(1 - q)}{(2q - 1)}\frac{\psi^A_L}{1 - h_R} + \frac{2q}{(2q - 1)} \left(1 - 2q(1 - q) - \frac{1}{\alpha}\right)u_{AS}(0) + (1 - q^2)u_{AS}(0)$$

In the numerical examples, $\psi^A_L = x \times \psi_0(w, \alpha, q)$ where $\psi_0(w, \alpha, q)$ is given by equation (20) and $x$ is an exogenous parameter.

If when-issued trading opens without prior information gathering (Proposition 1): The expected underpricing is given by equation (44), where $\psi^+_- = \psi^A_L$ and

$$\psi^{++}_L = \psi^D_L = x(1 - \alpha(q^2 + (1 - q)^2))w_L$$
References


