Competitor Collaboration and Product Distinctiveness

Arghya Ghosh and Hodaka Morita*

University of New South Wales

First version: June 11, 2005
This version: January 26, 2006

Abstract

Competitors often collaborate by sharing a part of their value-creating activities such as technology development, product design and distribution, which are important elements for creating product distinctiveness across firms. This paper explores economic consequences of competitor collaborations under the location framework in which competitors can save their fixed costs by forming a collaboration at the expense of the reduction in their product distinctiveness. In our framework, collaboration between competitors reduces the product distinctiveness between themselves, while it increases the distinctiveness between their products and the non-collaborator’s product. At the same time, intensified competition between collaborators lowers their prices and imposes downward pressure on non-collaborator’s pricing strategy. The interaction between these two effects, together with the heterogeneity of consumers, yields rich welfare implications. Furthermore, we demonstrate, contrary to the standard intuition, that partial ownership arrangements could enhance consumers’ welfare by inducing competitors to collaborate. Our analysis yields significant and novel welfare implications of competitor collaborations, which have recently attracted substantial attention from antitrust authorities.

JEL classification numbers: L13, L40.

Keywords: Antitrust, competitor collaboration, cooperative R&D, location model, oligopoly, partial ownership arrangement, product distinctiveness.

* School of Economics, University of New South Wales, Sydney, NSW 2052, Australia; Ghosh: a.ghosh@unsw.edu.au, Morita: h.morita@unsw.edu.au

We wish to thank Simon Anderson, Robert Cairns, Jay Pil Choi, Nisvan Erkal, Joshua Gans, Tom Holmes, John Panzar, Robert Russell, Bill Schworm, Mike Waldman, and seminar participants at University of Melbourne, UNSW, 23rd Australasian Economic Theory Workshop and Sydney/UNSW Theory Workshop for comments and discussions. We would also like to thank Jonathan Lim and Pierre Richard for excellent research assistance. Financial support from the Faculty Research Grants Program at the University of New South Wales is gratefully acknowledged.
1 Introduction

How does collaboration among firms affect welfare? This paper offers a new perspective on this important question through exploring economic consequences of collaboration among competing firms. Collusion, which is an important example for collaboration among firms, has been intensively studied in the industrial organization literature. However, in reality firms often remain competitors in the product market, while they collaborate in a variety of different stages of their value-creating activities such as technology development, product design and distribution. The incidence and the importance of such inter-firm collaboration have substantially increased in the last couple of decades (Caloghirou et al., 2003), and antitrust authorities have recently recognized collaboration among competitors as crucial issues (see Canada, 1995; United States, 2000; European Commission, 2001).

Given that collaboration among competitors often results in the reduction of their product distinctiveness (see below for details), we will explore economic consequences of collaborations among firms in which competitors can save their fixed costs at the expense of the reduction in their product distinctiveness. Also, given that in reality competitor collaboration is often accompanied by partial ownership arrangements (POAs), we will explore economic consequences of POAs in our framework, where POAs between competitors affect their incentives to form a collaboration. Note, throughout the paper we use “collaboration among competitors” and “competitor collaboration” interchangeably.

Below we list several recent examples of competitor collaborations:

• IBM, Sony Group and Toshiba have recently developed a high-performance microprocessor (named Cell), where a team of engineers from these companies has collaborated on the development at a joint design center for four years since March 2001. Each of these companies expects to use the new microprocessor as a crucial component in a range of its own products.\(^2\)

• Hitachi, Toshiba and Matsushita have recently reached an agreement to jointly establish a company to manufacture Liquid Crystal Display (LCD) panels for flat-panel TVs. The jointly established manufacturer will provide the three companies with LCD panels for their LCD TV businesses.\(^3\)

---

\(^1\)That is, firms in an industry could collaborate by undertaking a collusive strategy to reduce industry output and raise prices.

\(^2\)Details are available at http://www.toshiba.co.jp/about/press/2005_02/pr3101.htm (visited on April 12, 2005).

\(^3\)Details are available at http://www.toshiba.co.jp/about/press/2004_08/pr3101.htm (visited on April 12,
In the automobile industry, “platform-sharing” across firms has recently become common practice. By sharing a common platform (which is the core framework of cars), automobile manufacturers share key components such as engines, axles and transmission, and save their product development costs (Tierney et al., 2000; Lockwood, 2001). Renault and Nissan have developed a common platform for the Nissan Micra and the Renault Clio, and they plan to reduce the number of platforms they use to 10 in 2010 from the 34 they had in 2000.4

Yokohama Rubber (Tokyo) and Continental (Hanover) have set up a joint venture company for tire sales in Japan, which distribute Continental-brand tires as well as Yokohama-brand tires using the same nationwide sales network in Japan.5

One of the most often cited objectives for competitor collaboration is to obtain cost advantage by jointly exploiting economies of scale (Barney, 2002). Ohmae (1993, p.47) emphasized, by pointing out a number of real-world examples, the importance of fixed-cost savings through competitor collaborations. In the pharmaceutical industry, it is common that a company distributes their foreign competitors’ products through its own domestic distribution channel, with reciprocal arrangements that its products being distributed by their competitors.6 By avoiding to build their own distribution networks from scratch, they can save large amount of fixed costs. Concerning research and development (R&D), Ohmae pointed out that a number of competitor collaborations among the U.S. and Japanese firms had been established in the semiconductor industry to save their fixed R&D costs.

As illustrated by examples mentioned above, competitors often collaborate by sharing a part of their value-creating activities which consist of technology development, product design, manufacturing, marketing, distribution, and services.7 Note that these activities are important elements for creating product distinctiveness across firms. For example, Caves and Williamson (1985) empirically derived five important bases of product differentiation:


6According to Ohmae, in the United States, Marion Laboratories distributes Tanabe’s Herbesser and Chugai’s Ulcermin; Merck, Yamanouchi’s Gaster; Eli Lilly, Fujisawa’s Cefamezin. In Japan, Shionogi distributes Lilly’s Ceclor as Kefral; Sankyo, Squibb’s Capoten; Takeda, Bayer’s Adalat.

7This is based on a value-chain model proposed by McKinsey and Company (Grant, 1991; Barney, 2002). See also Porter (1985) for another important value-chain model, which contains similar elements.
products customized for specific customers, product complexity, emphasis on consumer marketing, different distribution channels, and service and support.

This suggests that a crucial disadvantage of competitor collaboration for collaborators is the reduction in their product distinctiveness. By sharing key components such as high-performance microprocessors, LCD panels and engines, competitors can save their costs for technology development, product design and manufacturing at the expense of the reduction in their product distinctiveness. Sharing of distribution channels enables substantial fixed-cost savings at the expense of reduced product-distinctiveness, given that distribution channel is a crucial determinant of product differentiation.

This paper explores economic consequences of this trade-off between product distinctiveness and fixed-cost savings by analyzing models that incorporate competitor collaboration into the location framework. As a simplest possible way to capture the trade-off, we first consider a “linear city” duopoly model (Hotelling, 1929), in which the two firms have an option to form a competitor collaboration. Without collaboration, one firm is located at one end of the line segment of length one, and the other firm is located at the other end. If they choose to collaborate, they can save their fixed costs but the distance between the two firms is reduced from 1 to $1 - z$ ($z \in [0, 1]$). Here $z$ captures the degree of the reduction in their product distinctiveness due to the collaboration, and we find that the two firms choose to collaborate when the value of $z$ is relatively small. Collaboration reduces equilibrium prices by intensifying competition between the two firms. Because of this effect, we find that collaboration increases the net utility of all consumers, even though some consumers are negatively affected by the change of product characteristics associated with the collaboration.

In reality, competitor collaborations rarely involve all firms in the industry. Given this, next we explore economic consequences of competitor collaborations in presence of firms that are not involved in the collaboration. We analyze a “circular city” model (Salop, 1979) consisting of three firms, where two firms (firms 1 and 2) have an option to form a collaboration while the third firm (firm 3) is assumed to be not involved in the collaboration. Without collaboration, the three firms are equidistantly located around a circle of circumference 1 (that is, the distance between any two firms is $\frac{1}{3}$). If firms 1 and 2 choose to collaborate, each firm $i$ ($= 1, 2$) undertakes an equal amount of relocation $\frac{z}{2}$ and move closer towards each other on the adjacent side, while firm 3 stays at the same location.

The collaboration between firms 1 and 2 intensifies the competition between themselves by reducing their distance from $\frac{1}{3}$ to $\frac{1}{3} - z$. At the same time, the collaboration weakens the
competition between each firm \( i (= 1, 2) \) and firm 3 by increasing their distance from \( \frac{1}{3} \) to \( \frac{1}{3} + \frac{z}{2} \). Concerning equilibrium prices charged by firms 1 and 2 under their collaboration, we find that the former effect dominates the latter so that they charge lower prices under collaboration. Concerning firm 3’s equilibrium price, we find that the competitor collaboration induces the non-collaborator (firm 3) to charge a price that is higher than the price it would charge when firms 1 and 2 choose not to collaborate.\(^8\)

Strategic interactions among collaborators and the non-collaborator result in a rich implication on consumers’ welfare. Since firms 1 and 2 charge lower prices under collaboration, some consumers who are located relatively close to firms 1 and 2 get better off. On the other hand, the competitor collaboration reduces the welfare of other consumers who are located relatively close to firm 3, because the collaboration induces firm 3 to charge a higher price. Also, at the aggregate level, we find that the competitor collaboration increases the aggregate consumer surplus. These welfare results yield novel antitrust implications as we discuss below. Also, as a theoretical contribution to the analyses of product differentiation under “circular city” models, our analysis sheds a new light on the analysis of asymmetric-location cases by presenting non-monotone results on how the competitor collaboration affects the non-collaborator’s pricing strategy and profitability.

Under competitor collaborations, firms are often linked through partial ownership arrangements (POAs) in which some firms own partial share holdings of other firms within the collaboration (see Section 5 for real-world examples).\(^9\) This motivates us to consider competitor collaborations in the presence of POAs. We explore extensions of our models in which firms 1 and 2 decide whether or not to collaborate in the presence of a POA, where each firm \( i (= 1, 2) \) owns a minority share holding of the other firm. The POA reduces the degree of the competition between firm 1 and 2 by linking their profitability. However, in contrast to the standard intuition that POAs decrease consumers’ welfare by softening competition, we find that the POA could in fact increase consumers’ welfare. The logic here is as follows: As described above, the collaboration between firms 1 and 2 increases the aggregate consumer surplus in our framework. However, if the collaboration intensifies

\(^{8}\)To be precise, we find that firm 3 charges a higher price under the collaboration whenever firms 1 and 2 choose not to collaborate.

\(^{9}\)Barney (2002) categorized competitor collaborations (“strategic alliances”) into non-equity alliances, equity alliances and joint ventures. According to Barney, in an equity alliance cooperating firms agree to work together to develop, manufacturer, or sell products or services, where their agreements are supported by equity holdings in alliance partners.
their competition too much, they choose not to collaborate. In such a case, we find that POAs could induce firms 1 and 2 to collaborate in our “circular city” setup. Furthermore, we find that, although the POA weakens the competition, this negative impact on the welfare of consumers can be more than offset by its positive impacts associated with the induced collaboration. This result yields a novel antitrust implication as we discuss below.

Competitor collaborations have been recognized as crucial issues by antitrust authorities, and they have recently issued antitrust guidelines that explain the analytical frameworks through which the authorities determine the overall competitive effects of competitor collaborations (see Canada, 1995; United States, 2000; European Commission, 2001).

Our analysis of competitor collaboration under “circular city” setup offers new perspectives for antitrust authorities to investigate this issue. Our first contribution is to point out that competitor collaborations may affect consumers in more complex manner than the antitrust guidelines indicate, by demonstrating that competitor collaborations can affect consumers differently depending on their preferences on product characteristics. In particular, we find that the competitor collaboration benefits some consumers who are located relatively close to the collaborators, while it harms the other consumers by inducing the non-collaborator to charge a higher price. This finding indicates that lower prices charged by the collaborators do not automatically mean that the competitor collaboration benefits all consumers, and suggests the importance for antitrust authorities to investigate how competitor collaborations affect the behavior of firms that are not involved in the collaboration.

Our second contribution concerns the basic logic behind potential antitrust harms associated with competitor collaborations. The authorities indicated in their guidelines that, although competitor collaborations themselves do not harm consumers, they could be a driving force of anticompetitive behavior such as collusion that harms consumers. This logic suggests that partial ownership arrangements (POAs) be a potential anticompetitive harm arising from competitor collaborations. In a sharp contrast, our analysis on POAs outlined above suggests that it can be a driving force (rather than an antitrust consequence) of competitor collaborations that benefit consumers. See Section 6 for more details on antitrust implications of our analysis.

The rest of the paper is organized as follows: Section 2 discusses the related literatures and our contributions to them. Section 3 analyzes a simple model that captures the trade-off between product distinctiveness and fixed-cost savings by incorporating competitor collaboration in a “linear city” duopoly model. Section 4 explores economic consequences
of competitor collaborations in a “circular city” model that captures strategic interactions among collaborators and the non-collaborator. Section 5 considers extensions of our models that incorporate partial ownership arrangements. Section 6 explores contributions of our analysis to antitrust investigations of competitor collaborations. Section 7 concludes.

2 Contributions to the literatures

The present paper is related to the literatures on theoretical analyses of cooperative research and development (R&D) and research joint ventures (RJVs), horizontal merger, and partial ownership arrangements. In this section we will discuss our contributions to these literatures.

Our analysis is related to the previous analyses on cooperative R&D and RJVs, given that they are important examples of competitor collaborations.\textsuperscript{10} Katz (1986) explored a four-stage model in which firms in an industry form a cooperative (cost-reducing) R&D agreement and determine sharing rules of the R&D cost and output before they compete against each other by choosing levels of their R&D efforts and production outputs. D’Aspremont and Jacquemin (1988) considered a two-stage model in which each firm determines the level of its cost-reducing R&D (with spillover) investment and then chooses the level of its production, where the cooperative R&D is modelled as the joint determination of the levels of their R&D investments. Following these seminal papers, a number of papers have explored economic consequences of cooperative R&D and RJVs. See Kamien et al. (1992), Suzumura (1992), and Choi (1993), among others.\textsuperscript{11}

The fundamental difference between the present paper and the previous papers in this literature is that ours explores economic consequences of competitor collaborations that affect

\textsuperscript{10}Although competitors collaborate in a variety of ways in reality, most previous papers on economic theoretical analyses of competitor collaborations have focused on cooperative R&D and research joint ventures (RJVs), as pointed out by Chen and Ross (2003). Important exceptions include Chen and Ross (2000) which analyzed a type of strategic alliance that involves the sharing of production capacity. Also, Chen and Ross (2003) analyzed a kind of joint venture in which the parents jointly produce a crucial input that they each subsequently use to produce final goods.

\textsuperscript{11}Building upon the d’Aspremont and Jacquemin model, Kamien et al. (1992) elaborated on the information-sharing aspect of R&D cooperation by considering a case in which, if firms cooperate in their R&D, the rate of R&D spillover increases to its maximal possible level. Suzumura (1992) examined the positive and normative effects of cooperative R&D \textit{vis-à-vis} noncooperative R&D, socially first-best R&D, and socially second-best R&D. See the next footnote on Choi (1993).
product characteristics. In our framework, competitors can save their fixed costs by forming a collaboration, which reduces the degree of their product distinctiveness and hence intensifies their competition. Furthermore, we consider competitor collaborations in presence of a firm that is not involved in the collaboration. Our analysis demonstrates that the strategic interactions among collaborators and the non-collaborator, together with the heterogeneity of consumers inherent in the location framework, yield rich welfare consequences and novel antitrust implications. Note, a few previous papers have captured the idea that a cooperative R&D agreement may intensify product-market competition among participants.\textsuperscript{12} However, they have not explored the interactions between participants and non-participants in the collaboration, and their consequences on consumers’ welfare.

Our contribution here is related to “external effects” that have been previously explored in the literature on horizontal mergers (see Salant et al., 1983; Deneckere and Davidson, 1985; Perry and Porter, 1985; Farrell and Shapiro, 1990a, among others). Given that any proposed merger presumably increases the joint profit of merging firms, Farrell and Shapiro (1990a) analyzed its effect on the profitability of firms that do not participate in the merger and on the welfare of consumers. That is, they emphasized the external effects of mergers on nonparticipant firms and consumers. Similar effects were also captured by Salant et al. (1983), Deneckere and Davidson (1985) and Perry and Porter (1985), although the focus of their analyses was on the profitability of merging firms.

These previous analyses captured external effects of collaboration among firms, given that horizontal merger here can be interpreted as a collaboration (or a collusive arrangement) among merging firms in their determination of prices and/or quantities. Our analysis, however, is fundamentally different from theirs, because ours analyzes collaboration among competitors (rather than collusive firms) in which firms share a part of their value-creating activities while they remain competitors in the product market. Our contribution in this

\textsuperscript{12}Choi (1993) captured this idea in his two-firm model of stochastic R&D. The two firms jointly choose their levels of R&D investment under a cooperative R&D agreement, where such an agreement increases the spillover rate of R&D results. When only one firm succeeds in R&D, the greater portion of the R&D result spills over to the other firm under a cooperative R&D agreement, which intensifies the product-market competition between the two firms. Hence the model captures the idea that a cooperative R&D agreement intensifies product-market competition. Lambertini et al. (2002) considered a two-firm model in which firms can form an RJV to share their product-development costs by jointly developing a single product. Their model also captures the idea that product-market competition can be intensified under RJVs, although their focus was on the effects of an RJV formation on the sustainability of implicit price collusion.
context is to show that the external effects, when analyzed in the context of competitor collaboration, yield welfare consequences and antitrust implications that have not been previously explored.

Finally, we discuss our contribution to the literature on partial ownership arrangements (POAs). Economic consequences of POAs have been previously explored under oligopoly models with homogeneous goods in the literature. Reynolds and Snapp (1986) demonstrated, using a modified Cournot oligopoly model, that POAs could result in less output and higher prices because such arrangements reduce the degree of competition among participants by linking their profitability. Farrell and Shapiro (1990b) explored the effects of changes in the level of partial ownership on price, output, profits, industry performance, and market concentration under a Cournot oligopoly. See Malueg (1991) and Gilo et al. (2005) for analyses of the effects that POAs have on the incentives for firms to engage in tacit collusion under infinitely repeated oligopoly models (Cournot in Malueg while Bertrand in Gilo et al.).

We contribute to the literature by incorporating a POA in a model of differentiated goods with an option for competitors to collaborate. This theoretical innovation allows us to capture the idea that POAs could induce a collaboration among competitors when the collaboration intensifies their competition by reducing the degree of their product distinctiveness. This results in a novel antitrust implication of competitor collaborations as we elaborate in Section 6.

3 Competitor Collaboration in a Duopoly Setup

A simplest possible way to capture the trade-off between product distinctiveness and fixed-cost savings is to incorporate competitor collaboration in a duopoly model with product differentiation. In this section, we explore the economic consequences of competitor collaboration in the “linear city” model (Hotelling, 1929). In the next section, using the “circular city” model (Salop, 1979), we will analyze competitor collaboration in presence of the third firm that is not involved in the collaboration.

13See e.g. Bresnahan and Salop (1986), Kwoka (1992), Flath (1992) and Reitman (1994) for related analyses.
3.1 The Model

Assume that a unit mass of consumers are uniformly distributed on the line segment [0, 1]. Each consumer is indexed by her location \( y \in [0, 1] \) on the line which represents her ideal point in the product characteristic space. Each consumer buys at most one unit of exactly one of the two varieties sold in the market. The price and location of variety \( i \) (=1,2) on the line are denoted by \( p_i \) and \( x_i \) respectively where \( x_i \in [0, 1] \) for all \( i (= 1, 2) \). The indirect utility of consumer \( y \in [0, 1] \) from purchasing one unit of variety \( i \) is given by \( V_i(y) = A - p_i - |x_i - y|^2 \), where \( A \) is the gross utility from consuming one unit of any variety, \( |x_i - y| \) denotes the distance between \( x_i \) and \( y \). The utility from not purchasing any variety is normalized to zero.

There are two firms with firm \( i \) producing variety \( i \). In what follows we use “the location of variety \( i \)” and “the location of firm \( i \)” interchangeably. The extent of differentiation between varieties 1 and 2 is captured by the distance between their locations, denoted \( d_{12} \equiv |x_1 - x_2| \). We assume that initially one firm is located at one end of the line segment while the other firm is located at the other end, and without loss of generality let \( x_1^0 = 0 \) and \( x_2^0 = 1 \) where the superscript ‘0’ denotes initial configuration. Each firm \( i \)’s cost for producing \( q_i \) units of variety \( i \) is given by \( \mu q_i + F_i \), where \( \mu \) and \( F_i \) denote the constant marginal cost and fixed cost for production respectively. We normalize \( \mu \) to be zero.

We incorporate an option for firms 1 and 2 to form a competitor collaboration in the following way: Assume that \( F_1 = F_2 = F \) if firms 1 and 2 do not collaborate while \( F_1 = F_2 = (1 - \phi)F \) if they collaborate, where \( 0 < \phi \leq \frac{1}{2} \) and \( F > 0 \). Here, parameter \( \phi \) captures the degree of cost saving; that is, by forming the collaboration each firm can save their fixed costs by \( \phi F \). But the cost saving comes at the expense of the reduction in their product distinctiveness. We capture this effect by assuming that each firm \( i \) (=1,2) undertakes an equal amount of relocation \( \frac{z}{2} \) and move closer towards each other under the collaboration. That is, the collaboration reduces the distance between firms 1 and 2 from \( d_{12} = 1 \) to \( d_{12} = 1 - z \), where \( z \in [0, 1) \) captures the degree of loss in their product distinctiveness.

(Figure 1 to be inserted here)

We consider the two stage game described below (see also Figure 1):

**Stage 1:** Firms 1 and 2 jointly decide whether or not to collaborate. If they do not collaborate, then \( F_1 = F_2 = F \) and \( (x_1, x_2) = (x_1^0, x_2^0) = (0, 1) \), and accordingly \( d_{12} = 1 \). If they collaborate, the distance between firms 1 and 2 is reduced to \( d_{12} = 1 - z \), where \( z \) captures the degree of loss in their product distinctiveness.
$|x_1^0 - x_2^0| = 1$. On the other hand, if they collaborate, then $F_1 = F_2 = (1 - \phi)F$ and $d_{12} = 1 - z \leq 1$, where $|x_i - x_i^0| = \frac{z}{2}$ for $i = 1, 2$. That is, each firm $i$ ($i = 1, 2$) undertakes an equal amount of relocation $\frac{z}{2}$ and move closer towards each other.

**Stage 2:** Given the location $(x_1, x_2)$, each firm $i$ chooses $p_i$ to maximize its own profit, taking its rivals’ prices $p_j$ as given.

### 3.2 Analysis

The game described above has two Stage 2 subgames. One is the subgame in which firms 1 and 2 collaborate at Stage 1 (call it collaboration subgame) while the other is the one in which they do not collaborate (non-collaboration subgame). We consider an equilibrium in which firms 1 and 2 jointly decide whether or not to collaborate at Stage 1 so that each firm’s profit in the subsequent Nash equilibrium outcome of the Stage 2 subgame is maximized. For expositional simplicity, we assume that, if firms 1 and 2 are indifferent between collaborating and not collaborating, they choose to collaborate. Note, all proofs are presented in the Appendix.

Throughout the analysis, we assume that the fixed cost for production is low enough so that each firm produces a strictly positive amount in equilibrium. We also assume that the gross utility from consuming a variety (captured by $A$) is high enough so that every consumer purchases a variety from a firm in equilibrium. If the value of $A$ is relatively low, consumers located beyond a certain distance from firms find it optimal to buy no products. In such cases, each firm becomes a local monopolist and does not directly compete with each other. Given our focus on the collaboration among *competitors*, we make these assumptions to ensure that the two firms directly compete with each other in equilibrium.

Each stage 2 subgame has a unique and symmetric equilibrium. Let $p_i(z)$ and $\pi_i(z)$ ($i = 1, 2$) denote each firm $i$’s equilibrium price and profit respectively, in the collaboration subgame. Given that the equilibrium is symmetric, we find that $p_1(z) = p_2(z) = 1 - z \equiv p(z)$ and $\pi_1(z) = \pi_2(z) = \frac{1-z}{2} \equiv \pi(z)$. Note, using the same notations, each firm’s equilibrium price and profit in the non-collaboration subgame are $p(0)$ and $\pi(0)$ respectively.

**Proposition 1:** In the equilibrium of the collaboration subgame, each firm’s equilibrium price and profit decrease as the loss of their product distinctiveness due to the collaboration increases. That is, $p(z)$ and $\pi(z)$ are strictly decreasing in $z$ for all $z \in [0, 1)$.
The competitor collaboration intensifies the competition between firms 1 and 2 by reducing the degree of their product distinctiveness. This is the disadvantage of the competitor collaboration for the collaborators. Since the loss of the product distinctiveness is captured by \( z \), each firm’s equilibrium price and profit are decreasing in \( z \). On the other hand, the degree of each firm’s fixed-cost saving (captured by \( \phi \)) is not affected by \( z \). This suggests that the two firms choose to collaborate at Stage 1 if the disadvantage of the collaboration, captured by \( z \), is relatively small.

**Proposition 2:** There exists a function \( z^*(\phi) \) such that firms 1 and 2 choose to collaborate at Stage 1 if and only if \( z \leq z^*(\phi) \), where \( z^*(\phi) \in (0, \frac{1}{2}] \). Furthermore, \( z^*(\phi) \) is strictly increasing in \( \phi \) for all \( \phi \in (0, \frac{1}{2}] \).

As discussed above, Proposition 2 tells us that firms 1 and 2 choose to collaborate if the loss of their product distinctiveness due to the collaboration is relatively small. The threshold \( z^*(\phi) \) is increasing in \( \phi \), which captures the degree of fixed-cost saving due to the collaboration. That is, the competitor collaboration becomes more likely as its advantage increases.

Next we consider how the competitor collaboration affects consumers.

**Proposition 3:** Let \( CS_C(y, z) \) and \( CS_{NC}(y) \) denote the consumer surplus of consumer \( y \in [0, 1] \) in the equilibrium of the collaboration subgame and the non-collaboration subgame respectively. Then, for any given \( z \in (0, 1) \), \( CS_C(y, z) > CS_{NC}(y) \) holds for all \( y \in [0, 1] \).

Collaboration between firms 1 and 2 affects consumers’ utility through two channels. On the one hand, collaboration reduces equilibrium prices through intensifying competition between the two firms, and this increases the utility of all consumers equally. On the other hand, consumers are also affected by the change of product characteristics, given that collaboration involves “relocation” of the firms. This effect decreases the utility of consumers who are located towards the ends of the line (precisely, consumers with address \( y \in [0, \frac{1}{4}] \) and \( y \in (1 - \frac{1}{4}, 1] \)) and increases the utility of other consumers, where the reduction of utility is largest for consumers located at the ends of the line (consumers with address \( y = 0 \) or 1).

We have found that the net of these two effects is positive even for consumers located at both ends of the line, and hence competitor collaboration increases the utility of all consumers as stated in Proposition 3. Note, this immediately implies that the competitor collaboration increases the aggregate consumer surplus as stated in the corollary below.
Corollary 1: Let $\widehat{CS}_C(z) \equiv \int_0^1 CS_C(y, z)\,dy$ and $\widehat{CS}_{NC} \equiv \int_0^1 CS_{NC}(y)\,dy$ denote the aggregate consumer surplus in the equilibrium of the collaboration subgame and the non-collaboration subgame respectively. Then, $\widehat{CS}_C(z) > \widehat{CS}_{NC}$ holds for all $z \in [0, 1)$.

4 Collaboration in Presence of Non-collaborator

In reality, competitor collaborations rarely involve all firms in the industry. For instance, all examples of competitor collaborations listed in Introduction involve only a fraction of firms in the industry. What are economic consequences of competitor collaborations in presence of firms that are not involved in the collaboration? In this section, we explore this question through analyzing a “circular city” model (Salop, 1979) consisting of three firms, where two firms have an option to form a collaboration while the third firm is assumed to be not involved in the collaboration. We demonstrate that the strategic interaction between the collaborators and the non-collaborator yields rich economic and policy implications. Also, as a theoretical contribution to the analyses of product differentiation under the location framework, we shed a new light on the analysis of asymmetric-location cases by presenting interesting results on how the competitor collaboration affects the non-collaborator’s pricing strategy and profitability.

4.1 The Model

Assume that a unit mass of consumers are uniformly distributed on a circle of circumference 1. Each consumer is indexed by her location $y \in [0, 1)$ on the circumference which represents her ideal point in the product characteristic space. Each consumer buys at most one unit of exactly one of the three varieties sold in the market. The price and location of variety $i$ ($=1,2,3$) on the circle are denoted by $p_i$ and $x_i$ respectively where $x_i \in [0, 1)$ for all $i (= 1, 2, 3)$. The indirect utility of consumer $y \in [0, 1)$ from purchasing one unit of variety $i$ is given by

$$V_i(y) = A - p_i - |x_i - y|^2,$$

where $A$ is the gross utility from consuming one unit of any variety, $|x_i - y|$ denotes the minimum distance (i.e. the closer arc length around the circle) between $x_i$ and $y$. The utility from not purchasing any variety is normalized to zero.
There are three firms with firm \( i \) producing variety \( i \). In what follows we use “the location of variety \( i \)” and “the location of firm \( i \)” interchangeably, as in the previous section. The extent of differentiation between two varieties \( i \) and \( j \), denoted by \( d_{ij} \), is captured by \( d_{ij} \equiv |x_i - x_j| \) — the minimum distance between the location of the two varieties \( i \) and \( j \). We assume that initially the firms are equidistantly located around the circle, i.e. \( d_{ij}^0 \equiv |x_i^0 - x_j^0| = \frac{1}{3} \) for all \( i, j \in \{1, 2, 3\}, i \neq j \) where the superscript ‘0’ denotes initial configuration. As in the previous section, each firm \( i \)'s cost for producing \( q_i \) units of variety \( i \) is given by \( \mu q_i + F_i \), where \( \mu \) and \( F_i \) denote the constant marginal cost and fixed cost for production respectively. We normalize \( \mu \) to be zero.

We incorporate an option for firms 1 and 2 to form a competitor collaboration. As in the previous section, we assume that \( F_1 = F_2 = F \) if firms 1 and 2 do not collaborate while \( F_1 = F_2 = (1 - \phi)F \) if they collaborate (\( 0 < \phi \leq \frac{1}{2} \) and \( F > 0 \)), where parameter \( \phi \) captures the degree of cost saving due to the collaboration. At the same time, the collaboration reduces the distance between firms 1 and 2 from \( d_{12} = \frac{1}{3} \) to \( d_{12} = \frac{1}{3} - z \), where \( z \in [0, \frac{1}{3}) \) captures the degree of the loss in their product distinctiveness. On the other hand, Firm 3’s location and fixed cost are unaffected by formation of the collaboration between firms 1 and 2, where \( F_3 = F \).

We consider the two stage game described below (see also Figure 2):

**Stage 1**: Firms 1 and 2 jointly decide whether or not to collaborate. If they do not collaborate, then \( F_1 = F_2 = F \) and \( (x_1, x_2) = (x_1^0, x_2^0) \), and accordingly \( d_{12} = d_{12}^0 = |x_1^0 - x_2^0| = \frac{1}{3} \). On the other hand, if they collaborate, then \( F_1 = F_2 = (1 - \phi)F \) and \( d_{12} = \frac{1}{3} - z \leq \frac{1}{3} = d_{12}^0 \), where \( |x_i - x_i^0| = \frac{z}{2} \) for \( i = 1, 2 \). That is, each firm \( i \) (= 1, 2) undertakes an equal amount of relocation \( \frac{z}{2} \) and move closer towards each other on the adjacent side. Irrespective of formation of the collaboration between firms 1 and 2, firm 3’s location and fixed cost are \( x_3 = x_3^0 \) and \( F_3 = F \) respectively. In absence of the collaboration the firms’ locations are equidistant from each other implying \( d_{23} = d_{31} = \frac{1}{3} \). Under the collaboration, \( d_{23} = d_{31} = \frac{1}{3} + \frac{z}{2} \).

**Stage 2**: Given the locations \( x \equiv (x_1, x_2, x_3) \) and accordingly the distance \( d \equiv (d_{12}, d_{23}, d_{31}) \), each firm \( i \) chooses \( p_i \) to maximize its own profit, taking rivals’ prices \( p_j \) and \( p_k \) as given \( (i, j, k \in \{1, 2, 3\}, i \neq j \neq k) \).

(Figure 2 to be inserted here)

There are two assumptions implicit in the model which are made to simplify the analy-
sis. Below we discuss the rationale behind them. First, the initial distance between any two varieties is assumed to be equal. This assumption seems natural, given the same cost specification for all the firms in absence of the competitor collaboration, same gross utility for all the varieties and uniform distribution of consumers. Moreover, in an extended game where firms choose locations and then prices, the symmetric configuration is indeed the subgame perfect Nash equilibrium.14 Second, we have assumed that, if firms 1 and 2 choose to form the collaboration, then each firm $i (=1,2)$ moves closer towards each other by the equal distance $\frac{z}{2}$ from the initial location, and firm 3 does not relocate. It can be shown that this is an equilibrium outcome of the following alternative model with relocation costs: Suppose that, if firms 1 and 2 choose to collaborate, then they cooperatively choose their locations $x_1$ and $x_2$ with a restriction that the distance between themselves must be less than or equal to $\frac{1}{3} - z$, and at the same time firm 3 chooses its own location. Here, each firm $i (i=1,2,3)$ must incur a relocation cost $c(r_i)$, where $r_i \equiv |x_i - x_0^i|$ captures the magnitude of relocation, $c(0) = c'(0) = 0$, and $c'(r) > 0$ and $c''(r) > 0$ for all $r > 0$.15

4.2 Analysis

Analogous to the analysis presented in the previous section, we consider an equilibrium in which firms 1 and 2 jointly decide whether or not to collaborate at Stage 1 so that each firm’s profit in the subsequent Nash equilibrium outcome of the Stage 2 subgame is maximized. The game has two Stage two subgames: collaboration subgame and non-collaboration subgame, where the former (latter) is the subgame in which firms 1 and 2 collaborate (do not collaborate) at Stage 1.16 Throughout the analysis, we assume that the fixed cost for production is low enough so that each firm produces a strictly positive amount in equilibrium, and that the gross utility from consuming a variety (captured by $A$) is high enough so that every consumer purchases a variety from a firm in equilibrium. Given our focus on the collaboration among competitors, as in the previous section we make these assumptions to ensure that the two firms directly compete with each other in equilibrium. Note, all proofs are presented in the Appendix.

14See Economides (1989).
15Examples of the relocation costs include costs for advertising the change of product characteristics and other adjustment costs associated with the change.
16For expositional simplicity, as in the previous section we assume that, if firms 1 and 2 are indifferent between collaborating and not collaborating, they choose to collaborate.
Consider Stage 2 price subgames, in which firms noncooperatively choose prices given their locations \( x \equiv (x_1, x_2, x_3) \) (which determines the distances \( d \equiv (d_{12}, d_{23}, d_{31}) \)). Given any \( x \), consider \( p \equiv (p_1, p_2, p_3) \) such that each firm \( i \) sells its product to consumers located in both sides of \( x_i \). Let \( y_{ij} \) denote the address of the consumer who is indifferent between purchasing variety \( i \) and \( j \). Then \( y_{ij} \) satisfies \( V_i(y_{ij}) = V_j(y_{ij}) \Rightarrow p_i + |x_i - y_{ij}|^2 = p_j + |x_j - y_{ij}|^2 \), solving which yields
\[
|y_{ij} - x_i| = \frac{p_j - p_i}{2d_{ij}} + \frac{d_{ij}}{2}.
\] (2)

Clearly all consumers located between \( x_i \) and \( y_{ij} \) will purchase variety \( i \). Since the density of consumer is unity and each consumer buys only one unit, the demand for variety \( i \) from the consumers located in the right side of \( x_i \) is \( |x_i - y_{ij}| \). Similarly the demand from the consumers in the left side of \( x_i \) is \( |x_i - y_{ki}| \). Summing them up yields the total demand for variety \( i \), denoted by \( q_i \):
\[
q_i = \frac{p_j - p_i}{2d_{ij}} + \frac{p_k - p_i}{2d_{ki}} + \frac{1 - d_{jk}}{2}.
\] (3)

Given \( d \equiv (d_{12}, d_{23}, d_{31}) \), \( p_j \) and \( p_k \), firm \( i \)'s stage 2 profit (which is \( p_i q_i = \pi_i \) ) is maximized when
\[
p_i = \frac{d_{ij}d_{ki}}{2} + \frac{p_j d_{ki} + p_k d_{ij}}{2(d_{ki} + d_{ij})} \quad (i, j, k \in \{1, 2, 3\}, i \neq j \neq k),
\] (4)
and this system of equations determines the unique equilibrium of the price subgame (see Economides (1989) for details).

Let \( p_i(z) \) and \( \pi_i(z) \) \((i = 1, 2, 3)\) denote each firm \( i \)'s equilibrium price and profit respectively, in the collaboration subgame. Noting that \( d_{12} = \frac{1}{3} - z \) and \( d_{23} = d_{31} = \frac{1}{3} + \frac{z}{2} \) under the collaboration subgame, we find
\[
p_1(z) = p_2(z) = \frac{(2^3 + z)(1^3 - z)(1^3 - z)}{8(\frac{2}{3} - z)}; \quad p_3(z) = \frac{5(2^3 + z)(1 - \frac{3}{2}z - \frac{9}{20}z^2)}{36(\frac{2}{3} - z)}; \quad \pi_1(z) = p_2(z) = \frac{25(2^3 + z)(1 - \frac{3}{2}z - \frac{9}{20}z^2)^2}{648(\frac{2}{3} - z)^2}.
\] (5)

On the other hand, in the non-collaboration subgame, equilibrium prices and profits are given by \( p_i = p_i(0) = p_i^0 \) and \( \pi_i = \pi_i(0) = \pi_i^0 \) \((i = 1, 2, 3)\) respectively.
4.2.1 Effects of competitor collaboration on collaborators

**Proposition 4:** In the equilibrium of the collaboration subgame, each firm \(i\)’s \((i = 1, 2)\) equilibrium price and profit decrease as the loss of their product distinctiveness due to the collaboration increases. That is, \(p_1(z) = p_2(z)\) and \(\pi_1(z) = \pi_2(z)\) are strictly decreasing in \(z\) for all \(z \in [0, \frac{1}{3}]\).

The collaboration between firms 1 and 2 intensifies the competition between themselves by reducing the degree of their product distinctiveness, as in the duopoly case analyzed in the previous section. Since each firm \(i\) (=1, 2) moves closer to each other by \(\frac{z}{2}\) under the collaboration, the loss of their product distinctiveness is captured by the reduction in their distance \(\frac{z}{2} + \frac{z}{2} = z\). At the same time, in presence of the non-collaborator (firm 3), the collaboration makes each firm \(i\)’s \((i = 1, 2)\) product more distinct from firm 3’s product, because each firm \(i\) moves away from firm 3 by \(\frac{z}{2}\). This effect, which was not captured in the duopoly analysis, weakens the competition between each collaborating firm and the non-collaborating firm.

Observe that the competitor collaboration reduces the distance between collaborators by \(z\) while it increases the distance between a collaborator and the non-collaborator only by \(\frac{z}{2}\), which is because a collaborating firm (say firm 2)’s relocation does not affect the distance between its collaboration partner (firm 1) and the non-collaborator (firm 3). In other words, the extent to which the competitor collaboration intensifies the competition between collaborators is greater than the extent to which it weakens the competition between a collaborator and the non-collaborator. It turns out, as a consequence, that the former effect dominates the latter so that firms 1 and 2 charge lower prices in the equilibrium of the collaboration subgame.\(^{17}\) This results in Proposition 4.

Given Proposition 4, proposition 5 below identifies the condition in which firms 1 and 2 choose to collaborate.

\(^{17}\)To see this algebraically, let \(i = 1, j = 2\) and \(k = 3\) in equation (3), where the price competition between firms 1 and 2 (between firms 1 and 3) is captured by the first term \(\frac{p_2 - p_1}{d_{12}}\) (the second term \(\frac{p_3 - p_1}{d_{31}}\)). By reducing its price by \(\Delta\), firm 1 can increase its quantity by \(\frac{\Delta}{d_{12}}\) and \(\frac{\Delta}{d_{31}}\) from “stealing” customers from firms 2 and 3 respectively. Given that \(d_{12} = \frac{1}{3} - z\) and \(d_{31} = \frac{1}{3} + \frac{z}{2}\) under the collaboration, we find that \(\frac{\Delta}{d_{12}} + \frac{\Delta}{d_{31}}\) is increasing in \(z\) for all \(z \in [0, \frac{1}{3}]\). That is, firm 1’s incentive to increase its demand by reducing its price is increasing in \(z\) for all \(z \in [0, \frac{1}{3}]\).
**Proposition 5:** There exists a function \( z^*(\phi) \) such that firms 1 and 2 choose to collaborate at Stage 1 if and only if \( z \leq z^*(\phi) \), where \( z^*(\phi) \in (0, \frac{1}{3}) \) for all \( \phi \in (0, 1) \). Furthermore, \( z^*(\phi) \) is strictly increasing in \( \phi \) for all \( \phi \in (0, \frac{1}{2}] \).

Proposition 4 tells us that each firm \( i \)'s (\( i = 1, 2 \)) post-collaboration profit is decreasing in \( z \), while the advantage of the collaboration (fixed-cost savings) is not affected by \( z \). Hence, firms 1 and 2 choose to collaborate when \( z \) is relatively small as stated in Proposition 5. The threshold \( z^*(\phi) \) is increasing in \( \phi \), which says that the competitor collaboration becomes more likely as the degree of fixed-cost savings due to the collaboration (captured by \( \phi \)) increases.

### 4.2.2 Effects of competitor collaboration on the non-collaborator

How does the strategic interaction between collaborators (firms 1 and 2) and the non-collaborator (firm 3) affects the non-collaborator’s equilibrium price and profit? Proposition 6 tells us that the competitor collaboration raises the non-collaborator’s price and profit in the equilibrium. Also, Proposition 6 identifies interesting non-monotone relationships between the non-collaborator’s equilibrium price/profit and the value of \( z \).

**Proposition 6:** If firms 1 and 2 choose to collaborate in the equilibrium, the collaboration induces firm 3 to charge a higher price and earn a higher profit than it does in the non-collaboration subgame. More precisely,

1. There exists a threshold value \( z' \in (0, \frac{1}{3}) \) such that \( p_3(z) \) is strictly increasing in \( z \) for all \( z < z' \) and decreasing in \( z \) for all \( z > z' \), where \( p_3(z) > p_3^0 \) holds for all \( z \in (0, \frac{1}{3}) \).

2. There exist threshold values \( \widehat{z} \) and \( \check{z}, 0 < \check{z} < \widehat{z} < \frac{1}{3} \), such that \( \pi_3(z) \) is strictly increasing in \( z \) for all \( z < \check{z} \) and increasing in \( z \) for all \( z > \check{z} \), where \( \pi_3(z) > \pi_3^0 \) for all \( z \in (0, \check{z}) \) and \( \pi_3(z) < \pi_3^0 \) for all \( z \in (\check{z}, \frac{1}{3}) \). Furthermore, \( \pi_3(z) > \pi_3^0 \) for all \( z \in (0, z^*) \).

Our model captures the following two effects of the competitor collaboration on the non-collaborator. On the one hand, since the collaboration between firms 1 and 2 makes their products more distinct from firm 3’s product by increasing the distance between firm 3 and firm \( i \) (\( i = 1, 2 \)), it weakens the competition between firm 3 and firm \( i \) (\( i = 1, 2 \)). On the other hand, firms 1 and 2 charge lower prices as \( z \) increases (Proposition 4), which imposes downward pressure on firm 3’s pricing and reduces its profitability.

(Figure 3 to be inserted here)
Proposition 6 tells us that the former effect dominates the latter when the value of $z$ is relatively small, while the latter dominates the former when $z$ is relatively large. That is, as $z$ increases from 0, $p_3(z)$ and $\pi_3(z)$ increase in the beginning but eventually start to decrease. Although the relationship is non-monotone, firm 3’s price is higher in the collaboration subgame than in the non-collaboration subgame (that is, $p_3(z) > p_3^0$ holds) for all $z \in (0, \frac{1}{3})$. On the other hand, firm 3’s profit becomes lower in the collaboration subgame when $z$ is larger than the threshold $\hat{z}$. But whenever firms 1 and 2 choose to form the collaboration (that is, for all $z \leq z^*$), firm 3’s profit is higher in the collaboration subgame (in other words, $z^* < \hat{z}$ holds).

Note, to the best of our knowledge, the non-monotone relationships between $p_3(z)$ ($\pi_3(z)$) and $z$ are new results in the literature on the theoretical analyses of product differentiation under the location framework.\(^{18}\) Most previous papers on location models focused on the case of symmetric location, in which (i) two firms are symmetrically located on a line or (ii) $n$ firms are equidistantly located on a circle. In contrast, by incorporating the competitor collaboration, we highlight the importance of the analysis of asymmetric location, and find that the interaction between collaborators and the non-collaborator gives rise to the non-monotonicity result.

4.2.3 Effects of competitor collaboration on consumers

Now we turn to the effects of the competitor collaboration on consumers. The collaboration between firms 1 and 2 reduces the distinctiveness between their products, which directly affects consumers’ utility by changing product characteristics. At the same time, the collaboration also affects the intensity of competition among all three firms, which indirectly affects consumers through equilibrium prices. The interaction of the product-characteristics effect and the price effect, together with the heterogeneity of consumers inherent in the location framework, yields rich implications on how the competitor collaboration affects consumers. In particular, unlike the duopoly analysis presented in the previous section, we find that, in presence of the non-collaborator under the circular city model, the competitor collaboration makes some consumers better off while others worse off depending on their preferences represented by their locations on the circle.

\(^{18}\)See Anderson et al. (1992) for a comprehensive analysis of product differentiation under location framework.
We categorize consumers into three groups depending on the way in which they are affected by the collaboration between firms 1 and 2 (see Figure 4). First, there is a group of consumers, call them Group $\alpha$ consumers, for whom the following two properties hold: (i) Each Group $\alpha$ consumer purchases a product from firm 1 or 2 in the equilibrium of the non-collaboration subgame, and purchases a product from the same firm in the equilibrium of the collaboration subgame, and (ii) the product that each Group $\alpha$ consumer purchases in the collaboration subgame is closer to her most desired product characteristics than in the non-collaboration subgame. Lemma 1 (i) identifies the location of Group $\alpha$ consumers. Second, there are another group of consumers, call them Group $\beta$ consumers, who purchase a product from firm 3 in the equilibrium of both subgames. Lemma 1 (ii) identifies the location of Group $\beta$ consumers. Finally, there are the other group of consumers who belong to neither Group $\alpha$ or $\beta$. We call them Group $\gamma$ consumers.

(Figure 4 to be inserted here)

Lemma 1:
(i) Let $M_{12}$ denote the mid-point of $x_1^0$ and $x_2^0$. There exists a unique distance $d_\alpha(z) \in (0, \frac{1}{6})$ such that a consumer is a Group $\alpha$ consumer if and only if the distance between the consumer and $M_{12}$ is less than $d_\alpha(z)$.
(ii) There exists a unique distance $d_\beta(z) \in (0, \frac{1}{8})$ such that a consumer is a Group $\beta$ consumer if and only if the distance between the consumer and $x_3^0$ is less than $d_\beta(z)$.

We are now ready to present Proposition 7, which captures the effects of the competitor collaboration on each group of consumers.

Proposition 7: Let $CS_C(y, z)$ and $CS_{NC}(y)$ denote the consumer surplus of consumer $y \in [0, 1)$ in the equilibrium of the collaboration subgame and the non-collaboration subgame respectively. For any given $z \leq z^*(\phi)$, we have the following properties:
(i) For all Group $\alpha$ consumers, $CS_C(y, z) > CS_{NC}(y)$.
(ii) For all Group $\beta$ consumers, $CS_C(y, z) < CS_{NC}(y)$.
(iii) For Group $\gamma$ consumers, there exists a unique distance $d_\gamma(z) \in (d_\alpha(z), \frac{1}{2} - d_\beta(z))$ such that $CS_C(y, z) > CS_{NC}(y)$ if $|y - M_{12}| < d_\gamma(z)$ while $CS_C(y, z) < CS_{NC}(y)$ if $|y - M_{12}| > d_\gamma(z)$. 

19
Recall that firms 1 and 2 choose to collaborate at Stage 1 if $z \leq z^*$. Proposition 7 tells us that the competitor collaboration makes all Group $\alpha$ consumers better off while all Group $\beta$ consumers worse off. First consider Group $\alpha$ consumers, who purchase from firm 1 or 2 in the equilibrium of both subgames. From Proposition 4 we know that firms 1 and 2 charge lower prices in the collaboration subgame. Then, given that the product each Group $\alpha$ consumer purchases in the collaboration subgame is closer to her most desired product characteristics, the price effect as well as the product-characteristics effect work in Group $\alpha$ consumers’ advantage.

Next consider Group $\beta$ consumers, who purchase from firm 3 in the equilibrium of both subgames. From Proposition 6 we know that firm 3 charges a higher price in the collaboration subgame. Hence, the competitor collaboration makes all Group $\beta$ consumers worse off, given that they are unaffected by the product-characteristics effect. Finally consider Group $\gamma$ consumers. In the equilibrium of the collaboration subgame, each Group $\gamma$ consumer purchases a product from firm 1 or 2 at a lower price than the price it purchases a product in the non-collaboration subgame. But the product each Group $\gamma$ consumer purchases in the collaboration subgame is farther away from her most desired product characteristics than in the non-collaboration subgame. That is, the price effect and the product-characteristics effect work in the opposite direction for Group $\gamma$ consumers. Proposition 7 tells us that the price effect dominate the product-characteristics effect for some Group $\gamma$ consumers who are located relatively close to the collaborating firms, and hence the collaboration makes them better off. On the other hand, the collaboration makes the other Group $\gamma$ consumers worse off.

In summary, the interaction of the product-characteristics effect and the price effect in our framework implies that the collaboration between firms 1 and 2 benefit some consumers who are located relatively close to the collaborating firms, while it harms the other consumers who are located far from them and relatively close to the non-collaborator (firm 3). We will elaborate on this finding in Section 6 to offer a new perspective for antitrust authorities to investigate the welfare consequences of competitor collaborations.

Finally we consider the effect of the competitor collaboration on aggregate consumer surplus. Let $\widehat{CS}_C(z) \equiv \int_0^1 CS_C(y, z)dy$ denote the aggregate consumer surplus in the equilibrium of the collaboration subgame, and define $\widehat{CS}_{NC} \equiv \int_0^1 CS_{NC}(y)dy$ analogously for the non-collaboration subgame.
Proposition 8: For all $z \in (0, \frac{1}{3})$, $\widehat{CS}_C(z) > \widehat{CS}_{NC}$ holds.

From Proposition 7 we know that the competitor collaboration benefits some consumers while harms the others. However, at the aggregate level Proposition 8 tells us that the competitor collaboration increases the aggregate consumer surplus for all $z \in (0, \frac{1}{3})$. Note, alternatively one could aggregate the effects of the competitor collaboration on consumers in terms of the number (more precisely, the measure) of consumers affected positively or negatively. Under this alternative measure, it can be shown that the collaboration may make a majority of consumers better off or worse off, depending on parameterizations.

5 Partial Ownership Arrangements

Competitor collaborations are often accompanied by partial ownership arrangements (POAs) in which some firms own partial share holdings of other firms within the collaboration. For example, in the automobile industry, competitor collaborations (“platform-sharing” as described above) are often accompanied by POAs. Renault-Nissan alliance, in which platform sharing plays a central role, started in March 1999 when Renault acquired 36.8% stake in Nissan. Renault now owns 44.4% stake in Nissan, while Nissan has 15% stake in Renault. At the same time, the two companies preserve distinct identities, corporate culture, and brand identity. Platform sharing between Mitsubishi Motors and DaimlerChrysler as well as that between General Motors and Fiat are also accompanied by POAs.

Given this, in this section we consider extensions that incorporate POAs in our models, and explore their economic and welfare implications.

Partial ownership arrangements in the “linear city” model

In this extension, everything is the same as in the original model analyzed in Section 3, except that there is a POA represented by $\theta \in [0, \frac{1}{2})$ between firms 1 and 2, where firm 1 (firm 2) owns fraction $\theta$ of firm 2’s (firm 1’s) stake. Given this, at Stage 1 firms 1 and 2

---

19We have also found that the competitor collaboration increases the total surplus for all $z \leq z^*$. We focus on effects on consumers, given that the main concern of the antitrust authorities is how consumers are affected by competitor collaborations. See Section 6 for details.

jointly decide whether or not to collaborate, and at Stage 2 they compete against each other by choosing prices. Note, each firm \( i = 1, 2 \) continues to own fraction \( \theta \) of the other firm’s stake whether or not they choose to collaborate at Stage 1. Given that the POA represented by \( \theta \) is a minority ownership, we assume that each firm \( i = 1, 2 \) chooses the price \( p_i \) to maximize

\[
(1 - \theta)p_i q_i + \theta p_j q_j, \tag{7}
\]

where \( i, j = 1, 2, i \neq j \), and \( p_i \) and \( q_i \) denote variety \( i \)'s price and quantity respectively. For similar formulations of POAs, see Reynolds and Snapp (1986), Farrell and Shapiro (1990b), Malueg (1991) and Gilo et al. (2005), among others.

Proposition 9: There exists a function \( z^*(\theta, \phi) \) such that firms 1 and 2 choose to collaborate at Stage 1 if and only if \( z \leq z^*(\theta, \phi) \), where \( z^*(\theta, \phi) \in (0, 1) \) is strictly increasing in \( \phi \) for all \( \phi \in (0, \frac{1}{2}) \). Furthermore, \( z^*(\theta, \phi) \) is strictly decreasing in \( \theta \) for all \( \theta \in [0, \frac{1}{2}] \).

The first part of the proposition is analogous to Proposition 2 in Section 3. That is, firms 1 and 2 choose to collaborate if the loss of their product distinctiveness due to the collaboration (captured by \( z \)) is relatively small, and the threshold \( z^*(\theta, \phi) \) is increasing in the degree of fixed-cost saving due to the collaboration (captured by \( \phi \)).

How does the POA represented by \( \theta \) affect incentives for firms 1 and 2 to collaborate? Proposition 9 tells us that the threshold \( z^*(\theta, \phi) \) is decreasing in \( \theta \); that is, the POA makes the competitor collaboration less likely to occur in the duopoly setup. The logic here is as follows: The POA weakens the competition between firms 1 and 2, which in turn increases their profits by inducing them to charge higher prices. Hence, as \( \theta \) increases, each firm’s stage 2 profit increases in the collaboration subgame as well as in the non-collaboration subgame. Recall that, in the original model, each firm’s stage 2 profit is higher in the non-collaboration subgame than in the collaboration subgame (see Proposition 4). Given

\[21\] Reynolds and Snapp (1986) made a distinction between firms and plants in the description of their model (see also Gilo et al. (2005), who made a distinction between firms and controllers). Following this formulation, the extension here can be described as follows: Assume that there are two firms and two plants, where firms are profit-maximizing decision-making units that own and/or control plants. Each plant \( i = 1, 2 \) produces variety \( i \), where \( x_i, x_i^0, F_i \) and \( p_i \), defined for firm \( i \) in the original model, are now defined for plant \( i \) here. There is a POA represented by \( \theta \in [0, \frac{1}{2}] \) between firms 1 and 2, where firm 1 (firm 2) owns fraction \( 1 - \theta \) of plant 1’s (plant 2’s) stake and fraction \( \theta \) of plant 2’s (plant 1’s) stake. Given that each firm \( i = 1, 2 \) is the majority owner of plant \( i \) (i.e., \( 1 - \theta > \frac{1}{2} \)), we assume that each firm \( i = 1, 2 \) chooses the price \( p_i \) to maximize its profit, which is given by \((1 - \theta)p_i q_i + \theta p_j q_j\).
this, we find that the incremental profit associated with larger θ is also higher in the non-collaboration subgame. This implies that the disadvantage of collaboration, captured by π_{NC} − π_C, is increasing in θ, where π_C (π_{NC}) denotes each firm’s second-period profit in the collaboration (non-collaboration) subgame. Hence the POA makes the collaboration less likely to occur.

**Partial ownership arrangements in the “circular city” model**

Next we turn to the second extension which incorporates the POA in the circular city model considered in the previous section. The structure of this extension is same as the structure of the first extension, except for the existence of the non-collaborator (firm 3). Given the POA represented by θ ∈ [0, 1/2] between firms 1 and 2, at Stage 1 they decide whether or not to collaborate, and then all three firms compete against each other by choosing prices at Stage 2. Note, we find that if θ is large enough (in particular, if θ > 0.28), then firms 1 and 2 do not necessarily prefer maximum possible distinctiveness between varieties 1 and 2 of the products. In such a case, our modelling assumption, “The distance between varieties 1 and 2 is equal to 1/3 − z under their collaboration”, loses its rationale, and more algebraically involved analyses will be required. Hence, for an analytical simplicity, we conduct our analysis assuming that θ < 0.22.

**Proposition 10:** In the second extension, there exists a function z*(θ, φ) with the same properties as the ones presented in Proposition 9, except that z*(θ, φ) is now strictly increasing in θ for all θ ∈ [0, 0.28).

Proposition 10 says that the POA between firms 1 and 2 makes their collaboration more likely to occur in the presence of the non-collaborator (firm 3). As in the duopoly case analyzed above, the POA benefits firms 1 and 2 by weakening their competition which in turn induces them to charge higher prices. This benefit, however, is restricted by the competition between collaborators and the non-collaborator, because by charging higher prices firms 1 and 2 lose some of their customers to firm 3. This restriction becomes weaker

---

22Let Π_i(z, θ) denote each firm i’s (i = 1, 2) stage 2 profit in the equilibrium of the collaboration subgame. We find that there exists a value 0.28 such that Π_i(z, θ) (i = 1, 2) is strictly decreasing in z for all z ∈ (0, 1/3] if and only if θ < 0.28. If θ is large enough, then they do not necessarily prefer maximum possible distinctiveness between varieties 1 and 2 of the products. This results in the non-monotone relationship between Π_i(z, θ) (i = 1, 2) and z when θ > 0.28.
when firms 1 and 2 collaborate, because the collaboration weakens the competition between each firm \( i (i = 1, 2) \) and firm 3 by increasing their distance. Because of this effect, we find that the POA increases each firm \( i \)'s \( (i = 1, 2) \) profit more in the collaboration subgame than in the non-collaboration subgame. This implies that the disadvantage of collaboration, captured by \( \pi_{NC} - \pi_C \), is now decreasing in \( \theta \), where \( \pi_C (\pi_{NC}) \) denotes each firm \( i \)'s \( (i = 1, 2) \) second-period profit in the collaboration (non-collaboration) subgame. Hence, in contrast to the duopoly case, the POA makes the collaboration more likely to occur.

**Effects of partial ownership arrangements on the aggregate consumer surplus**

Next we consider how POAs affect consumers in these extensions. The standard result is that POAs between firms reduce the degree of their competition and hence negatively affect consumers. In contrast, we demonstrate that POAs could benefit consumers by inducing competitors to collaborate, given that the competitor collaboration increases the aggregate consumer surplus in our framework. In the first extension (duopoly setup), Proposition 9 tells us that if the firms do not collaborate in absence of POAs, then they do not collaborate under POAs either. Thus POAs can never increase the aggregate consumer surplus. However, in the second extension there is a possibility for POAs to benefit consumers, because POAs could induce firms 1 and 2 to collaborate as indicated by Proposition 10. This possibility is made precise in Proposition 11 below.

**Proposition 11:** In the second extension, the POA can increase the aggregate consumer surplus under a range of parameter values. To be precise, let \( CS(\theta) \) denote the aggregate consumer surplus in the equilibrium. There exist values \( \theta' \) and \( \theta'' \), \( 0 < \theta' \leq \theta'' \leq \bar{\theta} \), such that \( CS(\theta) > CS(0) \) holds if and only if \( \theta' \leq \theta < \theta'' \). There exists a value \( z' > z^*(0, \phi) \) such that \( \theta' < \theta'' \) holds if \( z^*(0, \phi) < z < z' \). Furthermore, \( CS(\theta) \) is strictly decreasing in \( \theta \) for all \( \theta \in [\theta', \theta''] \).

The logic here is as follows: Let us start from the circular city model without POAs. In the previous section, we found that the aggregate consumer surplus is higher in the collaboration subgame than in the non-collaboration subgame for all \( z \in [0, \frac{1}{3}) \) (see Proposition 8). If \( z < z^*(0, \phi) \), then POAs cannot increase the aggregate consumer surplus because firms 1 and 2 choose to collaborate even without POAs.

On the other hand, if \( z > z^*(0, \phi) \), then firms 1 and 2 do not collaborate without POAs because the collaboration intensifies their competition too much. In such a case, if the value
of $z$ is relatively close to $z^*(0, \phi)$, a POA can induce the firms to collaborate as indicated by Proposition 10. Proposition 11 tells us that, although POAs weaken the competition between firms 1 and 2, this negative impact on consumers can be more than offset by its positive impacts associated with the induced collaboration, where $\theta' \leq \theta < \theta''$ is the key condition. Here $\theta' \leq \theta$ guarantees that the level of the POA is high enough to induce firms 1 and 2 to collaborate. Given $\theta' \leq \theta$, the aggregate consumer surplus $CS(\theta)$ is decreasing in $\theta$ because the POA weakens the competition between firms 1 and 2, and $\theta < \theta''$ guarantees that the aggregate consumer surplus is higher under the POA.

In general, it may appear that POAs reduce consumer welfare by weakening the competition among firms. The validity of this intuition has been theoretically demonstrated in the literature under oligopoly models with homogeneous goods (see Section 2 for details). In contrast to this standard intuition, by incorporating POAs in the model of differentiated goods with an option for competitors to collaborate, we have demonstrated that POAs could enhance consumer welfare by inducing competitor collaboration. This yields a novel antitrust implication of competitor collaborations, which will be elaborated in the next section.

6 Antitrust Implications

6.1 Background

Collaborations among competitors have been recently recognized as crucial issues by antitrust authorities. In the United States, the Federal Trade Commission (FTC) and the U.S. Department of Justice (DOJ) (collectively, “the Agencies”) issued the new Antitrust Guidelines for Collaboration among Competitors in 2000, which intended to explain how the Agencies analyze certain antitrust issues raised by collaborations among competitors (see United States, 2000).

The guidelines define that a competitor collaboration comprises a set of agreements (other than merger agreements) between competitors to engage in business activities such as research and development, production, marketing, distribution, sales or purchasing. They recognize potential procompetitive benefits as well as potential antitrust harms of competitor collaborations, where the main issue is how consumers are affected by the collaborations. Concerning the potential benefits, the guidelines state, “The Agencies recognize that consumers may benefit from competitor collaborations in a variety of ways. For example, a
competitor collaboration may enable participants to offer goods or services that are cheaper, more valuable to consumers, or brought to market faster than could be possible absent the collaboration” (p. 6). On the other hand, competitor collaborations may harm competition and consumers because, “..., agreements (among competitors) may limit independent decision making or combine the control of or financial interests in production, key assets, or decisions regarding price, output, or other competitively sensitive variables, ... . Competitor collaborations also may facilitate explicit or tacit collusion through facilitating practices such as the exchange or disclosure of competitively sensitive information ... ” (the guidelines, p. 6).

Agreements among competitors of a type that always or almost always tends to raise price or to reduce output are challenged by the Agencies as per se illegal, while agreements not challenged as per se illegal are analyzed as the rule of reason to determine their overall competitive effect. The guidelines explain the analytical framework through which the Agencies determine the overall competitive effect, where, “The central question is whether the relevant agreement likely harms competition by increasing the ability or incentive profitably to raise price above or reduce output, quality, service, or innovation below what likely would prevail in the absence of the relevant agreement” (p. 4).

The European Commission issued Guidelines on the applicability of Article 81 of the EC Treaty to horizontal cooperation in 2001 (see European Commission, 2001), where horizontal cooperation here is analogous to competitor collaboration above. The guidelines stated, “In most instances, horizontal cooperation amounts to cooperation between competitors. It covers for example areas such as research and development (R&D), production, purchasing or commercialisation” (p. 1). The purpose and the logical structure of the guidelines are similar to those of the U.S. guidelines outlined above.  

6.2 Contributions

As pointed out above, the antitrust authorities have recognized that competitors often collaborate by sharing a part of their value-creating activities such as research and development, production, marketing, distribution, sales or purchasing. Note, importantly, that these activities are crucial elements for creating product distinctiveness across firms, and hence competitor collaborations often reduce the distinctiveness of their products, as elaborated

---

23See also Canada (1995) for the guidelines by the Canadian antitrust authorities.
in previous sections of this paper. However, the antitrust guidelines outlined above have not addressed the connection between competitor collaboration and product distinctiveness. Our analyses offer new perspectives on the antitrust implications of competitor collaborations by exploring this connection.

Our first contribution concerns ways in which competitor collaborations affect consumers. The antitrust guidelines state that a competitor collaboration may benefit consumers by enabling participants to offer goods or services at lower prices, or may harm consumers by facilitating participants to profitably raise prices. Our analysis offers a new perspective for antitrust authorities to investigate this issue by pointing out that competitor collaborations can affect consumers differently depending on their preferences on product characteristics. Under our “circular city” setup, we have demonstrated that lower prices charged by the collaborators do not mean that the competitor collaboration benefits all consumers, because it harms some consumers by inducing the non-collaborator to charge a higher price. This finding suggests the importance for antitrust authorities to investigate how competitor collaborations affect the behavior of firms that are not involved in the collaboration.

Our second contribution concerns the basic logic behind potential antitrust harms associated with competitor collaborations. According to the guidelines, their antitrust concern is that collaborations among competitors could facilitate them to behave in anticompetitive manner to raise prices or reduce outputs, quality, etc. According to the U.S. guidelines, this is possible because competitor collaborations may limit independent decision making on competitively sensitive variables or enable collusion through facilitating the exchange of competitively sensitive information.\textsuperscript{24} The logic here is that, although competitor collaborations themselves do not harm consumers, they could be a driving force of anticompetitive behavior such as collusion that harms consumers. This logic suggests that partial ownership arrangements (POAs), which are often accompanied by competitor collaborations, be a potential anticompetitive harm arising from competitor collaborations, given that POAs tend to reduce the degree of competition in the market. In fact, the U.S. guidelines list “participants’ financial interests in each other”, which is POAs in our terminology, as a potential antitrust harm arising from competitor collaborations.

In contrast, our analysis indicates a different possibility by reversing the direction of the

\textsuperscript{24}Similarly, the European Commission’s guidelines point out that some types of competitor collaborations may cause a certain degree of commonality in costs, which may facilitate relevant firms to coordinate market prices and output.
logic. In the previous section, we have demonstrated that POAs could benefit consumers by inducing competitors to collaborate, given that the competitor collaboration increases the aggregate consumer surplus in our framework. That is, in contrast to the U.S. guidelines, our analysis suggests that POAs can be a driving force (rather than an antitrust consequence) of competitor collaborations, which could benefit consumers. Note, this connection is not captured by the European Commission’s guidelines as well; they state that the guidelines are only concerned with those types of cooperation which potentially generate efficiency gains (such as agreements on R&D, production, purchasing, commercialization, standardization, and environmental agreements), and that other types of horizontal agreements such as minority shareholdings are to be addressed separately.

7 Summary and Conclusion

Competitors often collaborate by sharing a part of their value-creating activities (technology development, product design, manufacturing, marketing, distribution, and services) to save their fixed costs. Given that the value-creating activities are important elements for creating product distinctiveness across firms, a crucial disadvantage of competitor collaboration for collaborators is the reduction in their product distinctiveness. This paper has explored economic consequences of the trade-off between product distinctiveness and fixed-cost savings by analyzing models that incorporate competitor collaboration into the location framework.

In our framework, collaboration between competitors reduces the product distinctiveness between themselves, while it increases the distinctiveness between their products and the non-collaborator’s product (the product-characteristics effect). At the same time, intensified competition between collaborators lowers their prices and imposes downward pressure on non-collaborator’s pricing strategy (the price effect). Under the interaction between the price effect and the product-characteristic effect, we have demonstrated that the competitor collaboration benefits some consumers who are located relatively close to the collaborating firms while it harms the other consumers who are located far from them, and that it increases the aggregate consumer surplus. Furthermore, in extensions of the models that incorporate partial ownership arrangements (POAs), we have found that a POA can increase the aggregate consumer surplus by inducing the competitors to collaborate. This finding is in a sharp contrast to the standard intuition that POAs reduce consumer welfare by weakening the competition among firms, and has yielded a novel antitrust implication of competitor
Antitrust consequences of competitor collaborations have recently attracted substantial attention from antitrust authorities. The present paper has made two contributions in this regard. First, we have pointed out that competitor collaborations may affect consumers in more complex manner than the antitrust guidelines indicate, by demonstrating that competitor collaborations can affect consumers differently depending on their preferences on product characteristics. Lower prices charged by the collaborators do not mean that all consumers benefit from the competitor collaboration in our framework, because it harms some consumers by inducing the non-collaborator to charge a higher price. Second, our framework has suggested a logic behind antitrust implications of competitor collaborations that is quite different from the one indicated by antitrust authorities in their guidelines. In particular, the extensions of our models that incorporate POAs have suggested that POAs can be a driving force, rather than an antitrust consequence, of competitor collaborations, which could benefit consumers.
8 Appendix

Proof of Proposition 1: For any \((x_1, x_2), x_i \in [0,1], i \in \{1,2\}\), let \(\bar{p}_i(x_1, x_2)\) denote each firm \(i\)'s price in the unique Nash equilibrium. Following Neven (1985) we find that \(\bar{p}_1(x_1, x_2) = \frac{2(x_2-x_1)+(x_2^2-x_1^2)}{x_3}\) and \(\bar{p}_2(x_1, x_2) = \frac{4(x_2-x_1)-(x_2^2-x_1^2)}{x_3}\). Substituting \(x_1 = \frac{z}{2}, x_2 = 1 - \frac{z}{2}\) (where \(z \in [0,1]\)) yields \(\bar{p}_1(\frac{z}{2}, 1 - \frac{z}{2}) = 1 - z\). However, by definition, \(\bar{p}_i(\frac{z}{2}, 1 - \frac{z}{2}) \equiv p_i(z)\). Thus we have \(p_1(z) = p_2(z) = 1 - z \equiv p(z)\) for all \(z \in [0,1]\). Symmetric locations, together with identical equilibrium prices, imply that consumers in \([0, \frac{1}{2}]\) buy from firm 1 while those in \((\frac{1}{2}, 1]\) buy from firm 2. This in turn gives \(\pi_1(z) = \pi_2(z) = \frac{1-z}{2} \equiv \pi(z)\) for all \(z \in [0,1]\). The result then follows from noting that \(p'(z) = -1 < 0\) and \(\pi'(z) = -\frac{1}{2} < 0\). Q.E.D.

Proof of Proposition 2: Firms 1 and 2 choose to collaborate at stage 1 if and only if the following holds: \(\pi(0) - \pi(z) \leq \phi F\) or equivalently \(z \leq 2\phi F \equiv z^*(\phi)\). Since \(\phi \in (0, \frac{1}{2}]\), \(F \in (0, \frac{1}{2}]\), we have \(z^*(\phi) < (0,1)\). Furthermore, \(z^*(\phi) = 2F > 0\) which implies the result. Q.E.D.

Proof of Proposition 3: For all \(y \in [0, \frac{1}{2}]\) we have \(CS_{NC}(y) = A - p(0) - y^2 = A - 1 - y^2\), \(CS_C(y, z) = A - p(z) - y^2 = A - (1-z) - (y - \frac{z}{2})^2\) and accordingly \(CS_C(y, z) - CS_{NC}(y) = z + \frac{z}{2}(2y - \frac{z}{2})\). Since \(CS_C(y, z) - CS_{NC}(y)\) is strictly increasing in \(y\) and \(CS_C(0, z) - CS_{NC}(0) = \frac{3z}{4} > 0\) for all \(z \in (0,1]\), we have \(CS_C(y, z) > CS_{NC}(y)\) for all \(y \in [0, \frac{1}{2}]\). The analysis for consumers located in \((\frac{1}{2}, 1]\) is analogous. Q.E.D.

Proof of Corollary 1: The proof of the corollary follows immediately from Proposition 3 and the discussion preceding the corollary.

Proof of Proposition 4: We have that for \(i = 1, 2\), \(p_i'(z) = -\frac{(20+270z-297z^2+108z^3)}{12(5-6z)^3}\) and \(\pi_i'(z) = -\frac{(10-3z)(20+330z-594z^2+513z^3-162z^4)}{96(5-6z)^4}\), where all the bracketed expressions are strictly positive for all \(z \in [0, \frac{1}{3}]\). This implies the result. Q.E.D.

Proof of Proposition 5: Define \(h(z, \phi) \equiv \pi_i(0) - \pi_i(z) - \phi F\), \(i = 1, 2\) where \(z \in [0, \frac{1}{3}]\) and \(\phi \in (0, \frac{1}{2}]\). Firms 1 and 2 collaborate if and only if \(h(z, \phi) \leq 0\) holds, i.e. \(\pi_i(0) - \pi_i(z) \leq \phi F\) holds for \(i = 1, 2\). We have (i) \(h(0, \phi) = -\phi F < 0\), (ii) \(h(\frac{1}{3}, \phi) = \frac{1}{2} - \phi F > 0\) (since \(\phi \leq \frac{1}{2}\)) and \(F < \frac{1}{2}\) and (iii) \(\frac{dh}{dz} = -\pi_i'(z) > 0\). Together (i) - (iii) imply that for each \(\phi \in (0, \frac{1}{2}]\), there exists a unique \(z^*(\phi) \in (0, \frac{1}{3})\) such that \(h(z^*(\phi), \phi) = 0\) and \(h(z, \phi) \leq 0\) if and only if \(z \leq z^*(\phi)\). Finally we have \(z^*(\phi) = \frac{\pi_i'(z)}{\frac{1}{3}} = \frac{F}{\pi_i'(\phi)} > 0\) where \(i = 1, 2\). Q.E.D.
Proof of Proposition 6: (i) We have that \( p_3'(z) = \frac{(4-3z)(20-75z-36z^2)}{24(5-6z)^2} \). Since \( 4 - 3z > 0 \) for all \( z \in [0, \frac{1}{3}] \) we have \( p_3'(z) > 0 \) for all \( z \in [0, z'] \), \( p_3'(z) < 0 \) for all \( z \in (z', \frac{1}{3}) \) and \( p_3'(z') = 0 \), where \( z' \approx 0.239 \) is the unique value of \( z \in [0, \frac{1}{3}] \) satisfying \( 20 - 75z - 36z^2 = 0 \). This in turn implies, given \( p_3(\frac{1}{3}) = \frac{1}{6} > \frac{1}{7} = p_3''(z) \), that \( p_3(z) > p_3''(z) \) holds for all \( z \in (0, \frac{1}{3}) \).

(ii) We have that \( \pi_3'(z) = \frac{(20-30z-9z^2)(54z^3+9z^2-150z+20)}{24(5-6z)^3} \), where \( \frac{20-30z-9z^2}{24(5-6z)^3} > 0 \) for all \( z \in [0, \frac{1}{3}] \).

We then find that \( \pi_3'(z) > 0 \) for all \( z \in [0, \bar{z}] \), \( \pi_3'(z) < 0 \) for all \( z \in (\bar{z}, \frac{1}{3}] \), and \( \pi_3'(\bar{z}) = 0 \), where \( \bar{z} \approx 0.135 \) is the unique value of \( z \in [0, \frac{1}{3}] \) satisfying \( 54z^3 + 9z^2 - 150z + 20 = 0 \).

Hence there exists a unique threshold value \( \bar{z} \in (0, \frac{1}{3}) \) such that \( \pi_3(z) \) is strictly increasing in \( z \) for \( z < \bar{z} \) and strictly decreasing in \( z \) for \( z > \bar{z} \). This in turn implies, given \( \pi_3(\frac{1}{3}) = 0.03125 < \pi_3(\bar{z}) = 0.0370 < \pi_3(z) \approx 0.0395 \), that there exists a unique \( \tilde{z} \in (\bar{z}, \frac{1}{3}) \) such that \( \pi_3(z) > \pi_3(\tilde{z}) \) for all \( z < \tilde{z} \), and \( \pi_3(z) < \pi_3(\bar{z}) \) for all \( z > \tilde{z} \). Setting \( \pi_3(z) = \pi_3(\tilde{z}) \) yields \( \tilde{z} \approx 0.25 \). By definition of \( z^*(\phi) \) we have \( \pi_3(z^*(\phi)) = \frac{1}{27} - \phi F \), and \( F < \frac{1}{2} \) imply that \( \pi_3(z^*) < \frac{1}{2} \) \( \Leftrightarrow \) \( z^* < 0.247 \). Hence \( z^* \leq \tilde{z} \), which implies \( \pi_3(z) > \pi_3(\tilde{z}) \) for all \( z \in [0, z^*] \). Q.E.D

Proof of Lemma 1: In the equilibrium of the collaboration subgame, let \( x_i(z) \in [0, 1) \) denote the location of firm \( i \) and let \( y_{ij}(z) \in [0, 1) \) denote the location of the consumer indifferent between purchasing from firms \( i \) and \( j \). Define \( x_i^0 \) and \( y_{ij}^0 \) analogously for the non-collaboration subgame. Recall that each consumer is indexed by her location \( y \in [0, 1) \).

Without loss of generality, let \( x_3^0 = 0 \) in what follows. We then have that \( (x_1^0, x_2^0, x_3^0) = (\frac{1}{3}, \frac{2}{3}, 0), (x_1(z), x_2(z), x_3(z)) = (\frac{1}{3} + \frac{y}{3} - \frac{z}{3}, 0) \) and \( M_{12} = \frac{1}{2} \). Given this, (2) and (5) presented in the text imply that \( (y_{31}^0, y_{12}^0, y_{23}^0) = (\frac{1}{6}, \frac{1}{2}, \frac{5}{6}) \) and \( (y_{31}(z), y_{12}(z), y_{23}(z)) = (\frac{20-30z-9z^2}{24(5-6z)^2} - \frac{100-114z+9z^2}{24(5-6z)^2}) \).

Then the following two conditions hold for all \( z \in (0, \frac{1}{3}) \):
\[ 0 = x_3^0 = x_3(z) < y_{31}(z) < y_{31}^0 < x_1^0 < x_1(z) < M_{12} = y_{12}^0 = y_{12}(z) = \frac{1}{2}, \] (8)
\[ M_{12} = y_{12}^0 = y_{12}(z) = \frac{1}{2} < x_2(z) < x_2^0 < y_{23}^0 < y_{23}(z) < x_3^0 = 1. \] (9)

First consider consumers located in \( [x_3^0, M_{12}] \) (\( = [0, \frac{1}{3}] \)). Given (9) above, a consumer \( y \in [x_3^0, M_{12}] \) purchases from firm 1 in the equilibrium of the collaboration subgame as well as the non-collaboration subgame if and only if \( y \in [y_{31}^0, M_{12}] \). Then a consumer \( y \in [x_3^0, M_{12}] \) is a Group \( \alpha \) consumer if and only if \( y \in [y_{31}^0, M_{12}] \) and \( |x_1(z) - y| < |x_1^0 - y| \) hold, where the latter condition is equivalent to \( |M_{12} - y| < \frac{2 - 3z}{12} \equiv d_\alpha(z) \). Also, given \( y_{31}(z) < y_{31}^0 \), consumer \( y \in [x_3^0, M_{12}] \) is a Group \( \beta \) consumer if and only if \( y < y_{31}(z) \), which is equivalent to \( |y - x_3^0| < \frac{20 - 30z - 9z^2}{24(5 - 6z)^2} \equiv d_\beta(z) \). Given the symmetry of the equilibrium locations, the
Proof of Proposition 7: Given the symmetry of the equilibrium locations, we focus on consumers located in \([y_{31}(z), y_{32}(z)]\). For all Group \(\alpha\) consumers we have \(CS_C(y, z) - CS_{NC}(y) = p_0^i - p_1(z) + (x_1(z) - x_3)(2y - x_3 - x_1(z)), \) and for \(y \in (y_{31}(z), y_{32}(z)]\) we have \(CS_C(y, z) - CS_{NC}(y) = p_0^i - p_1(z) + |y - x_1(z) - x_3(2y - x_3 - x_1(z))|\). Also, \(CS_C(y, z) - CS_{NC}(y)\) is continuous at \(y_{31}(z)\) by the definition of \(y_{31}(z)\). Then, given \(x_1(z) > x_3\) and \(x_1(z) > x_3\), we find that \(CS_C(y, z) - CS_{NC}(y)\) is continuous and strictly increasing in \(y\) for all \(y \in [y_{31}(z), \frac{x_0^i + x_1(z)}{2}]\). Furthermore, we have that \(CS_C(y_{31}(z), z) - CS_{NC}(y_{31}(z)) = p_0^i - p_3(z) < 0\) and \(CS_C(\frac{x_0^i + x_1(z)}{2}, z) - CS_{NC}(\frac{x_0^i + x_1(z)}{2}) = p_1^i - p_1(z) > 0\). Then the intermediate value theorem implies Proposition 7(iii). \(Q.E.D\)

Proof of Proposition 8: Let \(Y_i^C(z) (Y_i^NC(z))\) denote the set of consumers who buy from firm \(i\) in the equilibrium of the collaboration subgame (non-collaboration subgame). We have 
\[
\overline{CS}_C(z) - \overline{CS}_{NC} \equiv \left[\sum_{i=1}^{3} \int_{y \in Y_i^C(z)} (y - x_i(z))^2 dy - \int_{y \in Y_i^NC} (y - x_i)^2 dy\right] - \left[\sum_{i=1}^{3} \int_{y \in Y_i^C(z)} (y - x_i(z))^2 dy - \int_{y \in Y_i^NC} (y - x_i)^2 dy\right] = \left[\sum_{i=1}^{3} \int_{y \in Y_i^C(z)} (y - x_i(z))^2 dy - \sum_{i=1}^{3} \int_{y \in Y_i^NC} (y - x_i)^2 dy\right].
\]
Using (5) and (6) we find \(\sum_{i=1}^{3} \int_{y \in Y_i^C(z)} (y - x_i(z))^2 dy = \frac{3x^2(112 - 369x^2 - 9x^2)}{32(5 - 6z)^2}\). Recall that we have \((x_1, x_2, x_3) = (\frac{1}{5}, \frac{3}{3}, 0), (x_1(z), x_2(z), x_3(z))\) \(= (\frac{1}{3} + \frac{2}{3} - \frac{5}{3}, 0), (y_{31}(z), y_{12}(z), y_{23}(z)) = (\frac{1}{3}, \frac{5}{3}, 0)\) and \((y_{31}(z), y_{12}(z), y_{23}(z)) = (\frac{20 - 30 - 9z}{24(5 - 6z)}, 2 - 100 - 114 - 9z^2)\). We then find that \(CS^C(z) - CS^NC = \frac{z^2(996 + 580z + 336z^2 - 63z^3)}{64(5 - 6z)^2}\), which is strictly positive for all \(z \in [0, \frac{1}{3}]\). \(Q.E.D.\)

Proof of Proposition 9: Let \(p_i(z, \theta)\) and \(\Pi_i(z, \theta)(i = 1, 2)\) denote each firm \(i\)'s equilibrium price and profit respectively in the collaboration subgame. Through an analysis analogous to the one in Neven (1985) and the proof of Claim 1 we find that \(p_1(z, \theta) = p_2(z) = \frac{(1 - \theta)(1 - z)}{1 - 2\theta} \equiv \frac{(1 - \theta)(1 - z)}{1 - 2\theta}\).
\(p(z)\) and \(\Pi_1(z) = \Pi_2(z) = \frac{(1-\theta)(1-z)}{2(1-2\theta)} \equiv \pi(z)\) for all \(z \in [0, 1]\), where \(\theta < \tilde{\theta} \equiv \frac{4A-5}{8A-6}\).25

Firms 1 and 2 choose to collaborate at stage 1 if and only if the following holds: \(\Pi(0, \theta) - \Pi(z, \theta) \leq \phi F\) or equivalently \(z \leq \frac{4\phi F(1-\theta)}{1-\theta} \equiv z^*(\theta, \phi)\). We have \(\phi \in (0, \frac{1}{2})\), \(F \in (0, \frac{1}{2})\) and \(\frac{1-\theta}{1-\phi} > 0\) for all \(\theta < \frac{1}{2}\), which imply \(z^*(\theta, \phi) \in (0, 1)\). Furthermore, given that \(\theta < \frac{1}{2}\) we have \(\frac{\partial z^*}{\partial \phi} = \frac{4F(1-\theta)}{1-\theta} > 0\). Q.E.D.

**Proof of Proposition 10:** Let \(z \in [0, \frac{1}{4})\) (which determines \(d \equiv (d_{12}, d_{23}, d_{31})\)) and \(\theta \in [0, \tilde{\theta})\) be given, and consider \(p \equiv (p_1, p_2, p_3)\) such that each firm \(i\) sells its product to consumers located in both sides of \(x_i\). Analogous to (4), we find the following: (i) Given \(p_3\), firm \(i\)'s (\(i = 1, 2\)) profit (which is \((1-\theta)p_iq_i + \theta p_jq_j\), \(i, j = 1, 2, i \neq j\)) is maximized when \(p_i = \frac{d_{ij}d_{ji}}{2} + \frac{d_{ij}p_{ij}}{2d_{ji}}\) \((i, j = 1, 2, i \neq j)\) and (ii) Given \(p_1\) and \(p_2\), firm 3's profit (which is \(p_3q_3\)) is maximized when \(p_3 = \frac{d_{23}d_{32}}{2} + \frac{d_{23}p_{31}+p_{32}d_{21}}{2(d_{23}+d_{32})}\). Through an analysis analogous to the one found in Economides (1989), this system of equations determines the unique equilibrium of the price subgame, where equilibrium prices and profits of firm \(i\), denoted by \(p_i(z, \theta)\) and \(\Pi_i(z, \theta)\) respectively are given by

\[
p_i(z, \theta) \equiv \frac{(1-\theta)(\frac{3}{8} + z)(\frac{1}{8} - z)(\frac{10}{7} - z)}{8(\frac{7}{6} - z - \theta(\frac{7}{6} - \frac{5}{8}))},
\]

\[
\Pi_i(z, \theta) \equiv \frac{(1-\theta)(1-\frac{3z}{7} - 6\theta)(\frac{3}{7} + z)(\frac{1}{8} - z)(\frac{10}{7} - z)^2}{96(\frac{7}{6} - z - \theta(\frac{7}{6} - \frac{5}{8}))^2},
\]

for \(i = 1, 2\), and for firm 3, we have \(p_3(z, \theta) = \frac{5(\frac{3}{4} + z)(1 - \frac{3}{4} - \frac{3z}{7} - \theta(\frac{7}{6} - \frac{5}{8}))}{36(\frac{7}{6} - z - \theta(\frac{7}{6} - \frac{5}{8}))^2}\) and \(\Pi_3(z, \theta) = \frac{25(\frac{3}{4} + z)(1 - \frac{3}{4} - \frac{3z}{7} - \theta(\frac{7}{6} - \frac{5}{8}))^2}{64(\frac{7}{6} - z - \theta(\frac{7}{6} - \frac{5}{8}))^2}\).

We now establish the following claim:

**Claim 1:** For any given \(\theta \in [0, \tilde{\theta})\) \((\tilde{\theta} \equiv 0.275)\), \(\Pi_i(z, \theta) \ (i = 1, 2)\) is strictly decreasing in \(z\) for all \(z \in [0, \frac{1}{3})\). Also, \(\Pi_1(0, \theta) - \Pi_1(z, \theta)\) is (i) continuous in \(\theta\) for all \(\theta \in [0, \tilde{\theta})\) and \(z \in [0, \frac{1}{3})\) (ii) strictly decreasing in \(\theta\) for all \(\theta \in [0, \tilde{\theta})\) and \(z \in [0, z')\) where \(z' \approx 0.272\).26

**Proof:** We have that \(\frac{\partial \Pi_i(z, \theta)}{\partial z} = \frac{1}{96(5-6z-7\theta+3\theta z)^2}R_1(z, \theta)\), where \(R_1(z, \theta) \equiv \left(\frac{116 + 774z - 486z^2 + 108z^3}{5-6z-7\theta+3\theta z}\right)^2\), \(\frac{1}{(104 + 984z - 1188z^2 + 675z^3 - 81z^4)}\theta^2 - (\frac{1}{(20 + 330z - 594z^2 + 513z^3 - 162z^4})\) is quadratic in \(\theta\) and strictly positive for all \(z \in [0, \frac{1}{3})\) and \(\theta \in [0, \tilde{\theta})\). In addition, since \(1 - \theta > 0, 10 - 3z > 0\) and \(5 - 6z - 7\theta + 3\theta z > 0\), we have \(\frac{\partial \Pi_i(z, \theta)}{\partial z} < 0\) for all \(z \in [0, \frac{1}{3})\) and \(\theta \in [0, \tilde{\theta})\).

---

25Note that for all \(A > \frac{5}{4}\), \(\frac{4A-5}{8A-6} \in (0, \frac{1}{2})\) holds.

26For \(z \in [z', \frac{1}{3})\) the sign of \(\frac{\partial \Pi_1(0, \theta) - \Pi_1(z, \theta)}{\partial \theta}\) becomes ambiguous.
strictly decreasing in $z$ that firms 1 and 2 collaborate in the equilibrium if and only if

$\theta$

Proof: Let $Y_i^C(z, \theta)$ denote the set of consumers who buy from firm $i$ ($i = 1, 2, 3$) in the equilibrium of the collaboration subgame. We have $\bar{C}S_C(z, \theta) = A - \sum_{i=1}^{3} p_i(z, \theta) q_i(z, \theta) - \sum_{i=1}^{3} \int_{y \in Y_i^C(z, \theta)} (y - x_i(z))^2 dy$, where $p_i(z, \theta)$ and $q_i(z, \theta) = \frac{\pi_i(z, \theta)}{p_i(z, \theta)}$ are obtained using (10) and

$27$The proof is available on request.

Observe that (see (11)) $\Pi_i(0, \theta)$ and $\Pi_i(z, \theta)$ are continuous in $\theta$ for all $\theta \in [0, \bar{\theta})$ and $z \in [0, \frac{1}{3})$, which in turn imply that $\Pi_i(0, \theta) - \Pi_i(z, \theta)$ is continuous in $\theta$. We also have that

$\frac{\partial \Pi_i(0, \theta) - \Pi_i(z, \theta)}{\partial \theta} = \frac{z R_2(z, \theta)}{96(5 - 7 \theta)^5(5 - 6 \theta - 7 \theta + 3 \theta^2)}$, where $R_2(z, \theta)$ is a polynomial in $z$ and $\theta$. For all $z \in [0, \frac{1}{3})$ and $\theta \in [0, \bar{\theta})$ we have that $5 - 7 \theta > 0$, $5 - 6 \theta - 7 \theta + 3 \theta^2 > 0$, and using mathematical software MAPLE we prove that $R_2(z, \theta) > 0$ for all $\theta \in [0, \bar{\theta})$ and $z \in [0, z')$ where $z' \approx 0.272$. This implies the result. Q.E.D.

Firms 1 and 2 collaborate in the equilibrium if and only if $\Pi_i(0, \theta) - \Pi_i(z, \theta) \leq \phi F$ ($i = 1, 2$). We have that $\Pi_i(0, \theta) - \Pi_i(z, \theta) = 0 < \phi F$ when $z = 0$ and $\Pi_i(0, \theta) - \Pi_i(z, \theta) > \phi F$ for all $z > z''$ where $z'' \approx 0.26$. Then, since $z'' \approx 0.26 < z' \approx 0.272$ and $\Pi_i(0, \theta) - \Pi_i(z, \theta)$ is strictly decreasing in $\theta$ for all $\theta \in [0, \bar{\theta})$ and $z \in [0, z')$ (see Claim 1) this implies that there exists a unique value $z'(\theta, \phi) \in (0, z'')$ such that $\Pi_i(0, \theta) - \Pi_i(z, \theta) = \phi F$ if $z = z'(\theta, \phi)$ and that firms 1 and 2 collaborate in the equilibrium if and only if $z \leq z'(\theta, \phi)$.

Recall that $F < \frac{1}{27}$ and for collaboration to occur $\Pi_i(0, \theta) - \Pi_i(z, \theta) \leq \phi F$ or equivalently $F > \frac{\Pi_i(0, \theta) - \Pi_i(z, \theta)}{\phi}$ must hold, where $\phi \leq \frac{1}{27}$. The range $\left[\frac{\Pi_i(0, \theta) - \Pi_i(z, \theta)}{\phi}, \frac{1}{27}\right]$ is non-empty for some $\phi \in (0, \frac{1}{27})$ only if $z \geq z''$ where $z'' \approx 0.259$ which imply $z'(\theta, \phi) \leq z''$. Now, since $z'' < z'$ (where $z'$ is as defined in Claim 1), Claim 1 implies that $\Pi_i(0, \theta) - \Pi_i(z, \theta)$ is strictly decreasing in $\theta$ for all $\theta \in [0, \bar{\theta})$ and $z \in [0, z')$. Thus $\Pi_i(0, \theta) - \Pi_i(z(\theta, \phi), \theta) = \phi F \Rightarrow \Pi_i(0, \theta) - \Pi_i(z(\theta, \phi), \theta) \leq \phi F$ for all $z \leq z'(\theta, \phi)$. This implies that firms 1 and 2 collaborate in equilibrium if and only if $z \leq z'(\theta, \phi)$. The Implicit Function Theorem implies that $z'(\theta, \phi)$ is a continuously differentiable function of $\theta$ for all $\theta \in [0, \bar{\theta})$, and that $\frac{dz'(\theta, \phi)}{d\theta} > 0$ for all $\theta \in [0, \bar{\theta})$ (given $\frac{\partial \Pi_i(z, \theta)}{dz} < 0$ and $\frac{\partial \Pi_i(0, \theta) - \Pi_i(z, \theta)}{d\theta} < 0$). Q.E.D.

Proof of Proposition 11: For any given $z \in [0, \frac{1}{3})$ and $\theta \in [0, \bar{\theta})$, let $CS_C(y, z, \theta)$ and $CS_{NC}(y, \theta)$ denote the consumer surplus of consumer $y \in [0, 1)$ in the equilibrium of the collaboration subgame and the non-collaboration subgame respectively. Define $\bar{C}S_C(z, \theta) \equiv \int_0^1 CS_C(y, z, \theta) dy$ and $\bar{C}S_{NC}(\theta) \equiv \int_0^1 CS_{NC}(y, \theta) dy$. Note that $\bar{C}S_{NC}(\theta) \equiv \bar{C}S_C(0, \theta)$.

Claim 2: $\bar{C}S_C(z, \theta)$ is continuous in $z$ and $\theta$ for all $z \in [0, \frac{1}{3})$ and $\theta \in [0, \bar{\theta})$. Furthermore, for any given $z \in [0, \frac{1}{3})$, $\bar{C}S_C(z, \theta)$ is strictly decreasing in $\theta$ for all $\theta \in [0, \bar{\theta})$.

Proof: Let $Y_i^C(z, \theta)$ denote the set of consumers who buy from firm $i$ ($i = 1, 2, 3$) in the equilibrium of the collaboration subgame. We have $\bar{C}S_C(z, \theta) = A - \sum_{i=1}^{3} p_i(z, \theta) q_i(z, \theta) - \sum_{i=1}^{3} \int_{y \in Y_i^C(z, \theta)} (y - x_i(z))^2 dy$, where $p_i(z, \theta)$ and $q_i(z, \theta) = \frac{\pi_i(z, \theta)}{p_i(z, \theta)}$ are obtained using (10) and
(11). Through a procedure similar to the one presented in the proof of Proposition 8, we find that \( \overline{CS}_C(z, \theta) = A - \frac{R_3(z, \theta)}{R_4(z, \theta)} \), where 
\[
R_3(z, \theta) = (7920 - 9936z - 4536z^2 + 2916z^3)\theta^2 - (12960 + 23328z + 10368z^2 - 3888z^3 - 2916z^4) + (5200 - 12480z - 26642z^2 + 6372z^3 + 5994z^4 - 243z^5) \\
R_4(z, \theta) = 1728(5 - 6z - \theta(7 - 3z))^2(\not{0} \text{ for all } z \in [0, \frac{1}{3}) \text{ and } \theta \in [0, \theta)).
\]
Hence \( \overline{CS}_C(z, \theta) \) is continuous in \( z \) and \( \theta \) for all \( z \in [0, \frac{1}{3}) \) and \( \theta \in [0, \theta) \). Also, we have that 
\[
\frac{\partial \overline{CS}_C(z, \theta)}{\partial \theta} = \frac{(1-3z)(10-3z)(2+3z)(100-114z+9z^2+\theta(54z-144))}{864(5-6z-\theta(7-3z))^3},
\]
where the numerator and the denominator are strictly positive for all \( z \in [0, \frac{1}{3}) \) and \( \theta \in [0, \theta) \) which implies \( \frac{\partial \overline{CS}_C(z, \theta)}{\partial \theta} < 0 \). Q.E.D

By the definition of \( CS(\theta) \), we have that \( CS(\theta) = \overline{CS}_C(z, \theta) \) if \( z \leq z^*(\theta, \phi) \) and \( CS(\theta) = \overline{CS}_{NC}(\theta) = \overline{CS}_C(0, \theta) \) if \( z > z^*(\theta, \phi) \). Below we establish Claims 3 and 4.

**Claim 3:** Suppose that there exists a value \( \hat{\theta} \in (0, \theta) \) such that \( CS(\hat{\theta}) > CS(0) \). Then there exist values \( \hat{\theta}' \) and \( \hat{\theta}'' \), \( 0 < \hat{\theta}' < \hat{\theta} < \hat{\theta}'' \), such that \( CS(\theta) > CS(0) \) holds if and only if \( \theta \in [\hat{\theta}', \hat{\theta}''] \).

**Proof:** Suppose \( z \leq z^*(0, \phi) \), which implies \( z < z^*(\theta, \phi) \). We then have that \( CS(\hat{\theta}) = \overline{CS}_C(z, \hat{\theta}) < \overline{CS}_C(z, 0) = CS(0) \), where the inequality is implied by Claim 2. Now suppose \( z > z^*(\theta, \phi) \), which implies \( z > z^*(0, \phi) \). We then have that \( CS(\hat{\theta}) = \overline{CS}_{NC}(\hat{\theta}) < \overline{CS}_{NC}(0) = CS(0) \), where the inequality is implied by Claim 2. Hence \( z^*(0, \phi) < z \leq z^*(\theta, \phi) \) must hold to have \( CS(\hat{\theta}) > CS(0) \).

Pick any \( z \in (z^*(0, \phi), z^*(\theta, \phi)) \]. Recall that we have that \( z^*(\theta, \phi) \) is continuous and strictly increasing in \( \theta \) for all \( \theta \in [0, \hat{\theta}) \). Then there exists a unique value \( \hat{\theta}' \in (0, \hat{\theta}) \) such that \( z^*(\hat{\theta}', \phi) = z \). Given that \( z > z^*(\theta, \phi) \) for all \( \theta \in [0, \hat{\theta}') \), we have that \( CS(\theta) = \overline{CS}_{NC}(\theta) \leq \overline{CS}_{NC}(0) = CS(0) \) for all \( \theta \in [0, \hat{\theta}') \). Also, given that \( \overline{CS}_C(z, \theta) \) is strictly decreasing in \( \theta \), there exists a unique value \( \hat{\theta}'' \in (\hat{\theta}', \hat{\theta}) \) such that \( \overline{CS}_C(z, \theta) > CS(0) \) if and only if \( \theta \in [\hat{\theta}', \hat{\theta}''] \). This implies the desired result, given that \( CS(\theta) = \overline{CS}_C(z, \theta) \) for all \( \theta \in [\hat{\theta}', \hat{\theta}] \). Q.E.D

**Claim 4:** There exists a value \( z' \in (z^*(0, \phi), \frac{1}{3}) \) such that the following property holds: For any given \( z \in (z^*(0, \phi), z') \), there exists a value \( \hat{\theta} \in (0, \hat{\theta}) \) such that \( CS(\hat{\theta}) > CS(0) \).

**Proof:** We have that \( \overline{CS}_C(z^*(0, \phi), 0) > \overline{CS}_C(0, 0) (\equiv \overline{CS}_{NC}(0)) \) by Proposition 8. Given \( z^*(\theta, \phi) \) is continuous in \( \theta \), Claim 2 implies that \( \overline{CS}_C(z^*(\theta, \phi), \theta) \) is continuous in \( \theta \) for all \( \theta \in [0, \hat{\theta}) \). The continuity implies that there exists a value \( \hat{\theta} \in (0, \hat{\theta}) \) such that \( \overline{CS}_C(z^*(\theta, \phi), \theta) > \overline{CS}_C(0, 0) \) for all \( \theta \in [0, \hat{\theta}] \). Noting that \( z^*(0, \phi) < z^*(\hat{\theta}, \phi) \), pick any \( z \in (z^*(0, \phi), z^*(\hat{\theta}, \phi)) \).

Since \( z^*(\theta, \phi) \) is continuous and strictly decreasing in \( \theta \) for all \( \theta \in [0, \hat{\theta}) \), there exists a unique value \( \hat{\theta} \in (0, \hat{\theta}) \) such that \( z = z^*(\hat{\theta}, \phi) \). We have that \( CS(\hat{\theta}) = \overline{CS}_C(z^*(\hat{\theta}, \phi), \hat{\theta}) > \overline{CS}_C(0, 0) = CS(0) \). Given this, let \( z' = z^*(\hat{\theta}, \phi) \) and \( \hat{\theta} = \hat{\theta} \), which implies the desired result.
Q.E.D.

Finally, for any given $z \in [0, \frac{1}{3})$, let $\theta' = \tilde{\theta}$ and $\theta'' = \tilde{\theta}$ if there exists a value $\tilde{\theta} \in (0, \tilde{\theta})$ such that $CS(\tilde{\theta}) > CS(0)$, let $\theta' = \theta'' = \tilde{\theta}$ otherwise, and note that $CS(\theta) = \hat{CS}_C(z, \theta)$ for all $\theta \in [\theta', \theta'']$ by Claim 3. Then Claims 2-4 together imply the desired result. Q.E.D.
References


Figure 2
Figure 3
Figure 4