Debt and Equity*

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Abstract

Which is better for a small firm: debt financing or equity financing? A simple intertemporal model suggests that the two have very different potential incentive problems. With equity financing, the manager (an employee) may expend too little effort, while with debt financing, the manager (the owner) may keep the entire cash flow and default on the debt. Depending on the relative severity of these two incentive problems, either debt or equity may dominate the other. These problems are exacerbated by the possibility of an MBO and are reduced by sinking funds, stock or option compensation and non-vested pensions.

Comments are welcome

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1 Introduction

The conflict between a manager and outside financial claimants is a well-known source of economic inefficiency. In a proprietorship with outside debt, the conflict of interest may induce the owner-manager to take assets out of the firm or to choose too much risk, in either case collecting value at the expense of the bondholders. In a corporation with outside equity, an employee-manager may have the incentive to expend too little effort or to take projects that please the manager but do not add value for shareholders. It is our goal to explore what different incentive problems imply for the efficiency of different organizational forms. We explore these issues in a simple intertemporal model with a stark choice between a proprietorship raising money only through debt and a simple corporate form with outside equity and a fixed wage for the employee-manager. We find that the two forms have very different incentive problems, and either can be more efficient depending on the relative severity of the incentive problems. Allowing the possibility of an MBO tends to erode incentives in both forms, since the manager can “trash” the firm and buy it at a discount. Institutions such as employee share ownership, sinking funds and non-vested pensions can ameliorate the incentive problems both for the corporate form and for the proprietorship.

For the corporate form, the manager and equityholder both know the effort level. The effort is certainly known to the employee-manager who chose it, and the equityholder can infer effort from the output, because there is no uncertainty in our model. However, effort is assumed not to be publicly observable, which rules out the use of direct incentive contracts for the manager’s compensation. We can think of this as a result of the absence of accounting systems to record the output or the manager’s ability to divert the output for private benefits. In a simple firm, perhaps output is collected by the equityholder, like cash at the door of a night club or a crop planted and cared for by the manager but harvested by the equityholder. At the outset, the manager signs a wage contract that promises a fixed wage in each period. The equityholder is obligated to pay the fixed wage for as long as the manager is in charge of the project, but has full discretion to fire the manager at any time. Given that the manager’s wage does not depend on the output, the manager may have an incentive to expend no effort and just
collect the wage. Depending on the parameter values, the manager’s shirking may or may not be deterred by the threat of dismissal.

For the proprietorship, the manager is also the equityholder of the firm. In this case, the owner-manager has no incentive to shirk, but may have an incentive to take assets out of the firm and default on the debt. Depending on the parameter values, removal of assets followed by default may or may not be deterred by a threat to liquidate the firm when the interest payment is missed.

Whether the corporate form or the proprietorship is better depends on the relative severity of their incentive problems, which depends in turn on such determinants as the size of the initial investment, the future profitability of the project, how difficult effort is for the manager and the interest rate. For the corporate form, an increase in the employee-manager’s difficulty of effort or a decrease in the project’s future profitability increases the manager’s benefit from shirking, and makes the employee-manager’s incentive problem more severe. For the proprietorship, an increase in the interest rate or a decrease in the project’s future profitability increases the owner-manager’s benefit from defaulting, and therefore makes the owner-manager’s incentive problem more severe.

The possibility of an MBO may jeopardize funding of the project, because investors may be concerned the manager will expend too little effort and then purchase the firm at a discount. In other words, firing the manager or liquidating the project is not a punishment when an MBO is possible, because the manager can take over the firm and reinstate the project right after the dismissal or liquidation.

Other institutions can be used to mitigate the incentive problems and support the financing of the project. Debt financing can be supported by sinking funds. Equity financing can be supported by employee share ownership or managers’ non-vested pensions. The intuition behind using sinking funds to support debt financing is that sinking funds can be used to retire debt. If a large enough proportion of the debt is retired early enough, the payment of the remaining debt can be guaranteed by the liquidation value of the project. The intuition behind using employee share ownership to support equity financing is that compensating the manager in the company’s shares
can both transfer the ownership of the firm from the investor to the manager and eliminate the manager’s incentive to shirk. The intuition behind using managers’ non-vested pensions to support equity financing is that managers’ non-vested pensions can be held as collateral against the shirking. An important issue, which we have to resolve when using employee share ownership, is to find a feasible way to divide the firm’s output between the two parties (according to what they are entitled to receive) when the output itself is not publicly verifiable.

Our model has two important predictions on the two organizational forms. One prediction is that the corporate form is more likely to be used by non-effort-intensive firms and the proprietorship is more likely to be used by non-capital-intensive firms,\(^1\) whatever industries these firms may operate in. The other prediction is that it can be more difficult and tedious to arrange equity financing than to arrange debt financing, because of certain requirements that need to be satisfied in equity financing, such as the investor’s direct involvement in the operation of the firm. As a result, debt may be the preferred choice of the two when both can be used, and we may see debt being used more often than equity.

There is a large literature on the conflict between the manager and outside investors. Papers that are closely related to this paper are Townsend (1979), Diamond (1984), Gale and Hellwig (1985), Bolton and Scharfstein (1990), Hart and Moore (1998) and DeMarzo and Fishman (2000), which consider financing with outside debt, Myers (2000), which considers financing with outside equity, and Fluck (1998), which compares the two. With a few exceptions, most these papers consider financing of projects whose cash flows are not publicly observable.

Fluck (1998) compares debt with equity, and finds that both debt and equity can be used to finance projects with infinite life, and that equity (weakly) dominates debt. An interesting idea of the paper is that giving equityholders the right to fire the manager whenever they want to can be an effective way to support outside equity. While the ideas in Fluck (1998) are

\(^1\)Effort intensive firms and capital intensive firms here refer to respectively firms whose production requires a lot of managers’ effort and firms whose production requires a large initial investment.
interesting, the model is rather complicated and the analysis is hard to follow. The main result of the paper that equity dominates debt seems to be driven by an assumption on bankruptcy law, which does not allow the ownership of the project to be transferred to debtholders (even in bankruptcy). The (assumed) difference in debtholders’ and equityholders’ ability to replace the (current) manager (with a new manager) makes debt inferior. In this paper, we show that debt and equity induce different incentive problems, and that depending on the relative severity of the incentive problems, either debt or equity can dominate the other.

Our paper is related most closely to Myers (2000), a paper designed to illustrate Fluck’s ideas in a simple model. Myers (2000) considers equity financing in two organizational forms: partnership and corporation, and finds that while the partnership form can be used for both projects with infinite life and projects with finite life, the corporation form can be used only for projects with infinite life. It also finds that equity financing in both organizational forms requires co-investment by the manager. An important contribution of the paper is that it develops a simple non-stochastic model to study incentives. This paper compares another two different organizational forms, and finds that either organizational form can dominate the other. We further extend the results in Myers (2000) by showing that reputation effect, together with other institutions such as employee share ownership and non-vested pensions, is sufficient to support equity financing, and that co-investment by the manager is not necessary. An important difference between the equity contract in this paper and the one in Fluck (1998) and Myers (2000) is that equity contract gives the control over the firm’s output to equityholders in this paper but to the manager in the other two papers.

Bolton and Scharfstein (1990) and Hart and Moore (1998) consider financing of a project with finite life, and find that debt, but not equity, can be used to finance the project, and that inevitable early liquidation of the project can create certain inefficiency in debt financing. We show in this paper that with sinking funds, employee share ownership and non-vested pensions, both projects with infinite life and projects with finite life can be financed with both debt and equity, and that inefficient liquidation of the project can be avoided.

Townsend (1979), Diamond (1984) and Gale and Hellwig (1985) consider
financing of a one-period project whose cash flow can be verified at a cost, and show that outside debt is the optimal contract. DeMarzo and Fishman (2000) consider financing of a project whose cash flows are nonobservable even to investors, and also find that debt is the optimal contract.

The recent literature has made clear that the role of various contracts is much different in intertemporal settings than in the textbook one-period models. We choose to work with an intertemporal model because such a model makes it easier for us to study the incentives under reputation effect than a multi-period model. Our model is very similar to that in Myers (2000), which has a clever way of looking at incentives without any randomness. This means we cannot look at the asset substitution problem (i.e., the problem associated with the perverse incentive to substitute into a higher risk project to transfer value from bondholders to shareholders) that is often analyzed in single-period models, but we can still look at an interesting set of incentive issues in a tractable intertemporal model. An important difference between our model and the one in Myers (2000) is that the output depends on the manager’s effort in our model, but not in Myers’. The model in this paper is designed to highlight the differences between the two organizational forms and to keep our analysis reasonably simple. In doing so, we rule out the existence of other factors that can affect debt and equity, such as taxes.

The rest of the paper is organized as follows. Section 2 introduces the model in the paper, and considers equity financing of the project. Section 3 considers debt financing of the project. Section 4 compares debt with equity. Our analysis in Section 2 through Section 4 is based on the assumptions that the threat of dismissal and the threat of liquidation always exist and that the MBO is not possible. Section 5 considers the impact of relaxing these two assumptions on the financing of the project, and provides the intuitive reasoning for why the possibility of an MBO may jeopardize funding of the project and why the three institutions mentioned above can support the financing of the project. Section 6 formally defines the manager’s and investor’ choice problems and proves the results (stated) in Section 5. Section 7 concludes the paper.
2 The Model

This paper compares two different organizational forms for a profitable project whose cash flows are observable to both the manager and the investor, but not publicly verifiable. For illustrative purposes, we will tell the economic story of the paper by considering financing of a farm. There is nothing special about the farm upon which we have to rely to derive our results. Many firms, especially small firms, have the characteristics that are important in our model.

We assume that a manager, who is also the entrepreneur who owns the project idea initially, is capable of operating a farm. The cash flows from the farm are as follows. Initial investment in the land occurs at time 0 and costs an amount $K$. If the initial investment is made, the farm is in operation starting in year 1 through some date $T_{\text{sold}}$ (perhaps $T_{\text{sold}} = \infty$ indicating no sale) when the land is sold for $K$. The manager knows how to make the plants grow more than anyone else by talking to the plants but cannot communicate this knowledge. During each year $t$ of operation, the farm generates income

\begin{equation}
    y_t = rK + \alpha e_t,
\end{equation}

where $r$ is the riskfree rate, $\alpha > 1$ is a constant, and $e_t \in [0, \overline{e}]$ is the manager’s effort (talking to the plants) at time $t$. Here $\alpha$ measures both the manager’s efficiency and the farm’s profitability. Absent effort, the farm earns a normal return $rK$ and the net present value of the project is zero. Since $\alpha > 1$, the first-best has the manager undertaking the maximum effort $\overline{e}$ in all periods. If the manager leaves the farm for a year, the speech that makes the plants grow is forgotten. In this case, effort afterwards will forever be 0 and the farm may as well be sold since it will never earn more than the riskfree rate. We assume that the manager earns a normalized zero wage working outside the farm and that all the financial transactions between the manager and the investors can be verified at zero cost, but that contractual terms depending on nonpublic information rely on the agents’ incentive and cannot be enforced directly in court.
The manager’s utility function is

\[ U^M(C^M_1, E_1) = \sum_{t=1}^{\infty} \frac{u(c^M_t, e_t)}{(1 + r)^t} \]

where the pure rate of time discount equals the riskfree rate \( r \), the felicity function \( u \) is

\[ u(c, e) = \begin{cases} 
  c - e & \text{for } c \geq 0 \\
  -\infty & \text{otherwise}
\end{cases} , \]

and we use the conventions that

\[ C^M_t \equiv \{ c^M_t, c^M_{t+1}, \ldots \} \]

is the manager’s consumption from time \( t \) onwards and

\[ E_t \equiv \{ e_t, e_{t+1}, \ldots \} \]

is the effort profile from time \( t \) onwards. The manager has no endowment and therefore must turn to an outside investor to finance the project.

Outside investors are truly risk-neutral (unlike the manager who is risk-neutral for nonnegative consumption and infinitely risk averse for negative consumption). They care only about the expected net present value of any investment. Our formal model will assume there is a single (representative) investor, but assumptions we make in the model about bargaining power and sharing of rents will be motivated by competition among investors. The investor’s utility function is

\[ U^I(C^I_0) = \sum_{t=0}^{\infty} \frac{c^I_t}{(1 + r)^t} , \]
where, as for the entrepreneur, we define the investor’s consumption from \( t \) onwards by \( C'_t \). The investor’s endowment (of consumption) is normalized to 0. The actual endowment is irrelevant; global linearity implies that the investor’s actual endowment does not affect the feasibility of net trades or preferences over them.

What makes contracting difficult is that neither effort nor income is publicly verifiable and therefore a contract depending on either cannot be enforced in court. However, it is possible to sell a share allowing the investor to go in and harvest the crop on the land; it is assumed that this can be done costlessly.\(^2\) In this case, the investor observes the size of the crop directly, and the manager knows it implicitly because the manager knows the level of effort. However, an outside court cannot verify what is common knowledge to the manager and the investor. Together with control rights to fire the manager and sell off the firm, this will constitute an equity claim. General nonlinear claims are not available in the model, because the output is observed only by the person who harvests it.

**Corporate Form**

We will assume that when the manager-entrepreneur sets up the firm, because there are many potential investors, the manager can capture all the rents. One way to do so, at least for some parameter values, is to sell equity. At the outset (time 0), the investor pays \( K \) to buy equity in the firm and the manager uses the proceeds to buy the parcel of land. In this case, the investor is the equityholder of the firm, and the entrepreneur works as an employee (manager) of the firm.

The game between the employee-manager and the investor is a game of sequential move with perfect information. In each period \( t \) (such that \( t \geq 1 \)), we have the following stages:

1. The manager chooses effort \( e_t \in [0, \bar{e}] \) (how long to talk to the plants).

\(^2\)The cost of harvesting the crop, if not zero, can be incorporated into the firm’s cash flow.
2. The crop grows to produce output $y_t = rK + \alpha e_t$.

3. The investor harvests the entire crop and pays the manager the pre-agreed wage $w_t$ which is consumed by the manager $c_t = w_t$. (Later we will allow for saving by the manager, but for now saving by the manager would be redundant.) We will consider a candidate equilibrium in which $w_t = \alpha e$, the manager’s marginal product assuming maximal effort. We assume that the equityholder is obligated to pay the manager the fixed wage (unlimited liability), and that the equityholder always has the ability to do so (abundant wealth). We will replace these two assumptions with the assumption of limited liability in our analysis in Section 6.

4. The investor chooses the indicator $l_t$ where $l_t = 1$ if the investor chooses to liquidate the firm, fire the manager and sell the land for the amount $K$. Similarly, $l_t = 0$ if the investor retains the manager to operate the farm for another period. In our candidate equilibrium, the manager will be fired if less than maximal effort is exerted.

At each stage, the decision can depend on the entire history known to the agent. When the manager chooses $e_t$, that can depend on the entire previous history

$$h_t \equiv (E_{1,t-1}, L_{1,t-1}),$$

where

$$E_{1,t-1} \equiv (e_1, e_2, \ldots, e_{t-1})$$

3 The assumption that the manager consumes all the income in each period rules out the possibility that the manager will ever be able to take over the firm.

4 The assumptions of unlimited liability and abundant wealth are made here to keep this part of the analysis simple. It can be shown that when $\alpha e \leq K$, these two assumptions would give us the same result on the subgame perfect Nash equilibrium of the game between the manager and the investor as the assumption of limited liability.
and

\[(9) \quad L_{1,t-1} \equiv (l_1, l_2, ..., l_{t-1}),\]

while the investor’s choice \(l_t\) to sell the land and fire the manager depends on this history and this period’s effort:

\[(10) \quad (h^t, e_t) \equiv (E_{1,t}, L_{1,t-1}).\]

In other words, \(e_t\) is chosen as a function of \(h^t\) and \(l_t\) is a function of \(h^t\) and \(e_t\).

The strategies for the two agents interact to create a trajectory through the game. We will describe this more formally in Section 6. Here, we state the choice problems at a time when the equity has already been issued and the proceeds \((K)\) have been used to purchase the parcel of land. The decision problem faced by the manager is:

**Problem 1 Employee-manager’s choice (no MBO)** Given the investor’s strategy for dismissal \(\{l_t(h^t, e_t)\}\), the employee-manager chooses the strategy for effort \(\{e_t(h^t)\}\) to maximize the expected lifetime utility of consumption and effort

\[(11) \quad \sum_{t=1}^{\infty} \frac{c_t - e_t}{(1 + r)^t}\]

on the resultant trajectory, subject to

\[(12) \quad (\forall t) 0 \leq e_t \leq \varpi\]

and

\[(13) \quad (\forall t)c_t = \alpha \varpi \prod_{s=1}^{t-1} (1 - l_s).\]
The decision problem to be faced by the investor is (again leaving the more formal description to Section 6):

**Problem 2 Investor’s choice (no MBO)** Given the employee-manager’s strategy for effort \( \{e_t(h^t)\} \), the investor chooses the strategy for dismissal \( \{l_t(h^t, e_t)\} \) to maximize the expected lifetime cash flow

\[
-K + \sum_{t=1}^{\infty} \frac{((rK + \alpha e_t) - \alpha \sigma) \prod_{s=1}^{t-1}(1 - l_s)}{(1 + r)^t} + \frac{K}{(1 + r)\sum_{t=1}^{\infty} \prod_{s=1}^{t}(1 - l_s)}
\]

along the resultant trajectory, with the usual convention that an empty product is defined to be equal to 1.

Our equilibrium concept is subgame perfect Nash equilibrium, so there will be a similar problem to be solved at each date and for every possible history, and not just the histories that would be realized along the equilibrium path. This standard equilibrium concept seems like a reasonable one in this model (what further justification do we have for different equilibrium concepts?) and puts reasonable discipline on expectations about how the other player will continue if one player deviates from the candidate optimal strategy. Here is the result about the equilibrium in this case.

**Theorem 1 Equilibrium (Outside Equity)** Consider the corporate form with the employee-manager’s wage equal to the first-best marginal product \( \alpha \sigma \). We have the following result on the subgame perfect Nash equilibrium (henceforth “SPNE”) of the game between the employee-manager and the investor:

(i) **A bad equilibrium:** The candidate bad equilibrium defined below is always a SPNE in pure strategies;

(ii) **Good equilibrium with the most severe punishment for deviations:** The candidate good equilibrium defined below is a SPNE in pure strategies when \( \alpha \geq 1 + r \); It is the good equilibrium with the most severe punishment for deviations from the equilibrium path;
(iii) If the candidate good equilibrium defined below is a SPNE in pure strategies, then $\alpha \geq 1 + r$;

(iv) Only the good equilibrium makes the enterprise viable, since an investor anticipating the bad equilibrium would not expect to recover the initial investment of $K$.

The candidate bad equilibrium is defined by the employee-manager’s strategy for effort

$$e_t(h^t) = 0$$

and the investor’s strategy for dismissal

$$l_t(h^t, e_t) = 1.$$ 

The candidate good equilibrium is defined by the employee-manager’s strategy for effort

$$e_t(h^t) = \begin{cases} \tau & \text{if } (\forall s < t)e_s = \tau \\ 0 & \text{otherwise} \end{cases}$$

and the investor’s strategy for dismissal

$$l_t(h^t, e_t) = \begin{cases} 0 & \text{when } (\forall s \leq t)e_s = \tau \\ 1 & \text{otherwise} \end{cases}.$$ 

Proof: See Appendix A.

The agents’ strategies in the candidate good equilibrium consist of two parts: the good strategies (played on the equilibrium path) and the bad or punishment strategies (played on every off-equilibrium path), which are the
same as the strategies in the candidate bad equilibrium. The first-best effort in the good equilibrium is supported by the threat to play the punishment strategies forever once one party deviates from the good strategy, and it can be supported only when \( \alpha \geq 1 + r \), i.e., when the farm is profitable enough.

The agents’ strategies in the two candidate equilibria contain not only the agents’ plans of actions on every decision node of the game but also the agents’ beliefs about the future behavior of the other agent on each off-equilibrium path.\(^5\) The equilibrium concept which we use in our analysis requires the consistency between the agents’ beliefs and their actions in the equilibrium. In the candidate bad equilibrium, both agents plan to play the bad strategies on the equilibrium path, and they believe that the other agent will continue to play the bad strategy even if a deviation from the equilibrium path has occurred. As a result, it is optimal for both agents to play the bad strategies on all the decision nodes of the game. In the candidate good equilibrium, both agents plan to play the good strategies on the equilibrium path, and they believe that once a deviation from the equilibrium path has occurred, the other agent will play the bad strategy forever. As a result, it is optimal for both agents to play the good strategies on the equilibrium path and the bad strategies on each off-equilibrium path.

\(^5\)See Section 6.4 of Osborne and Rubinstein (1995) for more detailed discussion of this interpretation of the agent’s strategies.
3 Proprietorship

Another way to finance the project is to use debt. At the outset (time 0), the manager borrows $K$ from an investor to buy the parcel of land. In return, the manager promises the investor an interest payment in every subsequent period. In this case, the manager is also the equityholder of the firm, and the investor is the debtholder of the firm.

Many different debt can be used to finance the project. The required interest payments and debtholders’ rights to recall the outstanding debt can be quite different for different debt. Some debt, such as (some) bank debt, allow debtholders to recall part of or all the outstanding debt any time they want to. Some other debt, such as many long-term debt, won’t allow debtholders to do so unless a promised interest payment is missed. In this section, we focus on the debt that allows debtholders to recall all the outstanding debt whenever they want to.\(^6\) It can be shown that many other debt would produce the same results as this debt.

The game between the owner-manager and the investor is a game of sequential move with perfect information. In each period $t$ (such that $t \geq 1$), we have the following stages:

1. The manager chooses effort $e_t \in [0, \bar{e}]$. It is always optimal to choose the maximal effort level: $e_t = \bar{e}$.

2. The crop grows to produce output $y_t = rK + \alpha \bar{e}$.

3. The owner-manager harvests the entire crop, and then chooses the interest payment $i_t$ to the investor and the manager’s own saving and consumption in the period. Here we focus on a scenario in which the manager consumes all the income in the period (i.e. $c_t = y_t - i_t$) and does not save.\(^7\) (Later we will allow for saving by the manager.)

\(^6\)One advantage this debt has over many other debt is that this debt gives the investor more control over the manager than those debt do.

\(^7\)The assumption that the manager consumes all the income in each period rules out the possibility that the manager will ever be able to take over the firm.
4. The investor chooses the indicator \( l_t \) where \( l_t = 1 \) if the investor chooses to force the firm into bankruptcy, which leads to the firing of the manager, liquidation of the firm and sale of the land.\(^8\) Similarly, \( l_t = 0 \) if the investor decides to give the manager the control of the farm for another period.

5. In case the firm is forced into the bankruptcy, the land will be sold in a second-price auction, proceeds will be used to pay off the debt, and the remainder (if any) will go to the owner-manager.

Same as in the game in equity financing, a decision at any stage of the game can depend on the entire history known to the agent. The manager’s choice of \( i_t \) can depend on the entire previous history

\[
(19) \quad h^t \equiv (I_{1,t-1}, L_{1,t-1}),
\]

where

\[
(20) \quad I_{1,t-1} \equiv (i_1, i_2, \ldots, i_{t-1})
\]

and

\[
(21) \quad L_{1,t-1} \equiv (l_1, l_2, \ldots, l_{t-1}),
\]

while the investor’s choice \( l_t \) to force the firm into bankruptcy depends on this history and this period’s interest payment

\[
(22) \quad (h^t, i_t) \equiv (I_{1,t}, L_{1,t-1}).
\]

The strategies for the two agents interact to create a trajectory through the game. Here, we state the agents’ choice problems at a time when the

\(^8\)The investor can force the firm into bankruptcy by recalling all the outstanding debt. The manager will not be able to get the firm out of bankruptcy by paying off all the debt because the manager does not save.
money has already been borrowed and the purchase of the parcel of land has already been done.

Problem 3 Owner-manager’s choice (no Debt-Repurchase) Given the investor’s strategy for liquidation \( \{ l_t(h^t, i_t) \} \), the owner-manager chooses the strategy for interest payment \( \{ i_t(h^t) \} \) to maximize the expected lifetime utility of consumption and effort

\[
\sum_{t=1}^{\infty} c_t - \bar{c} \prod_{s=1}^{t-1} (1 - l_s) \frac{1}{(1 + r)^t}
\]

along the resultant trajectory, subject to

\[
(\forall t) c_t = (rK + \alpha \bar{c} - i_t) \prod_{s=1}^{t-1} (1 - l_s).
\]

The decision problem to be faced by the investor is:

Problem 4 Investor’s choice (no Debt-Repurchase) Given the owner-manager’s strategy for interest payment \( \{ i_t(h^t) \} \), the investor chooses the strategy for liquidation \( \{ l_t(h^t, i_t) \} \) to maximize the expected lifetime cash flow

\[
-K + \sum_{t=1}^{\infty} i_t \prod_{s=1}^{t-1} (1 - l_s) \frac{1}{(1 + r)^t} + \frac{K}{(1 + r) \sum_{t=1}^{\infty} \prod_{s=1}^{t-1} (1 - l_s)}
\]

along the resultant trajectory.

The result about the equilibrium in this case is given in the following theorem.

Theorem 2 Equilibrium (Debt) Consider a proprietorship with outside debt that has the required interest payments \( \{ i^*_t, t \geq 1 \} \) over time. We have
the following result on the subgame perfect Nash equilibrium of the game between the owner-manager and the investor:\(^9\)

(i) A bad equilibrium: The candidate bad equilibrium defined below is always a SPNE in pure strategies;

(ii) A good equilibrium: The candidate good equilibrium defined below is a SPNE in pure strategies if and only if

\[
\begin{align*}
(i) & \quad i^*_t \in [0, rK + \alpha \varepsilon] \quad \forall t \geq 1, \\
(ii) & \quad \sum_{s=1}^{+\infty} \frac{i^*_s}{(1 + r)^s} = K \\
& \quad \text{and} \\
(iii) & \quad \sum_{s=t+1}^{+\infty} \frac{rK + (\alpha - 1)\varepsilon - i^*_s}{(1 + r)^s-t} \geq i^*_t \quad \forall t \geq 1; \\

\end{align*}
\]

(iii) Good equilibrium with the lowest possible \(\alpha\): The good equilibrium that allows the project with the lowest possible \(\alpha\) to receive financing is the equilibrium with a constant interest payment over time

\[
\begin{align*}
(iii) & \quad i^*_t = rK \quad \forall t \geq 1; \\
& \quad \text{Under this interest payment, a project should be able to receive financing as long as} \\
& \quad \frac{(\alpha - 1)\varepsilon}{r} \geq rK; \\

\end{align*}
\]

\(^9\)Here we focus on the subgame perfect Nash equilibrium strategies that give the owner-manager all the NPV of the project.
(iv) Only the good equilibrium makes the enterprise viable, since an investor anticipating the bad equilibrium would not expect to recover the initial investment of $K$.

The candidate **bad equilibrium** is defined by the owner-manager’s strategy for interest payment

(31) \( i_t(h^t) = 0 \)

and the investor’s strategy for liquidation

(32) \( l_t(h^t, i_t) = 1. \)

The **candidate good equilibrium** is defined by the owner-manager’s strategy for interest payment

(33) \[ i_t(h^t) = \begin{cases} i^*_t & \text{if } (\forall s < t) i_s \geq i^*_s \\ 0 & \text{otherwise} \end{cases} \]

and the investor’s strategy for liquidation

(34) \[ l_t(h^t, i_t) = \begin{cases} 0 & \text{when } (\forall s \leq t) i_s \geq i^*_s \\ 1 & \text{otherwise} \end{cases} \]

**Proof:** See Appendix A.
4 Corporate Form versus Proprietorship

Our analysis in the previous two sections shows that the two organizational forms can have very different incentive problems. In equity financing, the employee-manager signs a fixed-wage contract and works as an employee of the firm. Given that the manager’s income is guaranteed by the wage contract and does not depend on the output of the firm, the manager may have an incentive not to exert any effort on the project and just collect the wage. The incentive problem in the corporate form is that the employee-manager may have an incentive to shirk.

In debt financing, the owner-manager borrows from an investor and promises the investor an interest payment in each subsequent period. As the residual claimant on the firm’s cash flow and value, the manager has no incentive to shirk, but may have an incentive to take the entire cash flow out of the firm and default on the debt. The incentive problem in the proprietorship is that the owner-manager may have an incentive not to make any interest payment.

The analysis produces the following result on the feasibility of the two organizational forms:

Theorem 3 Corporate Form versus Proprietorship

*Given the condition for the good equilibrium in the corporate form*

\[(35) \quad \bar{\sigma} \leq \frac{(\alpha - 1)\bar{\sigma}}{r}\]

*and the condition for the good equilibrium in the proprietorship*¹⁰

\[(36) \quad rK \leq \frac{(\alpha - 1)\bar{\sigma}}{r},\]

¹⁰See Theorem 1 and Theorem 2 for these two conditions.
we have that (i) neither the corporate form nor the proprietorship can be used when

\[ \frac{(\alpha - 1)\bar{\epsilon}}{r} < \min\{rK, \bar{\epsilon}\}; \]  

(ii) the proprietorship, but not the corporate form, can be used when

\[ rK \leq \frac{(\alpha - 1)\bar{\epsilon}}{r} < \bar{\epsilon}; \]  

(iii) the corporate form, but not the proprietorship, can be used when

\[ \bar{\epsilon} \leq \frac{(\alpha - 1)\bar{\epsilon}}{r} < rK; \]  

and (iv) both the corporate form and the proprietorship can be used when

\[ \max\{rK, \bar{\epsilon}\} \leq \frac{(\alpha - 1)\bar{\epsilon}}{r}. \]  

It is worth explaining the intuition behind the result in the theorem. The term on the right-hand-sides of both (35) and (36) equals the value of the ßrm to the manager. It also equals the loss that the manager will suffer when leaving the ßrm. The terms on the left-hand-sides of (35) and (36) equal respectively the gain for the manager from shirking and the gain from not making the interest payment. The two conditions in (35) and (36) imply that the corporate form and the proprietorship each can be used only when the manager cannot beneÞt from shirking or defaulting on the debt, and that depending on the relative severity of the two incentive problem, either organizational form can dominate the other.

The model has the following predictions on the optimal choice between the two organizational forms:
1. The corporate form should be the preferred choice of the two when the production is not effort intensive, but capital intensive.

2. The proprietorship should be the preferred choice of the two when the production is not capital intensive, but effort intensive.

In our example of the farm, these predictions imply that the corporate form, not the proprietorship, should be used when it is not a lot of work for the manager to talk to the plants, but the initial investment on the farm is large relative to the future profit of the farm. The proprietorship, not the corporate form, should be used when it is a lot of work for the manager to talk to the plants, but the initial investment of the farm is small relative to the future profit of the farm.
5 Impact Of An Aftermarket

Our analysis in Section 2 to Section 4 is based on the assumptions that the project, the manager and the investor all have infinite life and that the manager does not save. These two assumptions ensure that both the threat of dismissal and the threat of liquidation always exist and that the MBO is not possible. In this section, we study how relaxing these two assumptions would affect the results in the previous sections. We find that when these two assumptions do not hold, the threat to fire the manager or to liquidate the project itself cannot prevent the manager from shirking or defaulting on the debt, and that the financing of the project can be supported by institutions such as sinking funds, stock or option compensation and managers’ non-vested pensions. We provide only the intuitive reasoning for these findings in this section, and we will formally prove these results in the next section.

Failure of Reputation Effect to Support Financing

Our analysis in the previous sections shows that to support the financing of the project, the investor has to be able to punish the manager for deviations from the good strategies in all periods. The investor will not be able to do so when the two assumptions stated above do not hold. When the project or the manager has a finite life, the investor will not be able to punish the manager for shirking or defaulting on the debt in the last period of the project’s or the manager’s life. When an MBO is possible, firing the manager or liquidating the project is not a punishment to the manager anymore, because the manager can simply take over the firm and reinstate the project right after being fired. In both cases, besides the threat of dismissal and the threat of liquidation, some institutions need to be used to support the financing of the project.
Financing with Different Institutions

We find that three institutions can be used to support the financing of the project: debt financing can be supported by sinking funds, and equity financing can be supported by stock or option compensation or managers’ non-vested pensions.11 In this subsection we provide the intuitive reasoning for this result.

Financing with Debt with Sinking Funds

When debt is used, the lender can require the borrower to retire a (pre-specified) fraction of the (principal of the) debt periodically through a sinking fund.12 If we design the sinking fund of a debt in such a way that a large percentage of the debt is retired early, then the debt can be used to finance the project. The intuition is that when a large enough proportion of the debt is retired early enough, there won’t be any default on the debt because the payment of the remaining debt can be guaranteed by the liquidation value of the project. The quickest way to retire all the debt is to ask the manager to spend all the income on debt-repurchase.

Financing with Non-Vested Pensions

Each employees’ pension can have a non-vesting period. An employee may lose the pension if leaving the firm during this period. Managers’ non-vested pensions can be used to support equity financing of the project. If we design the manager’s pension in such a way that the manager loses the pension to a third party (such as the Pension Benefit Guarantee Corporation) when leaving the firm before the pension becomes vesting, the investor should be able to deter the manager’s shirking by holding the pension against the

11All stock compensation in the paper are assumed to be in the company’s non-voting shares. In our example of the farm, each share of the company’s stock is a claim on (a fraction of) the farm’s land. Since the value of the land is known to everyone, claims on the ownership of the land can always be traded publicly.

12This provision in the debt contract is referred to as the sinking fund requirement.
shirking. When the life of the project is longer than the non-vesting period of the pension, it may be optimal to set up the manager’s pension in such a way that it covers the later part of the project’s life.

**Financing with Stock or Option Compensation**

Stock or option compensation can also be used to support equity financing of the project. Its ability to do so depends crucially on two important properties of this compensation. One property is that this compensation allows the complete transfer of the firm’s ownership from the investor to the manager. The other property is that it can eliminate the manager’s incentive to shirk: when the manager’s ownership of the firm is large enough, the manager has no incentive to shirk. The biggest challenge in using this compensation, which is also a basic requirement for being able to use this compensation, is to find a feasible way to divide the firm’s output between the two parties according to what they are entitled to receive in each period, when the output of the firm itself is not publicly verifiable.

In our analysis of equity financing in Section 2, the investor is given the control over the output of the firm. The wage payment to the manager is guaranteed by the liquidation value of the project (under the limited liability assumption). When the manager is compensated in the company’s shares, the control over the firm’s cash flows can no longer be given to any single party. Giving the control to the investor will create another incentive problem (the incentive problem for the investor): the investor may have an incentive not to make any dividend payment to the manager. It can be shown that it can be very difficult to deal with the two incentive problems and have them both under control at the same time.

One way to split the firm’s output between the two parties is to give the control over one part of the output to one party and the control over the other part of the output to the other party. This can be done by dividing the overall product market into many sub-markets and allowing each party to collect revenue from sales in different sub-markets (as dividends and possibly the wage). In our example of the farm, this is equivalent to letting each party harvest crops on different parts of the land. One way to ensure that
the manager’s shirking reduces the manager’s dividend income but not the wage income is to pay the manager the wage in the company’s shares and the dividends in cash (collected from sales in certain sub-markets).

It is possible to pay the manager the wage and dividends in almost every possible combination of the company’s shares and cash. This can be done by paying the manager the wage in the company’s shares and dividends in cash and then exchanging the shares for cash of equal value or vice versa.\footnote{The exchange of shares for cash of equal value or vice versa can easily be done because the two have the same value.}

This output-sharing plan can be both difficult and tedious to arrange at times.\footnote{One requirement that needs to be satisfied for arranging this plan is the investor’s direct involvement in the operation of the firm.} The potential difficulties and tediousness associated with arranging this plan may prevent certain projects from being financed with equity, and may also make debt the preferred choice of the two when both can be used. As a result, we may see debt being used more often than equity.

### Financing of Projects with Finite Life

All the three institutions discussed above can be used to support financing of (similar) projects with finite life. The intuition behind this result is that use of sinking funds (stock or option compensation) makes it possible to retire all the debt (complete the transfer of the firm’s ownership from the investor to the manager) before the final period of the project’s life, and that use of non-vested pensions makes it possible for the investor to punish the manager for shirking in the last period of the project’s life.

### Predictions on Financing of Firms with Non-Verifiable Cash Flows

The model has a few additional predictions on the financing of firms whose cash flows are not publicly observable:
1. The primary sources of financing are private equity and private debt including bank debt.

2. Owners are often actively involved in the operation of the firms. Owners have the control over the output of the firms. Some owners may also work as the managers of the firms.

3. When equity is used, the managers’ ownership of the firms tends to increase over time.

4. When debt is used, the amount of outstanding debt tends to decrease over time.

5. Debt may be the preferred choice of the two when both debt and equity can be used. As a result, we may see debt being used more often than equity.
6 Formal Models

In this section, we formally prove the results stated in Section 5. We show that when the manager is allowed to take over the firm or when the manager or the project has a finite life, the threat of dismissal or the threat of liquidation itself cannot prevent the manager from shirking or defaulting on the debt, and the financing of the project can be supported by sinking funds, stock or option compensation and managers’ non-vested pensions. First we consider equity financing. Then we consider debt financing.

The agents’ choice problems in this section are much more complicated than the ones in Section 2 and Section 3 mainly because without the restrictions on the manager’s ability to take over the firm, the manager needs to make a few more decisions in each period, and as a result, we need to keep track of the values of many more variables and consider all possible outcomes of the agents’ actions.

Agents’ Choice Problems in Equity Financing

In this subsection we set up the agents’ choice problems in equity financing for a project with an infinite life. First, we define the variables

\( e_t \) — manager’s effort in period \( t \): \( e_t \in [0, \bar{e}] \).

\( w_t \) — manager’s wage in period \( t \): \( w_t \geq 0 \).

\( d_t \) — manager’s dividend income in period \( t \): \( d_t \geq 0 \).

\( c_t \) — manager’s consumption in period \( t \): \( c_t \geq 0 \).

\( \Delta \beta_{w,t} \) — fraction of the ownership of the company that the manager receives as part of the wage income in period \( t \) (which is a choice variable for the investor): \( \Delta \beta_{w,t} \in [0, 1] \).\(^{15}\)

\(^{15}\) \( \Delta \beta_{w,t} K \) equals the value of the part of the wage that the manager receives in the
$\Delta \beta_{d,t}$—fraction of the ownership of the company that the manager receives as part of the dividend income in period $t$ (which is a choice variable for the investor): $\Delta \beta_{d,t} \in [0, 1]$.\footnote{$\Delta \beta_{d,t} K$ equals the value of the part of the dividend income that the manager receives in the company’s shares.}

$\beta_t$—fraction of the company owned by the manager at the end of period $t$: $\beta_t \in [0, 1]$ and $\beta_t \equiv \beta_{t-1} + \Delta \beta_{w,t} + \Delta \beta_{d,t}$.

$j_t$—manager’s total income from the project in period $t$, which includes both the manager’s wage income and dividend income.

$m_t$—cash balance in the manager’s saving account (or money market account) at the beginning of period $t$.

$W_t$—manager’s total wealth at the end of period $t$, which includes the manager’s ownership of the firm.

$y_t$—output of the project in period $t$.

$l_t$—investor’s decision on firing the manager in period $t$: $l_t = 1$ if the manager is fired in period $t$ and $l_t = 0$ not.

Next, we define the agents’ strategies:

$\sigma_{m,e}$—manager’s strategy on effort: $\sigma_{m,e} : h^t \mapsto [0, \bar{e}]$ and $e_t \equiv \sigma_{m,e}(h^t)$.

$\sigma_{m,c}$—manager’s strategy on consumption: $\sigma_{m,c} : h^t \mapsto [0, W_t]$ and $c_t \equiv \sigma_{m,c}(h^t)$.

$\sigma_m \equiv (\sigma_{m,e}, \sigma_{m,c})$.

$\sigma_{i,l}$—investor’s strategy on firing the manager: $\sigma_{i,l} : (h^t, e_t) \mapsto \{0, 1\}$ and $l_t \equiv \sigma_{i,l}(h^t, e_t)$.

$\sigma_{\Delta \beta_{w,t}}$—investor’s strategy on paying part of or all the manager’s wage in company’s shares.
the company’s shares instead of cash: \( \sigma_{\Delta b,w} : (h^t, e_t) \mapsto [0, 1] \) and \( \Delta \beta_{w,t} \equiv \sigma_{\Delta b,w}(h^t, e_t) \).

\( \sigma_{\Delta b,d} \) — investor’s strategy on paying part of or all the manager’s dividend income in the company’s shares instead of cash: \( \sigma_{\Delta b,d} : (h^t, e_t) \mapsto [0, 1] \) and \( \Delta \beta_{d,t} \equiv \sigma_{\Delta b,d}(h^t, e_t) \).

\( \sigma_i \equiv (\sigma_{\Delta b,w}, \sigma_{\Delta b,d}, \sigma_{i,t}) \).

Here we assume that the decision on compensating the manager in the company’s shares or cash is made by the investor and that the investor can always compensate the manager in the combination of the company’s shares and cash that the investor likes. According to our discussion in the previous section, this can be done by paying the manager the wage in the company’s shares and the dividends in cash and then exchanging the company’s shares for cash of equal value or vice versa.

Each pair of the agents’ strategies creates a trajectory of the outcomes of the game between the two agents. Now we define the trajectory of the outcomes determined by \( \sigma_m \) and \( \sigma_i \). For each triple \( (h^t, \sigma_m, \sigma_i) \), we define trajectory\(^{17}\)

\[
\{ (e_1, e_2, \ldots), (\Delta \beta_{w,1}, \Delta \beta_{w,2}, \ldots), (\Delta \beta_{d,1}, \Delta \beta_{d,2}, \ldots), (l_1, l_2, \ldots) \} \]

recursively:

\[
\begin{align*}
e_s &= \begin{cases} e_s & \text{for } s < t \\ \sigma_{m,e} ((E_{1,s-1}, (\Delta \beta_{1,s-1}, (L_{1,s-1}) & \text{otherwise} \end{cases} \\
c_s &= \begin{cases} c_s & \text{for } s < t \\ \sigma_{m,c} ((E_{1,s-1}, (\Delta \beta_{1,s-1}))(L_{1,s-1}) & \text{otherwise} \end{cases}
\end{align*}
\]

\(^{17}\)An implicit assumption made here is that the manager’s consumption is not observable to others and therefore cannot be used by the equityholder in the decision-making. In case the manager’s consumption is observable to the equityholder, the financing of the project can be supported by requiring that the manager consumes all or most of the income in each period.
\[ \Delta \beta_{w,s} = \begin{cases} \Delta \beta_{w,s} & \text{for } s < t \\ \sigma_{\Delta \beta_{w}}((E_{1,s}), (\Delta \beta_{1,s-1}, (L_{1,s-1})) & \text{otherwise} \end{cases} \]

\[ \Delta \beta_{d,s} = \begin{cases} \Delta \beta_{d,s} & \text{for } s < t \\ \sigma_{\Delta \beta_{d}}((E_{1,s}), (\Delta \beta_{1,s-1}, (L_{1,s-1})) & \text{otherwise} \end{cases} \]

\[ l_s = \begin{cases} l_s & \text{for } s < t \\ \sigma_{\Delta \beta_{l}}((E_{1,s}), (\Delta \beta_{1,s-1}, (L_{1,s-1})) & \text{otherwise} \end{cases} \]

\[ y_s \equiv \begin{cases} y_s & \text{for } s < t \\ rK + \alpha e_s & \text{otherwise} \end{cases} \]

where

(43) \( E_{1,t-1} \equiv (e_1, e_2, ..., e_{t-1}) \),

(44) \( \Delta \beta_{1,t-1} \equiv (\Delta \beta_{w,1,t-1}, \Delta \beta_{d,1,t-1}) \),

(45) \( \Delta \beta_{w,1,t-1} \equiv (\Delta \beta_{w,1}, \Delta \beta_{w,2}, ..., \Delta \beta_{w,t-1}) \),

(46) \( \Delta \beta_{d,1,t-1} \equiv (\Delta \beta_{d,1}, \Delta \beta_{d,2}, ..., \Delta \beta_{d,t-1}) \),

and

(47) \( L_{1,t-1} \equiv (l_1, l_2, ..., l_{t-1}) \).

The time when the manager is fired and the project is sold along the trajectory is given by

(48) \( T_{\text{sold}} \equiv \inf \{t|l_t = 1 \text{ and } W_t < K, t \geq 1\} \),

and the time when an MBO occurs along the trajectory is given by\(^{18}\)

(49) \( T_{\text{mbo}} \equiv \inf \{t|l_t = 1 \text{ and } W_t \geq K, t \geq 1\} \).

\(^{18}\)It is optimal for the manager to take over the firm before it is sold to others. Otherwise, the manager would forget the speech that can make the plants grow.
To simplify the notation, we assume that

\[(50) \quad l_t = 1, \quad \forall \ t \geq \min\{T_{\text{sold}}, T_{\text{mbo}}\}.\]

For the convenience of our discussion, now we define three sets of constraints: one on the manager’s strategy, one on the investor’s strategy, and one on the following set of variables: the firm’s output, the manager’s wage, the manager’s dividend income, the manager’s total income from the project, cash balance in the manager’s saving account, and the manager’s total wealth.

The constraints on the manager’s strategy, which specify the range and initial values of the manager’s choice variables, are

\[(51) \quad 0 \leq c_t \leq W_t,\]

\[(52) \quad 0 \leq e_t \leq \bar{e},\]

\[(53) \quad 0 \leq m_t,\]

and

\[(54) \quad m_0 \equiv 0.\]

The constraints on the investor’s strategy, which specify the range and initial values of the investor’s choice variables, are

\[(55) \quad \Delta \beta_{w,t} \in [0, 1],\]

\[(56) \quad \Delta \beta_{d,t} \in [0, 1],\]

\[(57) \quad \beta_t \in [0, 1],\]
\( \Delta \beta_{w,0} \equiv 0, \)

\( \Delta \beta_{d,0} \equiv 0, \)

\( \beta_0 \equiv 0, \)

\( \Delta \beta_{w,t} \begin{cases} \in \left[ \frac{\alpha e}{K} - r, \frac{\alpha e}{K} \right] & \text{when } t < \min\{T_{\text{mbo}}, T_{\text{sold}}\} \\ = 0 & \text{otherwise} \end{cases}, \)

\( \Delta \beta_{d,t} \begin{cases} \in \left[ 0, \frac{d h}{K} \right] & \text{when } t < \min\{T_{\text{mbo}}, T_{\text{sold}}\} \\ = 0 & \text{otherwise} \end{cases}, \)

and

\( \beta_t \equiv \beta_{t-1} + \Delta \beta_{w,t} + \Delta \beta_{d,t}, \)

where \( T_{\text{sold}} \) and \( T_{\text{mbo}} \) are given in (48) and (49) respectively. The lower bound of \( \Delta \beta_{w,t} \) in (61) guarantees the non-negativity of the firm’s dividend payment.

The constraints on the firm’s output, the manager’s wage, the manager’s dividend income, the manager’s total income from the project, cash balance in the manager’s saving account, and the manager’s total wealth are

\( y_t = rK + \alpha e_t, \)

\( w_t = \begin{cases} w_t^* & \text{when } t \leq \min\{T_{\text{sold}}, T_{\text{mbo}}\} \\ 0 & \text{when } t > \min\{T_{\text{sold}}, T_{\text{mbo}}\} \end{cases}, \)

\( d_t = \begin{cases} \beta_{t-1}(y_t - (w_t^* - \Delta \beta_{w,t}K)) & \text{when } t \leq \min\{T_{\text{sold}}, T_{\text{mbo}}\} \\ 0 & \text{when } t > \min\{T_{\text{sold}}, T_{\text{mbo}}\} \end{cases}. \)
\[ j_t = \begin{cases} 
    w_t + d_t & \text{when } t \leq \min\{T_{\text{sold}}, T_{\text{mbo}}\} \\
    y_t & \text{when } t > T_{\text{mbo}} \text{ and } T_{\text{mbo}} < T_{\text{sold}} \\
    0 & \text{when } t > T_{\text{sold}} \text{ and } T_{\text{sold}} < T_{\text{mbo}} 
\end{cases} \]

\[ m_{t+1} = \begin{cases} 
    (1 + r)(m_t + j_t - (\beta_t - \beta_{t-1})K - c_t) & \text{when } t < \min\{T_{\text{mbo}}, T_{\text{sold}}\} \\
    (1 + r)(m_t + j_t - (1 - \beta_t)K - c_t) & \text{when } t = T_{\text{mbo}} \text{ and } T_{\text{mbo}} < T_{\text{sold}} \\
    (1 + r)(m_t + j_t - c_t) & \text{when } t > T_{\text{mbo}} \text{ and } T_{\text{mbo}} < T_{\text{sold}} \\
    (1 + r)(m_t + j_t + \beta_{t-1}K - c_t) & \text{when } t = T_{\text{sold}} \text{ and } T_{\text{sold}} < T_{\text{mbo}} \\
    (1 + r)(m_t - c_t) & \text{when } t > T_{\text{sold}} \text{ and } T_{\text{sold}} < T_{\text{mbo}} 
\end{cases} \]

and

\[ W_t = m_t + j_t + \beta_{t-1}K. \]

The manager’s dividend income in (66) is determined based on the assumption that the manager’s wage has to be paid in full before any dividend payment can be made by the firm. (68) provides the cash balance of the manager’s saving account under different circumstances.

Now we define the agents’ choice problems at a time when the equity has already been issued and the parcel of land has already been purchased. The manager’s choice problem is

**Problem 5 Manager’s choice (MBO)** Given the wage contract \( \{w^*_t = \alpha \bar{e}, t \geq 1\} \) and the investor’s strategy satisfying the constraints in (55) to
(63), the manager chooses $\sigma_m$ to maximize the expected lifetime utility of consumption and effort

$$
\sum_{t=1}^{+\infty} \frac{c_t - e_t}{(1 + r)^t}
$$

subject to the constraints in both (51) to (54) and (64) to (69).

The investor’s choice problem is

**Problem 6 Investor’s choice (MBO)** Given the wage contract \( \{w_t^* = \alpha \bar{e}, t \geq 1\} \) and the manager’s strategy $\sigma_m$ satisfying the constraints in (51) to (54), the investor chooses $\sigma_i$ to maximize the expected lifetime cash flow

$$
\sum_{t=1}^{T_{\text{min},e}} y_t - w_t^* - d_t + (\beta_t - \beta_{t-1})K + \frac{(1 - \beta_t^{T_{\text{min},e}})K}{(1 + r)^t}
$$

subject to the constraints in both (55) to (63) and (64) to (69), where

$$
T_{\text{min},e} \equiv \min\{T_{\text{sold}}, T_{\text{mbo}}\},
$$

and $T_{\text{sold}}$ and $T_{\text{mbo}}$ are given in (48) and (49) respectively.

**Equity Financing**

We have the following result on equity financing of the project.

**Theorem 4 Equilibrium (Outside Equity with MBO)** Consider the corporate form with the employee-manager’s wage equal to first-best marginal
product $\alpha e$ and a project with an infinite life. We have the following result on equity financing of the project and the subgame perfect Nash equilibrium of the game between the employee-manager and the investor:

(i) Financing cannot be supported by the threat of dismissal itself: Equity financing of the project cannot be supported by the threat to fire the manager itself, when neither the employee share ownership nor the manager’s non-vested pension is used. The candidate bad equilibrium defined below is the only SPNE in pure strategies for all $\alpha > 0$.

(ii) Financing can be supported by employee share ownership: With employee share ownership, equity financing of the project can be supported when

\begin{equation}
1 + r < \frac{K}{\bar{e}}
\end{equation}

and

\begin{equation}
\beta_{T_{mbo} - 1} \geq \frac{1}{\alpha},
\end{equation}

where

\[ T_{mbo} \equiv \inf\{t|l_t = 1 \text{ and } W_t \geq K, \ t \geq 1\} \]
\[ = \inf\{t|\frac{\alpha \bar{e}((1 + r)^t - 1)}{r} \geq K\} \]
\[ \beta_{T_{mbo} - 1} = \frac{\alpha \bar{e}((1 + r)^{T_{mbo} - 1} - 1)}{rK}, \]

provided the investor and the manager can find a feasible way to divide the firm’s output between them according to what they are entitled to receive in each period. The candidate good equilibrium that provides the quickest transfer of the firm’s ownership from the equityholder to the manager is the candidate good equilibrium with zero consumption and compensation in only the company’s shares defined below.
(iii) Financing can be supported by non-vested pension: If the manager’s pension is designed in such a way that it has an infinite non-vesting period and that the manager loses the pension to a third party when leaving the firm, then equity financing of the project can be supported for all $\alpha > 0$.

(iv) Financing of projects with finite life: For a project with finite life $T$ and initial investment of $K(1 - \frac{1}{(1+r)^T})$, equity financing of the project can be supported with either employee share ownership or the manager’s non-vested pension. Employee share ownership can be used when

$$1 + \frac{r}{1 - (1+r)^1-T} \leq \alpha < (1 - \frac{1}{(1+r)^T}) \frac{K}{\bar{e}}$$

and

$$\beta_{t_{mbo} - 1} \geq \frac{1}{\alpha}$$

where

$$t_{mbo} \equiv \inf\{t|t = 1 \text{ and } W_t \geq (1 - \frac{1}{(1+r)^{T-t}})K, 1 \leq t \leq T\}$$

$$= \inf\{t|\frac{\alpha \bar{e} ((1+r)^t - 1)}{r} \geq (1 - \frac{1}{(1+r)^{T-t}})K\}$$

$$\beta_{t_{mbo} - 1} = \frac{\alpha \bar{e} ((1+r)^{t_{mbo} - 1} - 1)}{(1 - (1+r)^{t_{mbo} - 1 - T})rK}.$$

The candidate good equilibrium that provides the quickest transfer of the firm’s ownership from the equityholder to the manager is the candidate good equilibrium with zero consumption and compensation in only the company’s shares. The manager’s non-vested pension can be used when

$$L_{nvp} \geq T + 1 - \min\{t_{mbo}, t_{shirk}\}.$$\[19\]We have similar results for when the manager has a finite life.

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where

\[
t_{\text{shirk}} = \inf\{t | \bar{e} > \alpha \bar{e} (1 - \frac{1}{(1 + r)^{T-t}})K\}
\]

and \(L_{\text{nvnp}}\) is the length (in number of periods) of the non-vesting period of the manager’s pension.

The candidate bad equilibrium is defined by the employee-manager’s strategy for effort and consumption

(78) \(e_t(h^t) = 0\)

(79) \(c_t(h^t) \leq W_t\),

and the investor’s strategy for dismissal and manager’s share ownership

(80) \(l_t(h^t, e_t) = 1\)

(81) \(\beta_t(h^t, e_t) = 0\).

The candidate good equilibrium with the quickest transfer of the firm’s ownership is defined by the employee-manager’s strategy for effort and consumption

(82) \(e_t(h^t) = \begin{cases} \bar{e} & \text{if } (\forall s < t) e_s = \bar{e} \\ 0 & \text{otherwise} \end{cases}\)

(83) \(c_t(h^t) = 0 \text{ when } \beta_{t-1} < 1\)
and the investor’s strategy for dismissal and manager’s share ownership

\begin{equation}
\begin{aligned}
    l_t(h^t, e_t) = \begin{cases} 
    0 & \text{when } (\forall s \leq t) e_s = \bar{e} \\
    1 & \text{otherwise}
    \end{cases}
\end{aligned}
\end{equation}

\begin{equation}
\beta_t(h^t, e_t) = \min\{\beta_{t-1}(1 + r) + \frac{\alpha \bar{e}}{K}, 1\} \quad \text{when } \beta_{t-1} < 1
\end{equation}

**Proof:** See Appendix A.

Now we provide an intuitive explanation of some of the results in the theorem. To support equity financing with employee share ownership, two conditions need to be satisfied. One condition is that the project can neither be too profitable nor be too unprofitable. Equity financing cannot be supported if the future value of the project is smaller than the costly effort to the manager or the manager’s wage in a period is larger than the initial investment of the project. The manager is always better off shirking in the first circumstance, and the manager will shirk and then take over the project in the second circumstance. The other condition is that the manager’s ownership of the firm needs to be large enough when the manager has enough wealth to take over the project. The reason is that the manager has no incentive to shirk under this circumstance only when the manager owns a large enough fraction of the firm. To support equity financing with manager’s non-vested pension, the pension needs to have a long enough non-vesting period.

**Agents’ Choice Problems in Debt Financing**

In this subsection, we set up the agents’ choice problems in debt financing for a project with an infinite life. First, we define the variables:

- $i_t$—interest payment in period $t$: $i_t \in [0, y_t]$.
- $c_t$—manager’s consumption in period $t$: $c_t \geq 0$.  

$\beta_t$—fraction of the company’s total debt retired by the sinking fund at the end of period $t$ (which is a choice variable for the manager): $\beta_t \in [0, 1]$.

$j_t$—manager’s total income from the project in period $t$.

$m_t$—cash balance in the manager’s saving account at the beginning of period $t$.

$W_t$—manager’s total wealth at the end of period $t$.

$y_t$—output of the project in period $t$.

$l_t$—investor’s decision on forcing the firm into bankruptcy in period $t$: $l_t = 1$ if force the firm into bankruptcy in period $t$ and $l_t = 0$ if not.

Next, we define the agents’ strategies:

$\sigma_{m,i}$—manager’s strategy on interest payment: $\sigma_{m,i} : h^t \mapsto [0, y_t]$ and $i_t \equiv \sigma_{m,i}(h^t)$.

$\sigma_{m,c}$—manager’s strategy on consumption: $\sigma_{m,c} : h^t \mapsto [0, W_t]$ and $c_t \equiv \sigma_{m,c}(h^t)$.

$\sigma_{m,b}$—manager’s strategy on retiring part of or all the outstanding debt with the sinking fund: $\sigma_{m,b} : (h^t) \mapsto \{0, 1\}$ and $\beta_t \equiv \sigma_{m,b}(h^t)$.

$\sigma_m \equiv (\sigma_{m,i}, \sigma_{m,c}, \sigma_{m,b})$.

$\sigma_{i,l}$—investor’s strategy on forcing bankruptcy: $\sigma_{i,l} : (h^t, e_t, p_t) \mapsto \{0, 1\}$ and $l_t \equiv \sigma_{i,l}(h^t, e_t, p_t)$.

$\sigma_i \equiv \sigma_{i,l}$.

Each pair of the agents’ strategies creates a trajectory of the outcomes of the game between the two agents. Now we define the trajectory of the outcomes determined by $\sigma_m$ and $\sigma_i$. For each triple $(h^t, \sigma_m, \sigma_i)$, we define
trajectory

\[(86) \quad h_t^\infty = \{(i_1, i_2, \ldots), (\beta_1, \beta_2, \ldots), (l_1, l_2, \ldots)\}\]

recursively:

\[(87) \quad i_s = \begin{cases} \ i_s & \text{for } s < t \\ \sigma_{m,i}((I_{1,s-1}), (\beta_{1,s-1}), (L_{1,s-1})) & \text{otherwise} \end{cases}
\]

\[(88) \quad c_s = \begin{cases} \ c_s & \text{for } s < t \\ \sigma_{m,c}((I_{1,s-1}), (\beta_{1,s-1}), (L_{1,s-1})) & \text{otherwise} \end{cases}
\]

\[(89) \quad \beta_s = \begin{cases} \ \beta_s & \text{for } s < t \\ \sigma_{m,b}((I_{1,s-1}), (\beta_{1,s-1}), (L_{1,s-1})) & \text{otherwise} \end{cases}
\]

\[(90) \quad l_s = \begin{cases} \ l_s & \text{for } s < t \\ \sigma_{i,l}((I_{1,s}), (\beta_{1,s}), (L_{1,s-1})) & \text{otherwise} \end{cases}
\]

\[(91) \quad y_s = \begin{cases} \ y_s & \text{for } s < t \\ rK + \alpha \bar{e} & \text{otherwise} \end{cases}
\]

where

\[(88) \quad I_{1,t-1} \equiv (i_1, i_2, \ldots, i_{t-1}),\]

\[(89) \quad \beta_{1,t-1} \equiv (\beta_1, \beta_2, \ldots, \beta_{t-1})\]

and

\[(90) \quad L_{1,t-1} \equiv (l_1, l_2, \ldots, l_{t-1}).\]

The time when the manager is fired and the firm is forced into bankruptcy along the trajectory is given by

\[(91) \quad T_{\text{bankruptcy}} \equiv \inf\{t | l_t = 1 \text{ and } W_t < (1 + r)(1 - \beta_{t-1})K, \ t \geq 1\},\]
and the time when a take-over through debt-repurchase occurs along the trajectory is given by

\[ T_{\text{takeover}} \equiv \inf \{ t | l_t = 1 \text{ and } W_t \geq (1 + r)(1 - \beta_{t-1})K, \ t \geq 1 \}, \]

To simplify the notation, we assume that

\[ l_t = 1, \ \forall \ t \geq \min \{ T_{\text{bankruptcy}}, T_{\text{takeover}} \}. \]

For the convenience of our discussion, now we define two sets of constraints, one on the manager’s strategy, the other on the following set of variables: the firm’s output, the manager’s total income from the project, the cash balance in the manager’s saving account, and the manager’s total wealth.

The constraints on the manager’s strategy, which specify the range of the manager’s choice variables and the values of these variables under different circumstances, are

\[ 0 \leq e_t \leq \bar{e}, \]

\[ 0 \leq c_t \leq W_t, \]

\[ 0 \leq i_t \leq y_t, \]

\[ 0 \leq m_t, \]

\[ \beta_t \in [0, 1] \]

\[ m_0 \equiv 0, \]
(100) $\beta_0 = 0,$

(101) $e_t = \begin{cases} 
\tau & \text{when } t \leq T_{\text{bankruptcy}} \\
0 & \text{when } t > T_{\text{bankruptcy}} 
\end{cases},$

and

(102) $\beta_t \equiv \begin{cases} 
1 & \text{when } t \geq T_{\text{takeover}} \text{ and } T_{\text{takeover}} < T_{\text{bankruptcy}} \\
0 & \text{when } t \geq T_{\text{bankruptcy}} \text{ and } T_{\text{bankruptcy}} < T_{\text{takeover}} 
\end{cases},$

where $T_{\text{bankruptcy}}$ and $T_{\text{takeover}}$ are given in (91) and (92) respectively.

The constraints on the firm’s output, the manager’s total income from the project, the cash balance in the manager’s saving account, and the manager’s total wealth are

(103) $y_t = rK + \alpha e_t,$

(104) $j_t = \begin{cases} 
y_t - i_t & \text{when } t \leq \min\{T_{\text{bankruptcy}}, T_{\text{takeover}}\} \\
y_t & \text{when } t > T_{\text{takeover}} \text{ and } T_{\text{takeover}} < T_{\text{bankruptcy}} \\
0 & \text{when } t > T_{\text{bankruptcy}} \text{ and } T_{\text{bankruptcy}} < T_{\text{takeover}} 
\end{cases},$
\[ m_{t+1} = \begin{cases} 
(1 + r) (m_t + j_t - (\beta_t - \beta_{t-1})K - c_t) \\
\text{when } t < \min\{T_{\text{takeover}}, T_{\text{bankruptcy}}\} \\
-(1 + r) \min\{K, (1 + r)(1 - \beta_{t-1})K - i_t\} \\
\text{when } t = T_{\text{takeover}} \text{ and } T_{\text{takeover}} < T_{\text{bankruptcy}} \\
(1 + r) (m_t + j_t - c_t) \\
\text{when } t > T_{\text{takeover}} \text{ and } T_{\text{takeover}} < T_{\text{bankruptcy}} \\
(1 + r) (m_t + j_t - c_t) + (1 + r)(K - \min\{K, (1 + r)(1 - \beta_{t-1})K - i_t\}) \\
\text{when } t = T_{\text{bankruptcy}} \text{ and } T_{\text{bankruptcy}} < T_{\text{takeover}} \\
(1 + r) (m_t - c_t) \\
\text{when } t \geq T_{\text{bankruptcy}} \text{ and } T_{\text{bankruptcy}} < T_{\text{takeover}} 
\end{cases} \]

and

\[ W_t = m_t + j_t, \]

where (105) provides the cash balance of the manager’s saving account under different circumstances.

Now we define the agents’ choice problems at a time when the money has already been borrowed and the parcel of land has already been purchased. The manager’s choice problem is

Problem 7 Manager’s choice (Debt-Repurchase) Given the sinking fund requirement on debt-repurchase over time \( \{\beta_t^* | \beta_0^* = 0, \beta_t^* \in [0, 1] \text{ and } \beta_t^* \leq \beta_{t-1}^*(1+r) + \frac{a^*}{K}, t \geq 1\} \), the required interest payments \( \{i_t^* = (1 - \beta_{t-1})r K, t \geq 1\} \) over time and the investor’s strategy \( \sigma_i \), the manager chooses \( \sigma_m \) to maximize the expected lifetime utility of consumption and effort

\[ \sum_{t=1}^{\infty} \frac{c_t - e_t}{(1 + r)^t} \]
subject to the constraints in both (94) to (102) and (103) to (106).

The investor’s choice problem is

**Problem 8 Investor’s choice (Debt-Repurchase)** Given the sinking fund requirement on debt-repurchase over time \( \{ \beta^*_t | \beta^*_0 = 0, \beta^*_t \in [0, 1] \text{ and } \beta^*_t \leq \beta^*_{t-1} (1+r) + \frac{\alpha}{K}, t \geq 1 \} \), the required interest payments \( \{ i^*_t = (1-\beta^*_{t-1})rK, t \geq 1 \} \) over time and the manager’s strategy \( \sigma_m \) satisfying the constraints in (94) to (102), the investor chooses \( \sigma_i \) to maximize the lifetime cash flow

\[
\sum_{t=1}^{T_{\min,d}^{-1}} i_t + (\beta_t - \beta_{t-1})K \left( \frac{1}{1+r} \right)^t + \min\{ K, (1+r)(1-\beta T_{\min,d}^{-1})K - i_t \} \left( \frac{1}{1+r} \right)^{T_{\min,d}^{-1}}
\]

subject to the constraints in (103) to (106), where

\[
(108) \ T_{\min,d} = \min\{ T_{\text{bankruptcy}}, T_{\text{takeover}} \},
\]

and \( T_{\text{bankruptcy}} \) and \( T_{\text{takeover}} \) are given in (91) and (92) respectively.

**Debt Financing**

We have the following result on debt financing of the project.\(^{20}\)

**Theorem 5 Equilibrium (Outside Debt with Debt-Repurchase)** Consider a proprietorship with outside debt that has the required interest payments of \( \{ i^*_t = (1-\beta^*_{t-1})rK, t > 0 \} \) and a project with an infinite life. We have the following result on debt financing of the project and the subgame perfect Nash equilibrium of the game between the owner-manager and the investor:

\(^{20}\)Here we consider only debt contracts with constant interest payments over time.
(i) Financing cannot be supported by the threat to force bankruptcy itself: Debt financing of the project cannot be supported by the threat to force bankruptcy itself, when the sinking fund is not used. The candidate bad equilibrium defined below is the only SPNE in pure strategies for all $\alpha > 0$.

(ii) Financing can be supported by sinking fund: With sinking fund, debt financing of the project can be supported when

$$1 + \frac{r^2 K}{\bar{e}} \leq \alpha \leq \frac{(1 - r)K}{\bar{e}}.$$  \hfill (109)

The candidate good equilibrium that provides the quickest repurchase of the debt is the candidate good equilibrium defined below in which the manager is required to spend all the income on debt-repurchase.

(iii) Financing of projects with finite life: With sinking fund, debt financing of a project with finite life $T$ and initial investment of $K\left(1 - \frac{1}{(1 + r)^T}\right)$ can be supported when

$$1 + \frac{r^2 K}{\left(1 - \left(1 + r\right)^{1-T}\right)\bar{e}} \leq \alpha < \left(1 - r - \frac{1}{(1 + r)^{T-1}}\right)\frac{K}{\bar{e}}.$$  \hfill (110)

and

$$\beta_{\text{takeover}}^{-1} \geq \frac{r}{1 + r - (1 + r)^{t-T}},$$  \hfill (111)

where

$$t_{\text{takeover}} \equiv \inf\{t|\alpha \bar{e} \geq (1 - r - \frac{1}{(1 + r)^{T-t}})K, \ t \geq 1\}.$$  

The candidate good equilibrium that provides the quickest repurchase of the debt is the candidate good equilibrium in which the manager is required to spend all the income on debt-repurchase.
The candidate bad equilibrium is defined by the owner-manager’s strategy for interest payment, consumption and debt-repurchase

\[(112) \ i_t(h^t) = 0\]

\[(113) \ c_t(h^t) \leq W_t\]

\[(114) \ \beta_1(h^t) = 0 \quad \text{and} \quad \beta_t(h^t) = \beta_{t-1}\]

and the investor’s strategy for forcing bankruptcy

\[(115) \ l_t(h^t, i_t) = 1.\]

The candidate good equilibrium that provides the quickest repurchase of the debt is defined by the owner-manager’s strategy for interest payment, consumption and debt-repurchase

\[(116) \ i_t(h^t) = \begin{cases} 
  i_t^* & \text{if} \ (\forall s < t) \ i_s = i_s^* \quad \text{and} \ \beta_s = \beta_s^* < 1 \\
  0 & \text{otherwise}
\end{cases}\]

\[(117) \ c_t(h^t) = \begin{cases} 
  0 & \text{when} \ \beta_{t-1} < 1 \\
  \leq W_t & \text{otherwise}
\end{cases}\]

\[(118) \ \beta_t(h^t, e_t) = \begin{cases} 
  \beta_t^* & \text{if} \ (\forall s < t) \ i_s = i_s^* \quad \text{and} \ \beta_s = \beta_s^* < 1 \\
  \beta_{t-1} & \text{otherwise}
\end{cases}\]

where

\[(119) \ \beta_t^* = \begin{cases} 
  \min\{\beta_{t-1}(1+r) + \frac{\alpha e}{K}, 1\} & \text{when} \ \beta_{t-1} < 1 \\
  1 & \text{otherwise}
\end{cases},\]
and the investor’s strategy for forcing bankruptcy

\begin{equation}
(120) \quad l_t(h^t, e_t) = \begin{cases} 
0 & \text{when } (\forall s \leq t) \ i_s \geq i^*_s \text{ and } \beta_s = \beta^*_s < 1 \\
1 & \text{otherwise}
\end{cases}
\end{equation}

**Proof:** See Appendix A.

The intuition behind the result on debt financing is similar to that behind the result on equity financing. In order to receive debt financing, a project can neither be too profitable nor be too unprofitable. In order for a project with a finite life to be financed with debt, a large enough proportion of the debt needs to be retired when the manager has enough wealth to take over the project.

The comparison between the results in the previous two theorems shows that depending on the parameter values, either debt or equity can dominate the other.
7 Conclusion

This paper considers the optimal choice between two different organizational forms for a project whose cash flows are observable to both the manager and investor, but not publicly verifiable. The nonverifiability of the firm’s cash flows creates an extreme form of manager’s moral hazard problem, which can both make the financing of the project difficult to arrange and have a large impact on the allocation of the control over the output of the firm and the form of managerial compensation. We find that the two organizational forms can have very different incentive problems. For the corporation form, the employee-manager may have an incentive to expend too little effort on the project, because the manager’s income is guaranteed by a fixed-wage contract. For the proprietorship, the owner-manager has no incentive to shirk, but may have an incentive to keep the entire cash flow and default on the debt. Depending on the relative severity of the two incentive problems, either organizational form can dominate the other. The possibility of an MBO can exacerbate the incentive problems and make it impossible to prevent shirking or defaulting on the debt with only the threat of dismissal or the threat of liquidation. Certain institutions, such as sinking funds, employee share ownership and managers’ non-vested pensions, can be used to mitigate the incentive problems and support the financing of the project.

The paper has two important predictions on the financing of the firm. One prediction is that the corporate form is more likely to be used by non-effort-intensive firms and the proprietorship is more likely to be used by non-capital-intensive firms. The other prediction is that it is in general more difficult to arrange equity financing than to arrange debt financing, and as a result, debt may be the preferred choice of the two when both can be used and we may see debt being used more often than equity.
A Proofs of the Theorems

Proof of Theorem 1: The game between the employee-manager and the investor is a game of perfect information with sequential moves. Therefore the subgame starting at any node is a proper subgame. To show that a pair of agents' strategies are subgame perfect Nash equilibrium strategies, following the definition of such strategies, we need to show that this pair of strategies are Nash equilibrium strategies in every subgame, i.e., in each subgame, given the strategy of one agent, the strategy of the other agent is this agent’s best response or optimal strategy. Given the sequential structure, it suffices to show for each strategy that the agent who moves first in the subgame is moving optimally in the subgame given the other agent’s claimed equilibrium strategy. For example, if the firing decision is optimal in every subgame given some initial effort by the manager, it is necessarily optimal given the manager’s actual choice of effort, and therefore once we know the firing decision is optimal in every subgame starting with the firing decision, it is also optimal in the subgame starting with the previous effort decision.

First, we prove (i): the candidate bad equilibrium is always a subgame perfect Nash equilibrium. First, we want to ask whether, if the manager did deviate by choosing $e_t < \bar{\sigma}$ in period $t$, the investor would choose to liquidate the firm and fire the manager. The manager’s strategy indicates that once he chooses his effort level to be below $\bar{\sigma}$, he will always choose zero effort level. Liquidating the project in period $t$ gives the investor his expected lifetime cash flow

\begin{equation}
(121) \quad -K + \sum_{s=1}^{t-1} \frac{(rK + \alpha e_t) - \alpha \bar{\sigma}}{(1 + r)^s} + \frac{(rK + \alpha e_t) - \alpha \bar{\sigma}}{(1 + r)^t} + \frac{K}{(1 + r)^t}
\end{equation}

\begin{align*}
= -K + \sum_{s=1}^{t-1} \frac{rK}{(1 + r)^s} + \frac{(rK + \alpha e_t) - \alpha \bar{\sigma}}{(1 + r)^t} + \frac{K}{(1 + r)^t},
\end{align*}

while delaying the liquidation of the project until period $i$ for $i > t$ gives the

50
investor his expected lifetime cash flow

\[
(122) \quad -K + \sum_{s=1}^{t-1} \frac{(rK + \alpha e) - \alpha e}{(1 + r)^s} + \frac{\alpha e_t}{(1 + r)^t} + \sum_{s=t}^{i} \frac{rK - \alpha e}{(1 + r)^s} + \frac{K}{(1 + r)^i}
\]

\[
= -K + \sum_{s=1}^{t-1} \frac{rK}{(1 + r)^s} + \frac{\alpha e_t}{(1 + r)^t} + \sum_{s=t}^{i} \frac{rK - \alpha e}{(1 + r)^s} + \frac{K}{(1 + r)^i}.
\]

The change in expected lifetime cash flow is given by

\[
(123) \quad \sum_{s=t+1}^{i} \frac{rK - \alpha e}{(1 + r)^s} + \frac{K}{(1 + r)^t} - \frac{K}{(1 + r)^t}
\]

\[
= \frac{K}{(1 + r)^t} - \sum_{s=t+1}^{i} \frac{\alpha e}{(1 + r)^s} - \frac{K}{(1 + r)^t}
\]

\[
= - \sum_{s=t+1}^{i} \frac{\alpha e}{(1 + r)^s} < 0.
\]

So the investor’s optimal strategy after the manager’s deviation is to liquidate the firm immediately. Next, we want to ask whether the manager should choose zero effort level in the subgame in which the manager did deviate by choosing \(e_t < \bar{e}\) in period \(t\), but the investor did not liquidate the project and fire the manager in the same period. The investor’s strategy indicates that once the manager chooses \(e_t < \bar{e}\), the investor will always choose to liquidate the project. Expecting the investor to liquidate the project at the end of the period, the manager’s optimal decision is to play his candidate bad equilibrium strategy: choose zero effort level and just collect the wage.

Next, we prove (ii): the candidate good equilibrium is the SPNE for all \(\alpha \geq 1 + r\). Since the candidate good and bad equilibria agree after any deviation, we have already shown (in the proof of (i)) that the agents’ candidate-good-equilibrium strategies are Nash equilibrium strategies in all subgames off the equilibrium path. Here we only need to show that these strategies are also Nash equilibrium strategies in all subgames along the equilibrium path. Given the investor’s strategy, consider a manager contemplating a defection from the optimal strategy by choosing a different strategy and specifically
choosing effort level different from $\overline{e}$ first at time $t$. Since feasible effort must satisfy $e_t \in [0, \overline{e}]$, any feasible deviation must have $e_t < \overline{e}$. The agent’s utility from the proposed equilibrium strategy is

$$
(124) \sum_{s=1}^{\infty} \frac{\alpha e - \overline{e}}{(1 + r)^s} = \frac{\alpha e - \overline{e}}{r}
$$

which is the utility for salary of $\alpha e$ and effort of $\overline{e}$ in every period. If the manager does deviate and chooses $e_t < \overline{e}$ first in period $t$, the manager will then be fired and the utility is instead

$$
(125) \sum_{s=1}^{t-1} \frac{\alpha e - \overline{e}}{(1 + r)^s} + \frac{\alpha e - e_t}{(1 + r)^t} = \frac{\alpha e - \overline{e}}{r} \left( 1 - \frac{1}{(1 + r)^{t-1}} \right) + \frac{\alpha e - e_t}{(1 + r)^t}.
$$

The change in expected utility is given by

$$
(126) \frac{\alpha e - e_t}{(1 + r)^t} - \frac{\alpha e - \overline{e}}{r(1 + r)^{t-1}} = \frac{1}{r(1 + r)^t} \left( r(\alpha e - e_t) - (\alpha - 1)(1 + r)\overline{e} \right) \\
\leq \frac{\overline{e}}{r(1 + r)^t} \left( r\alpha - \alpha - r\alpha + 1 + r \right) \\
= \frac{\overline{e}}{r(1 + r)^t} (1 + r - \alpha) \\
\leq 0,
$$

(since by assumption $\alpha \geq 1 + r$,) so deviation does not pay. Given the manager’s strategy, the investor’s utility is $K$ if he liquidates the project. If he does not liquidate the project, his strategy, together with the manager’s strategy, would create a trajectory that is the same as the entire equilibrium path. The investor receives a payoff of $rK$ in every period along this trajectory, and his utility from not liquidating the project is

$$
(127) \sum_{s=t+1}^{\infty} \frac{rK}{(1 + r)^{s-t}} = K.
$$
So the manager’s strategy of not liquidating the project is his optimal response to the investor’s strategy.

Next, we prove (iii): if the candidate good equilibrium is a SPNE in pure strategies, then $\alpha \geq 1 + r$. The change in the manager’s expected lifetime cash flow from a deviation from the equilibrium path, as given in (126), is

\[
(128) \quad \frac{\alpha c - e_t}{(1 + r)^t} - \frac{\alpha c - c}{r(1 + r)^{t-1}} = \frac{1}{r(1 + r)^t} (r \alpha c - (\alpha - 1)(1 + r)c) - \frac{e_t}{(1 + r)^t} = \frac{c}{r(1 + r)^t} (1 + r - \alpha) - \frac{e_t}{(1 + r)^t}.
\]

In order for the right-hand-side of (128) to be non-positive for all $e_t \in [0, c)$, $\alpha$ has to be as least as large as $1 + r$.

At last, we prove (iv): only the good equilibrium makes the enterprise viable, since an investor anticipating the bad equilibrium would not expect to recover the initial investment of $K$. In case the bad equilibrium occurs after making the investment $K$, the investor will liquidate the project and fire the manager at the end of the first period, which gives him a payoff off $rK - \alpha c + K$. The present value of the investor’s income from the project is then

\[
(129) \quad \frac{rK - \alpha c + K}{1 + r} < \frac{rK + K}{1 + r} = K.
\]

So the investor will refuse to invest on the project if he expects the bad equilibrium to occur.

**Proof of Theorem 2:** Because of the similarity between this proof and the proof of Theorem 1, we provide only the key steps in this proof.

Since the proof of part (i) is very similar to the proof of part (i) in Theorem 1, we will not provide the proof (of part (i)) here.
Now we prove (ii): the candidate good equilibrium is a SPNE if and only if

(130) $i_t^* \in [0, rK + \alpha \bar{e}]$ \quad \forall t \geq 1,

(131) $\sum_{s=1}^{+\infty} \frac{i_s^*}{(1 + r)^s} = K$ and

(132) $\sum_{s=t+1}^{+\infty} \frac{rK + (\alpha - 1)e - i_s^*}{(1 + r)^s-t} \geq i_t^* \quad \forall t \geq 1.$

First, we show that the candidate good equilibrium is a SPNE when (130), (131) and (132) hold. Since the candidate good and bad equilibria agree after any deviation, we have already shown (in the proof of (i)) that the agents’ candidate-good-equilibrium strategies are Nash equilibrium strategies in all subgames off the equilibrium path. Here we only need to show that these strategies are also Nash equilibrium strategies in all subgames along the equilibrium path. Given the investor’s strategy, the manager’s utility from the proposed equilibrium strategy is

(133) $\sum_{s=1}^{+\infty} \frac{rK + \alpha \bar{e} - i_s^* - \bar{e}}{(1 + r)^s}.$

The manager’s utility from a deviation from the proposed equilibrium strategy at time $t$ by choosing $i_t < i_t^*$ is

(134) $\sum_{s=1}^{t-1} \frac{rK + \alpha \bar{e} - i_s^* - \bar{e}}{(1 + r)^s} + \frac{rK + \alpha \bar{e} - i_t - \bar{e}}{(1 + r)^t}.$
The change in expected utility is given by

\[
(135) \quad \frac{i_t^* - i_t}{(1 + r)^t} - \sum_{s=t+1}^{\infty} \frac{rK + \alpha \bar{e} - i_s^* - \bar{e}}{(1 + r)^s} \\
\leq \frac{1}{(1 + r)^t} \left( i_t^* - \sum_{s=t+1}^{\infty} \frac{rK + (\alpha - 1)\bar{e} - i_s^*}{(1 + r)^{s-t}} \right) \leq 0
\]

by (132). So the deviation does not pay.

Given the manager’s strategy, the investor’s expected lifetime cash flow from the proposed equilibrium strategy is

\[
(136) \quad \sum_{s=1}^{\infty} \frac{i_s^*}{(1 + r)^s} = K.
\]

The investor’s expected lifetime cash flow from a deviation from the proposed equilibrium strategy at time \( t \) by choosing \( l_t = 1 \) is

\[
(137) \quad \sum_{s=1}^{t-1} \frac{i_s^*}{(1 + r)^s} + \frac{1}{(1 + r)^t} \left( i_t^* + \min\left\{ \sum_{s=t+1}^{\infty} \frac{i_s^*}{(1 + r)^{s-t}}, K \right\} \right)
\]

The change in expected lifetime cash flow is given by

\[
(138) \quad \frac{1}{(1 + r)^t} \left( \min\left\{ \sum_{s=t+1}^{\infty} \frac{i_s^*}{(1 + r)^{s-t}}, K \right\} - \sum_{s=t+1}^{\infty} \frac{i_s^*}{(1 + r)^{s-t}} \right) \leq 0.
\]

So the deviation does not pay for the investor either.

Next, we show that if the candidate good equilibrium is a subgame perfect Nash equilibrium, then (130), (131) and (132) hold. Since both (130) and (131) hold by assumption, we need only to prove that (132) holds. The
change in the manager’s expected lifetime cash flow from a deviation from the equilibrium path, as given in (135), is

\[
\begin{align*}
(139) \quad \frac{i^*_t - i_t}{(1 + r)^t} - \sum_{s=t+1}^{\infty} \frac{rK + \alpha \bar{\pi} - i^*_s - \bar{\pi}}{(1 + r)^s} \\
= \frac{1}{(1 + r)^t} \left( i^*_t - \sum_{s=t+1}^{\infty} \frac{rK + (\alpha - 1) \bar{\pi} - i^*_s}{(1 + r)^{s-t}} \right) + \frac{i_t}{(1 + r)^t}
\end{align*}
\]

In order for the right-hand-side of (139) to be non-positive for all \(i_t \in [0, rK + \alpha \bar{\pi}]\), (132) has to hold.

Now we prove (iii): the good equilibrium that allows the project with the lowest possible \(\alpha\) to receive financing is the equilibrium with constant interest payment over time

\[
(140) \quad i^*_t = rK \quad \forall t \geq 1.
\]

From (ii), we know that both the necessary and sufficient conditions for the project to receive financing are that (130), (131) and (132) hold. From (131) and (132), we can determine the lower bound of \(\alpha\)

\[
(141) \quad \frac{(\alpha - 1) \bar{\pi}}{r} \geq i^*_t + \sum_{s=2}^{\infty} \frac{i^*_s}{(1 + r)^{s-1}} - \frac{rK}{r} = (1 + r)K - K = rK.
\]

It can easily be verified that with the interest payments in (140), a project with \(\alpha = 1 + \frac{r^2K}{r^2} \) should be able to receive financing, i.e., for \(i^*_t = rK, \forall t \geq 1\), and \(\alpha = 1 + \frac{r^2K}{r^2}\), (130), (131) and (132) hold.

Part (iv) can be proved with the same arguments as those in the proof of part (iv) in Theorem 1: in case the bad equilibrium occurs after making the initial investment \(K\), the investor will liquidate the project and fire the manager at the end of the first period, which would give him the payoff off
\[ i_1 + K \text{ where } i_1 < rK. \]

The present value of the investor’s income from the project is then

\[
\frac{i_1 + K}{1 + r} < \frac{rK + K}{1 + r} = K. \tag{142}
\]

So the investor should refuse to lend to the manager if he expects the bad equilibrium to occur.

**Proof of Theorem 4:** Because of the similarity between the proof of this theorem and the proofs of Theorem 1 and Theorem 2, we provide only the key steps in this proof.

First, we prove (i): the financing of the project cannot be supported by the threat to fire the manager itself. It suffices to show that the equityholder’s strategy for dismissal in the candidate good equilibrium in Theorem 1 itself cannot deter the manager from shirking. One way to prove this is to show that given the equityholder’s strategy for dismissal

\[
l_t(h^t, e_t) = \begin{cases} 
0 & \text{when } (\forall s \leq t)e_s = \bar{e} \\
1 & \text{otherwise}
\end{cases}, \tag{143}
\]

the manager is better off playing the following strategy on effort and consumption\(^{22}\)

\[
e_t(h^t) = \begin{cases} 
\bar{e} & \text{when } \beta_{t-1} = 0 \text{ and } W_t < K \\
0 & \text{when } \beta_{t-1} = 0 \text{ and } W_t \geq K \\
\bar{e} & \text{when } \beta_{t-1} = 1
\end{cases}, \tag{144}
\]

\[
c_t(h^t) = \begin{cases} 
0 & \text{when } \beta_{t-1} = 0 \\
\leq W_t & \text{when } \beta_{t-1} = 1
\end{cases} \tag{145}
\]

---

\(^{21}\)Here we focus on the equilibrium with constant interest payments over time: \(i_1^* = rK, \forall t \geq 1.\)

\(^{22}\)Since the investor does not compensate the manager in the company’s shares before the MBO if any, the manager’s ownership of the firm is going to be zero before the MBO and one after the MBO.
which involves shirking (when the manager is able to take over the firm) and taking over the firm through MBO, rather than the manager’s strategy in the candidate good equilibrium in Theorem 1

\[
(146) \quad c_t(h^t) = \begin{cases} \bar{e} & \text{if } (\forall s < t) e_s = \bar{e} \\ 0 & \text{otherwise} \end{cases}
\]

(147) \( c_t(h^t) \leq W_t \)

which requires the manager to expend the first-best effort in all periods.

Given the equityholder’s strategy in (143), the manager’s utility from playing the strategy in (146) and (147) is

\[
(148) \quad \sum_{s=1}^{\infty} \frac{(\alpha - 1)\bar{e}}{(1 + r)^s} = \frac{(\alpha - 1)\bar{e}}{r}.
\]

If instead playing the strategy in (144) and (145), the manager gets

\[
(149) \quad \sum_{s=1}^{T_{mbo} - 1} \frac{(\alpha - 1)\bar{e}}{(1 + r)^s} + \frac{\alpha\bar{e} - K}{(1 + r)^{T_{mbo}}} + \sum_{s=T_{mbo} + 1}^{\infty} \frac{rK + (\alpha - 1)\bar{e}}{(1 + r)^s} = \frac{(\alpha - 1)\bar{e}}{r} + \frac{\bar{e}}{(1 + r)^{T_{mbo}}}
\]

where

\[
(150) \quad T_{mbo} \equiv \inf\{t| l_t = 1 \text{ and } W_t \geq K, \ t \geq 1\}.
\]

The gain for the manager equals

\[
(151) \quad \frac{\bar{e}}{(1 + r)^{T_{mbo}}} > 0.
\]
Next, we prove (ii): the financing can be supported by employee share ownership. Here we only prove that the agents’ strategies given below are equilibrium strategies when

\[(152) \alpha \in \left[1 + r, \frac{K}{\bar{e}}\right]\]

and

\[(153) \beta_{T_{mb}} - 1 \geq \frac{1}{\alpha}\]

where

\[
T_{mb} = \inf \{t \mid \frac{\alpha \bar{e} ((1 + r)^t - 1)}{r} \geq K\}
\]

\[
\beta_{T_{mb}} - 1 = \frac{\alpha \bar{e} ((1 + r)^{T_{mb}} - 1)}{rK}.
\]

The manager’s strategy on effort and consumption is

\[(154) e_t(h^t) = \begin{cases} \bar{e} & \text{if } (\forall s < t) e_s = \bar{e} \\ 0 & \text{otherwise} \end{cases}\]

\[(155) c_t(h^t) = \begin{cases} 0 & \text{when } \beta_t < 1 \\ \leq W_t & \text{otherwise} \end{cases},\]

and the investor’s strategy for liquidation and manager’s share ownership is

\[(156) l_t(h^t, e_t) = \begin{cases} 0 & \text{when } (\forall s \leq t) e_s = \bar{e} \\ 1 & \text{otherwise} \end{cases}\]
\( (157) \beta_t(h^t, e_t) = \min\{\beta_{t-1}(1 + r) + \frac{\alpha \bar{e}}{K}, 1\} \) when \( \beta_{t-1} < 1 \).

First, it is quite clear that we need to have \( \alpha < \frac{K}{\bar{e}} \), otherwise the manager can shirk in the first period and then take over the project. The proof of that the strategy in (156) and (157) is the optimal response to the strategy in (154) and (155) is very similar to the proof of part (ii) in Theorem 1. Here we provide only the proof of that the strategy in (154) and (155) is the optimal response to the strategy in (156) and (157) when (152) and (153) hold.

Given the equityholder’s strategy in (156) and (157), the manager’s utility from playing the strategy in (154) and (155) is

\[
(158) \sum_{s=1}^{\infty} \frac{(\alpha - 1)e_s}{(1 + r)^s} = \frac{(\alpha - 1)e_t}{r}.
\]

Now we consider a deviation from the strategy in (154) and (155). Suppose the manager chooses \( e_t < \bar{e} \) in period \( t \) when \( \beta_{t-1} < 1 \) and \( W_t < K \). Then the manager’s utility from playing this new strategy is

\[
(159) \frac{\alpha \bar{e} + \beta_{t-1}(K + rK + \alpha e_t - \alpha \bar{e}) - e_t}{(1 + r)^t} = \frac{\alpha \bar{e}}{(1 + r)^t} + \sum_{s=1}^{t-1} \frac{\alpha e_s}{(1 + r)^s} + \frac{\alpha \beta_{t-1}(e_t - \bar{e}) - e_t}{(1 + r)^t} - \frac{\bar{e}}{(1 + r)^t} - \frac{1 - \bar{e}}{(1 + r)^{t-1}} + \frac{\alpha \beta_{t-1}(e_t - \bar{e}) - e_t}{(1 + r)^t}.
\]

The term \( \sum_{s=1}^{t-1} \frac{\alpha e_s}{(1 + r)^s} \) on the right-hand-side of this equation equals the value of the manager’s ownership of the firm at time \( t \). The term \( -\sum_{s=1}^{t-1} \frac{e_s}{(1 + r)^s} \)
equals the disutility the manager receives from generating this value. The change in expected utility is given by

\[
\begin{align*}
\alpha e^{(1 + r) t} & \quad \alpha \beta_{t-1}(e_t - \bar{e}) - e_t \\
= \frac{1}{r(1 + r)^t} & \quad (ra - (\alpha - 1)(1 + r)\bar{e}) + \frac{\alpha \beta_{t-1}(e_t - \bar{e}) - e_t}{(1 + r)^t} \\
= \frac{\bar{e}}{r(1 + r)^t} & \quad (1 + r - \alpha \alpha + 1 + r) + \frac{\alpha \beta_{t-1}(e_t - \bar{e}) - e_t}{(1 + r)^t} \\
\leq & \quad 0,
\end{align*}
\]

(since by assumption \(\alpha \geq 1 + r\),) so the deviation does not pay.

Now we show that we will have a smooth transfer of the firm’s ownership from the investor to the manager when (152) and (153) hold. It suffices to show that when (152) and (153) hold, the manager has no incentive to shirk even if he is able to take over the project (i.e. even if \(W_t \geq K\)). The manager’s utility from deviating from the good equilibrium strategy by choosing effort \(e_t < \bar{e}\) and then taking over the firm in period \(t\) is

\[
\begin{align*}
(161) \quad & \quad \frac{\alpha e \bar{e} + \beta_{t-1}(K + rK + \alpha e_t - \alpha \bar{e}) - e_t}{(1 + r)^t} - \sum_{s=1}^{t-1} \frac{\bar{e}}{(1 + r)^s} - \frac{K}{(1 + r)^t} \\
+ \sum_{s=t+1}^{\infty} & \quad \frac{rK + (\alpha - 1)\bar{e}}{(1 + r)^s} \\
= & \quad \frac{(\alpha - 1)\bar{e} + \beta_{t-1}(1 + r)K}{(1 + r)^t} - \sum_{s=1}^{t-1} \frac{\bar{e}}{(1 + r)^s} + \sum_{s=t+1}^{\infty} \frac{(\alpha - 1)\bar{e}}{(1 + r)^s} \\
+ & \quad \frac{(1 - \alpha \beta_{t-1})(\bar{e} - e_t)}{(1 + r)^t} \\
= & \quad \sum_{s=1}^{\infty} \frac{(\alpha - 1)\bar{e}}{(1 + r)^s} + \frac{(1 - \alpha \beta_{t-1})(\bar{e} - e_t)}{(1 + r)^t}.
\end{align*}
\]
The change in expected utility is given by

\[
(162) \quad \frac{(1 - \alpha \beta_{t-1})(\tau - e_t)}{(1 + r)^t} \leq 0
\]

if and only if

\[
(163) \quad 1 - \alpha \beta_{t-1} \leq 0, \quad \forall \ t \geq T_{mbo}
\]
or i.e.

\[
(164) \quad \beta_{t-1} \geq \frac{1}{\alpha}, \quad \forall \ t \geq T_{mbo}.
\]

Now we show (iii): the financing can be supported by the manager’s pension with an infinite non-vesting period. Specifically, we show that by holding an amount \( P \) no smaller than \( \tau \) in the manager’s pension account against the shirking, the strategies

\[
(165) \quad e_t(h^t) = \begin{cases} \tau & \text{if } (\forall s < t)e_s = \tau \\ 0 & \text{otherwise} \end{cases}
\]

\[
(166) \quad c_t(h^t) \leq W_t
\]

\[
(167) \quad l_t(h^t, e_t) = \begin{cases} 0 & \text{when } (\forall s \leq t)e_s = \tau \\ 1 & \text{otherwise} \end{cases}
\]

are SPNE strategies. Here we only verify that given the equityholder’s strategy in (167) and the amount \( P \geq \tau \) held in the manager’s pension account, the manager cannot benefit from shirking. Given the equityholder’s strategy
in (167), the manager’s utility from playing the strategy in (165) and (166) is

\[ (168) \sum_{s=1}^{\infty} \frac{(\alpha - 1)\bar{\sigma}}{(1 + r)^s} = \frac{(\alpha - 1)\bar{\sigma}}{r}. \]

Now we consider a deviation from the strategy in (165) and (166). Suppose
the manager chooses \( e_t < \bar{\sigma} \) in period \( t \). Then the manager’s utility from doing this new strategy is

\[ (169) \sum_{s=1}^{t-1} \frac{\alpha\bar{\sigma} - \bar{\sigma}}{(1 + r)^s} + \frac{\alpha\bar{\sigma} - e_t - P}{(1 + r)^t} \]
\[ = \frac{\alpha\bar{\sigma} - \bar{\sigma}}{r} \left( 1 - \frac{1}{(1 + r)^{t-1}} \right) + \frac{\alpha\bar{\sigma} - e_t - P}{(1 + r)^t} \]

if the manager is not able to take over the firm in period \( t \), and

\[ (170) \sum_{s=1}^{t-1} \frac{\alpha\bar{\sigma} - \bar{\sigma}}{(1 + r)^s} + \frac{\alpha\bar{\sigma} - e_t - P}{(1 + r)^t} - \frac{K}{(1 + r)^t} + \sum_{s=t+1}^{\infty} \frac{rK + \alpha\bar{\sigma} - \bar{\sigma}}{(1 + r)^s} \]
\[ = \frac{(\alpha - 1)\bar{\sigma}}{r} + \frac{\bar{\sigma} - e_t - P}{(1 + r)^t} \]

if the manager is able to do so. The change in expected utility is at most

\[ (171) \frac{\bar{\sigma} - e_t - P}{(1 + r)^t} \leq 0, \]

since \( P \geq \bar{\sigma} \). So the deviation does not pay.

At last, we prove (iv): projects with finite life can be financed with either the employee share ownership or the manager’s non-vested pension with a reasonably long non-vesting period. Since the proof for financing with non-vested pension is similar to that of part (iii), we will provide only the proof
for financing with employee share ownership here. The project considered is assumed to have a finite life of $T$ periods and an initial investment of $K(1 - \frac{1}{(1+r)^T})$. Here we only show that the strategies

$$
(172) \quad e_t(h^t) = \begin{cases} 
\bar{e} & \text{if } (\forall s < t)e_s = \bar{e} \\
0 & \text{otherwise}
\end{cases}
$$

$$
(173) \quad c_t(h^t) = \begin{cases} 
0 & \text{when } \beta_{t-1} < 1 \\
\leq W_t & \text{otherwise}
\end{cases}
$$

$$
(174) \quad l_t(h^t, e_t) = \begin{cases} 
0 & \text{when } (\forall s \leq t)e_s = \bar{e} \\
1 & \text{otherwise}
\end{cases}
$$

$$
(175) \quad \beta_t(h^t, e_t) = \min\{ \frac{\alpha \bar{e}((1+r)^t - 1)}{(1 - (1+r)^{t-T})rK}, 1 \} \text{ when } t \leq T - 1
$$

are equilibrium strategies when

$$
(176) \quad \alpha \in \left[ 1 + \frac{r}{1 - (1+r)^{1-T}}, (1 - \frac{1}{(1+r)^{T-1}}\frac{K}{\bar{e}}) \right]
$$

and

$$
(177) \quad \beta_{t_{mbo} - 1} \geq \frac{1}{\alpha},
$$

where

$$
\begin{align*}
t_{mbo} & = \inf\{ t | \frac{\alpha \bar{e}((1+r)^t - 1)}{r} \geq (1 - \frac{1}{(1+r)^{T-t}})K \} \\
\beta_{t_{mbo} - 1} & = \frac{\alpha \bar{e}((1+r)^{t_{mbo} - 1} - 1)}{(1 - (1+r)^{t_{mbo} - T-1})rK}.
\end{align*}
$$
First, it is quite clear that we need to have $\alpha < (1 - \frac{1}{(1+r)^T})\frac{K}{\bar{e}}$, otherwise the manager can shirk in the first period and then take over the project. Since one part of the proof is simple, we will omit it here and provide the other part by showing that the strategy in (172) and (173) is the optimal response to the strategy in (174) and (175) when (176) and (177) hold. The arguments used in this part of the proof are similar to those used in the proof of part (ii).

Given the equityholder’s strategy in (174) and (175), the manager’s utility from playing the strategy in (172) and (173) is

\[
(178) \sum_{s=1}^{T} \frac{(\alpha - 1)\bar{e}}{(1+r)^s} = \frac{(\alpha - 1)\bar{e}}{r} \left(1 - \frac{1}{(1+r)^T}\right).
\]

Now we consider a deviation from the strategy in (172) and (173). Suppose the manager chooses $e_t < \bar{e}$ in period $t$ when $\beta_{t-1} < 1$ and $W_t < K(1 - \frac{1}{(1+r)^{t-1}})$. Then the manager’s utility from playing this new strategy is

\[
(179) \frac{\alpha \bar{e} + \beta_{t-1}((1 - (1+r)^{t-T})K + rK + \alpha e_t - \alpha \bar{e}) - e_t}{(1+r)^t},
\]

\[
-\sum_{s=1}^{t-1} \frac{\bar{e}}{(1+r)^s} = \frac{\alpha \bar{e}}{(1+r)^t} + \sum_{s=1}^{t-1} \frac{\alpha \bar{e}}{(1+r)^s} - \sum_{s=1}^{t-1} \frac{\bar{e}}{(1+r)^s} + \frac{\alpha \beta_{t-1}(e_t - \bar{e}) - e_t}{(1+r)^t} \]

\[
= \frac{\alpha \bar{e}}{(1+r)^t} + \frac{(\alpha - 1)\bar{e}}{r} \left(1 - \frac{1}{(1+r)^{t-1}}\right) + \frac{\alpha \beta_{t-1}(e_t - \bar{e}) - e_t}{(1+r)^t}.
\]

The change in expected utility is given by

\[
(180) \frac{\alpha \bar{e}}{(1+r)^t} - \frac{(\alpha - 1)\bar{e}}{r(1+r)^{t-1}} + \frac{(\alpha - 1)\bar{e}}{r(1+r)^t} + \frac{\alpha \beta_{t-1}(e_t - \bar{e}) - e_t}{(1+r)^t} \]

\[
= \frac{\bar{e}}{r(1+r)^t} (1 + r - \alpha) + \frac{(\alpha - 1)\bar{e}}{r(1+r)^t} + \frac{\alpha \beta_{t-1}(e_t - \bar{e}) - e_t}{(1+r)^t}.
\]
\[
= \frac{(\alpha - 1) \bar{\epsilon}}{r} \left( \frac{1}{(1 + r)^T} - \frac{1}{(1 + r)^t} \right) + \frac{\bar{\epsilon}}{(1 + r)^t} + \frac{\alpha \beta_{t-1}(e_t - \bar{\epsilon})}{(1 + r)^t} - e_t.
\]

In order for the right-hand-side of (180) to be non-positive, \( \alpha \) needs to satisfy

\[
(181) \quad \alpha \geq 1 + r \max_{t \geq 1} \left( \frac{1}{(1 + r)^t} - \frac{1}{(1 + r)^{t+1}} \right) = 1 + r \left( \frac{1}{(1 + r)^t} - \frac{1}{(1 + r)^{T+1}} \right) |_{t=1}
= 1 + \frac{r}{1 - (1 + r)^{1-\bar{\epsilon}}}. \]

Now we show that we will have a smooth transfer of the firm’s ownership from the investor to the manager when (176) and (177) hold. It suffices to show that the manager has no incentive to shirk even if he is able to take over the project (i.e. even if \( W_t \geq K(1 - \frac{\bar{\epsilon}}{(1 + r)^{t+1}}) \)). The manager’s utility from deviating by choosing effort \( e_t < \bar{\epsilon} \) and then taking over in period \( t \) is

\[
(182) \quad \frac{\alpha \bar{\epsilon} + \beta_{t-1} \left((1 - (1 + r)^{t-T})K + rK + \alpha e_t - \alpha \bar{\epsilon}\right) - e_t}{(1 + r)^t} - \frac{\sum_{s=1}^{t-1} \bar{\epsilon} \frac{1}{(1 + r)^s}}{(1 + r)^t} - \frac{(1 - (1 + r)^{t+1-T})K}{(1 + r)^t} + \frac{\sum_{s=t+1}^{T} rK + (\alpha - 1) \bar{\epsilon}}{(1 + r)^s} - \sum_{s=1}^{t-1} \frac{\bar{\epsilon}}{(1 + r)^s}
+ \sum_{s=t+1}^{T} \frac{(\alpha - 1) \bar{\epsilon}}{(1 + r)^s} + \frac{(1 - \alpha \beta_{t-1})(\bar{\epsilon} - e_t)}{(1 + r)^t}
= \sum_{s=1}^{T} \frac{(\alpha - 1) \bar{\epsilon}}{(1 + r)^s} + \frac{(1 - \alpha \beta_{t-1})(\bar{\epsilon} - e_t)}{(1 + r)^t}
\]

The change in expected utility is

\[
(183) \quad \frac{(1 - \alpha \beta_{t-1})(\bar{\epsilon} - e_t)}{(1 + r)^t} \leq 0
\]
if and only if

\[(184) \beta_{t-1} \geq \frac{1}{\alpha}, \quad \forall t \geq t_{mbo}.\]

**Proof of Theorem 5:** Because of the similarity between the proof of this theorem and the proof of Theorem 4, we provide only the key steps in the proof.

Since the proof of (i) is very similar to the proof of (i) in Theorem 4, we will not provide the proof here.

First, we prove (ii): the financing can be supported by sinking fund. Here we only prove that the strategies given below are equilibrium strategies when

\[(185) 1 + \frac{r^2 K}{\bar{e}} \leq \alpha \leq \frac{(1 - r)K}{\bar{e}}.\]

The manager’s strategy for interest payment, consumption and debt-repurchase is

\[(186) i_t(h^t) = \begin{cases} i_t^* & \text{if } (\forall s < t) i_s = i_s^* \text{ and } \beta_s = \beta_s^* < 1 \\ 0 & \text{otherwise} \end{cases}\]

\[(187) c_t(h^t) = \begin{cases} 0 & \text{when } \beta_{t-1} < 1 \\ \leq W_t & \text{otherwise} \end{cases}\]

\[(188) \beta_t(h^t, e_t) = \begin{cases} \beta_t^* & \text{if } (\forall s < t) i_s = i_s^* \text{ and } \beta_s = \beta_s^* < 1 \\ \beta_{t-1} & \text{otherwise} \end{cases},\]

and the investor’s strategy for forcing bankruptcy is

\[(189) l_t(h^t, e_t) = \begin{cases} 0 & \text{when } (\forall s \leq t) i_s \geq i_s^* \text{ and } \beta_s = \beta_s^* \\ 1 & \text{otherwise} \end{cases},\]
where

\[(190) \beta_t^* = \begin{cases} 
\min\{\beta_{t-1}(1+r) + \frac{\alpha e}{K}, 1\} & \text{when } \beta_{t-1} < 1 \\
1 & \text{otherwise}
\end{cases}.\]

First, it is quite clear that we need to have \(\alpha \geq 1 + \frac{r^2 K}{e}\), otherwise the manager can benefit from defaulting on the debt in the first period. It is also quite clear that we need to have \(\alpha < \frac{(1-r)K}{e}\), otherwise the manager can default on the debt in the first period and then take over the project through debt-repurchase. Since one part of the proof is simple, we will omit it here and provide the other part by showing that the strategy in (186) to (188) is the optimal response to the strategy in (189) when (185) holds. Given the debtholder’s strategy in (189), the manager’s utility from playing the strategy in (186) to (188) is

\[(191) \sum_{s=1}^{\infty} \frac{(\alpha - 1)e}{(1+r)^s} = \frac{(\alpha - 1)e}{r}.
\]

Now we consider a deviation from the strategy in (186) to (188). Suppose the manager chooses \(i_t < rK\) and \(\beta_t = \beta_{t-1}\) in period \(t\) when \(\beta_{t-1} < 1\) and \(W_t < \min\{K, (1-\beta_{t-1})(1+r)K\}\). Then the manager’s utility from playing this new strategy is

\[(192) \frac{rK + \alpha e - i_t + \max\{K - (1+r)(1-\beta_{t-1})K, 0\}}{(1+r)^t} - \sum_{s=1}^{t-1} \frac{e}{(1+r)^s} = \frac{rK + (\alpha - 1)e - i_t + \max\{\beta_{t-1}(1+r)K - rK, 0\}}{(1+r)^t} - \sum_{s=1}^{t-1} \frac{e}{(1+r)^s} - \sum_{s=1}^{t-1} \frac{\alpha e}{(1+r)^s} - \frac{rK}{(1+r)^t} + \max\{\sum_{s=1}^{t-1} \frac{\alpha e}{(1+r)^s} - \frac{rK}{(1+r)^t}, 0\} - \sum_{s=1}^{t-1} \frac{e}{(1+r)^s}.
\]
\[\begin{align*}
&= rK + (\alpha - 1)e^r - i_t - \frac{e^r}{1 + r} \left( 1 - \frac{1}{(1 + r)^{t-1}} \right) \\
&\quad + \max\left\{ \frac{e^r (1 - \alpha)}{r} \left( 1 - \frac{1}{(1 + r)^t} \right) - \frac{rK}{(1 + r)^t}, 0 \right\}.
\end{align*}\]

The change in expected utility is given by

\[\begin{align*}
(193) \quad &\frac{(\alpha - 1)e^r - i_t}{(1 + r)^t} + \frac{(\alpha - 1)e^r}{r} \left( 1 - \frac{1}{(1 + r)^{t-1}} \right) - \frac{(\alpha - 1)e^r}{r} \\
&= \frac{(\alpha - 1)e^r - i_t}{(1 + r)^t} - \frac{(\alpha - 1)e^r}{r(1 + r)^{t-1}} \\
&= - \frac{(\alpha - 1)e^r}{r(1 + r)^t} - \frac{i_t}{(1 + r)^t} \leq 0
\end{align*}\]

when \( \frac{\alpha e^r (1 - \alpha)}{r} \left( 1 - \frac{1}{(1 + r)^t} \right) > \frac{rK}{(1 + r)^t} \), and

\[\begin{align*}
(194) \quad &\frac{rK + (\alpha - 1)e^r - i_t}{(1 + r)^t} - \frac{e^r}{r} \left( 1 - \frac{1}{(1 + r)^{t-1}} \right) - \frac{(\alpha - 1)e^r}{r} \\
&= \frac{r^2K + r(\alpha - 1)e^r - (1 + r)^t(\alpha - 1)e^r}{r(1 + r)^t} - \frac{i_t}{(1 + r)^t} \\
&\quad - \frac{e^r}{r} \left( 1 - \frac{1}{(1 + r)^{t-1}} \right) \\
&\leq \frac{r^2K + r(\alpha - 1)e^r - (1 + r)(\alpha - 1)e^r}{r(1 + r)^t} - \frac{i_t}{(1 + r)^t} \\
&\quad - \frac{e^r}{r} \left( 1 - \frac{1}{(1 + r)^{t-1}} \right) \\
&= \frac{r^2K - (\alpha - 1)e^r}{r(1 + r)^t} - \frac{i_t}{(1 + r)^t} - \frac{e^r}{r} \left( 1 - \frac{1}{(1 + r)^{t-1}} \right) \\
&\leq 0
\end{align*}\]

(since by assumption \( \alpha \geq 1 + \frac{r^2K}{e^r} \) when \( \frac{\alpha e^r (1 - \alpha)}{r} \left( 1 - \frac{1}{(1 + r)^t} \right) \leq \frac{rK}{(1 + r)^t} \).
Now we consider a deviation from the strategy in (186) to (188) when \( \beta_{t-1} < 1 \) and \( W_t \geq \min\{K, (1 - \beta_{t-1})(1 + r)K\} \). Since \( y_t = rK + \alpha \bar{e} \) (when the manager defaults on the debt) is always smaller than \( K \) by assumption \( \alpha < \frac{(1 - r)K}{e} \), the manager is going to be able to take over the firm through debt-repurchase only when \( (1 - \beta_{t-1})(1 + r)K < K \). Suppose the manager chooses \( i_t < rK \) and \( \beta_t = \beta_{t-1} \) in period \( t \). Then the manager’s utility from playing this new strategy is

\[
(195) \quad \frac{rK + \alpha \bar{e} - i_t}{(1 + r)^t} - \sum_{s=1}^{t-1} \frac{\bar{e}}{(1 + r)^s} + \sum_{s=t+1}^{\infty} \frac{rK + (\alpha - 1)\bar{e}}{(1 + r)^s} \\
= \frac{(1 + r)K + (\alpha - 1)\bar{e} - i_t}{(1 + r)^t} - \sum_{s=1}^{t-1} \frac{\bar{e}}{(1 + r)^s} + \sum_{s=t+1}^{\infty} \frac{(\alpha - 1)\bar{e}}{(1 + r)^s} \\
= \frac{\beta_{t-1}(1 + r)K + (\alpha - 1)\bar{e} - i_t}{(1 + r)^t} - \sum_{s=1}^{t-1} \frac{\bar{e}}{(1 + r)^s} + \sum_{s=t+1}^{\infty} \frac{(\alpha - 1)\bar{e}}{(1 + r)^s} \\
= \sum_{s=1}^{t-1} \frac{\alpha \bar{e}}{(1 + r)^s} + \frac{(\alpha - 1)\bar{e} - i_t}{(1 + r)^t} - \sum_{s=1}^{t-1} \frac{\bar{e}}{(1 + r)^s} + \sum_{s=t+1}^{\infty} \frac{(\alpha - 1)\bar{e}}{(1 + r)^s} \\
= \sum_{s=1}^{\infty} \frac{(\alpha - 1)\bar{e}}{(1 + r)^s} - \frac{i_t}{(1 + r)^t} \\
= \frac{(\alpha - 1)\bar{e}}{r} - \frac{i_t}{(1 + r)^t}.
\]

The change in expected utility is given by

\[
(196) \quad \frac{(\alpha - 1)\bar{e}}{r} - \frac{i_t}{(1 + r)^t} - \frac{(\alpha - 1)\bar{e}}{r} = -\frac{i_t}{(1 + r)^t} \leq 0.
\]

So the deviation does not pay.

At last, we prove (iii): projects with finite life can be financed with the sinking fund. The project considered here is assumed to have a finite life of
$T$ periods and an initial investment of $K(1 - \frac{1}{(1+r)^T})$. Here we only prove that the following strategies

(197) $i_t(h^t) = \begin{cases} i_t^* & \text{if } (\forall s < t) \ i_s = i_s^* \text{ and } \beta_s = \beta_s^* < 1 \\ 0 & \text{otherwise} \end{cases}$

(198) $c_t(h^t) = \begin{cases} 0 & \text{when } \beta_{t-1} < 1 \\ \leq W_t & \text{otherwise} \end{cases}$

(199) $\beta_t(h^t, e_t) = \begin{cases} \beta_t^* & \text{if } (\forall s < t) \ i_s = i_s^* \text{ and } \beta_s = \beta_s^* < 1 \\ \beta_{t-1} & \text{otherwise} \end{cases}$

(200) $l_t(h^t, e_t) = \begin{cases} 0 & \text{when } (\forall s \leq t) \ i_s \geq i_s^* \text{ and } \beta_s = \beta_s^* \\ 1 & \text{otherwise} \end{cases}$

are equilibrium strategies when

(201) $1 + \frac{r^2K}{(1 - (1+r)^{1-T})} \leq \alpha < \left(1 - r - \frac{1}{(1+r)^{T-1}}\right) \frac{K}{\bar{e}}$

and

(202) $\beta_{\text{takeover}-1} = \frac{r}{1 + r - (1+r)^{t-T}}$, 

where

$t_{\text{takeover}} = \inf\{t | \alpha \bar{e} \geq (1 - r - \frac{1}{(1+r)^{T-t}})K, \ t \geq 1\}$

$\beta_t^*(h^t, e_t) = \min\left\{\frac{\alpha \bar{e} ((1+r)^{T-t} - 1)}{(1 - (1+r)^{t-T}) rK}, 1\right\}$ when $t \leq T - 1$. 

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First, it is quite clear that we need to have $\alpha < (1 - r - \frac{1}{(1+r)^{T-t}})K\bar{e}$, otherwise the manager can default on the debt in the first period and then take over the project through debt-repurchase. Since one part of the proof is simple, we will omit it here and provide the other part by showing that the strategy in (197) to (199) is the optimal response to the strategy in (200) when (201) and (202) hold. Given the debtholder’s strategy in (200), the manager’s utility from playing the strategy in (197) to (199) is

$$\sum_{s=1}^{T} \frac{(\alpha - 1)e}{(1+r)^s} = \frac{(\alpha - 1)e}{r} \left(1 - \frac{1}{(1+r)^T}\right).$$

Now we consider a deviation from the strategy in (197) to (199). Suppose the manager chooses $i_t < rK$ and $\beta_t = \beta_{t-1}$ in period $t$ when $\beta_{t-1} < 1$ and $W_t < \min\{(1 - \frac{1}{(1+r)^{t+T-1}})K, (1 - \frac{1}{(1+r)^{t+T-1}})(1 - \beta_{t-1})K + (1 - \beta_{t-1})rK\}$. Then the manager’s utility from playing this new strategy is

$$rK + (\alpha - 1)e - i_t \frac{\bar{e}}{(1+r)^t} - \sum_{s=1}^{t-1} \frac{\bar{e}}{(1+r)^s} \frac{(1 - \frac{1}{(1+r)^{t+T-1}})K - (1 - \beta_{t-1})rK}{(1+r)^t}.$$

The change in expected utility is given by, when $K \geq (1 - \beta_{t-1})K + (1 - \beta_{t-1})rK$,

$$\frac{rK + (\alpha - 1)e - i_t}{(1+r)^t} + \frac{(1 - \frac{1}{(1+r)^{t+T-1}})\beta_{t-1}K - (1 - \beta_{t-1})rK}{(1+r)^t}$$

$$- \sum_{s=1}^{t-1} \frac{\bar{e}}{(1+r)^s} - \frac{(\alpha - 1)e}{r} \left(1 - \frac{1}{(1+r)^T}\right)$$

$$\beta_{t-1}(1 + r - (1 + r)^{t-T})K + (\alpha - 1)e - i_t$$

$$- \sum_{s=1}^{t-1} \frac{\bar{e}}{(1+r)^s} - \frac{(\alpha - 1)e}{r} \left(1 - \frac{1}{(1+r)^T}\right).$$
\[
\begin{align*}
&= \sum_{s=1}^{t} (\alpha - 1) e - \frac{i_t}{(1+r)^t} = \frac{(\alpha - 1) e}{r} \left( 1 - \frac{1}{(1+r)^t} \right) \\
&= \frac{(\alpha - 1) e}{r} \left( 1 - \frac{1}{(1+r)^t} \right) - \frac{i_t}{(1+r)^t} - \frac{(\alpha - 1) e}{r} \left( 1 - \frac{1}{(1+r)^T} \right) \\
&\leq 0,
\end{align*}
\]

The change in expected utility is given by, when \( K < (1 - \beta_{t-1}) K + \frac{(1 - \beta_{t-1}) rK}{1 + \gamma_t(1+r)^T} \).

\[
\begin{align*}
(206) \quad &\frac{rK + (\alpha - 1) e - i_t}{(1+r)^t} = \frac{t-1}{s=1} \frac{e}{(1+r)^s} - \frac{(\alpha - 1) e}{r} \left( 1 - \frac{1}{(1+r)^t} \right) \\
&= \frac{rK + (\alpha - 1) e - i_t}{(1+r)^t} - \frac{e}{r} \left( 1 - \frac{1}{(1+r)^t} \right) \\
&\leq \left[ \frac{rK}{(1+r)^t} - \frac{e}{r} \left( 1 - \frac{1}{(1+r)^t} \right) \right]_{t=1} \\
&\leq \left[ \frac{rK}{(1+r)^t} - \frac{e}{r} \left( 1 - \frac{1}{(1+r)^t} \right) \right]_{t=1} - \frac{i_t}{(1+r)^t} \\
&= \frac{rK}{(1+r)} - \frac{(\alpha - 1) e}{r} \left( 1 - \frac{1}{(1+r)^t} \right) - \frac{i_t}{(1+r)^t}.
\end{align*}
\]

In order for the right-hand-side of (206) to be non-positive, \( \alpha \) needs to satisfy

\[
(207) \quad \alpha \geq 1 + \left( \frac{r^2 K}{e} \right) \left( \frac{1}{(1+r)^t} - \frac{1}{(1+r)^T} \right) = 1 + \left( \frac{r^2 K}{e} \right) \left( 1 - \frac{1}{(1+r)^T} \right).
\]

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Now we consider a deviation from the strategy in (197) to (199) when 
\( \beta_{t-1} < 1\) and \( W_t \geq \min\{(1 - \frac{1}{(1+r)^{t-1}})K, (1 - \frac{1}{(1+r)^{t-1}})(1 - \beta_{t-1})K + (1 - \beta_{t-1})rK\} \). We show that debt financing of the project can be supported by sinking fund when \( \beta_{t-1} \geq \frac{r}{1+r-(1+r)^{t-1}} \) and cannot otherwise.

Suppose the manager chooses \( i_t < rK \) and \( \beta_t = \beta_{t-1} \) in period \( t \). Then the manager’s utility from playing this new strategy is, when

\[
K \geq (1 - \beta_{t-1})K + \frac{(1 - \beta_{t-1})rK}{1 - (1 + r)^{t-T}},
\]
i.e. when

\[
\beta_{t-1} \geq \frac{r}{1 + r - (1 + r)^{t-T}},
\]

\[
\frac{rK + \alpha \tau - \tau - i_t}{(1 + r)^t} - \sum_{s=1}^{t-1} \frac{\tau}{(1 + r)^s} + \sum_{s=t+1}^{T} \frac{rK + (\alpha - 1)\tau}{(1 + r)^s}
\]

\[
= \frac{rK + (\alpha - 1)\tau - i_t}{(1 + r)^t} - \sum_{s=1}^{t-1} \frac{\tau}{(1 + r)^s} + \sum_{s=t+1}^{T} \frac{\alpha - 1)\tau}{(1 + r)^s}
\]

\[
+ \frac{(1 - \frac{1}{(1+r)^{t-1}})K}{(1 + r)^t} - \frac{(1 - \frac{1}{(1+r)^{t-1}})(1 - \beta_{t-1})K + (1 - \beta_{t-1})rK}{(1 + r)^t}
\]

\[
= \beta_{t-1}(1 - \frac{1}{(1+r)^{t-1}})K + \beta_{t-1}(1 + rK - i_t)
\]

\[
+ \frac{T}{\sum_{s=1}^{T} (1 + r)^s} - \sum_{s=1}^{T} \frac{(\alpha - 1)\tau}{(1 + r)^s}
\]

\[
= \sum_{s=1}^{T} \frac{(\alpha - 1)\tau}{(1 + r)^s} - \frac{i_t}{(1 + r)^t}.
\]
The change in expected utility is given by

\[
(211) \quad \frac{(\alpha - 1)\bar{e}}{r} \left( 1 - \frac{1}{(1 + r)^T} \right) - \frac{i_t}{(1 + r)^t} - \frac{(\alpha - 1)\bar{e}}{r} \left( 1 - \frac{1}{(1 + r)^T} \right) = -\frac{i_t}{(1 + r)^t} \leq 0.
\]

So the deviation does not pay.

The manager’s utility from playing this new strategy is, when

\[
(212) \quad K < (1 - \beta_{t-1})K + \frac{(1 - \beta_{t-1})rK}{1 - (1 + r)^{t-T}}
\]

i.e. when

\[
(213) \quad \beta_{t-1} < \frac{r}{1 + r - (1 + r)^{t-T}}.
\]

\[
(214) \quad \frac{rK + \alpha\bar{e} - \bar{e} - i_t}{(1 + r)^t} - \sum_{s=1}^{t-1} \frac{\bar{e}}{(1 + r)^s} + \sum_{s=t+1}^{T} \frac{rK + (\alpha - 1)\bar{e}}{(1 + r)^s} - (1 - \beta_{t-1})K \leq \frac{rK + \alpha\bar{e} - \bar{e} - i_t}{(1 + r)^t} - \sum_{s=1}^{t-1} \frac{\bar{e}}{(1 + r)^s} + \sum_{s=t+1}^{T} \frac{rK + (\alpha - 1)\bar{e}}{(1 + r)^s} \frac{rK + (\alpha - 1)\bar{e}}{1 - (1 + r)^{t-T}}
\]
by (210). The change in expected utility is given by

\[
\begin{align*}
    &= \sum_{s=1}^{T} \frac{(\alpha - 1)\bar{c}}{(1+r)^s} - \frac{i_t}{(1+r)^t}.
\end{align*}
\]

when \( i_t = 0 \). So the manager will benefit from the deviation.
References


